Let $u = \tan x$. Then $\dfrac{du}{dx} = \sec^2 x$. Also, $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$, so

$y' = \dfrac{d}{dx}\int_1^{\tan x} \sqrt{3t + \sqrt{t}}\, dt = \dfrac{d}{du}\int_1^{u} \sqrt{3t + \sqrt{t}}\, dt \cdot \dfrac{du}{dx} = \sqrt{3u + \sqrt{u}}\, \dfrac{du}{dx}$

$= \sqrt{3\tan x + \sqrt{\tan x}}\sec^2 x.$