

$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{8n} \tan \frac{i\pi}{8n}$ can be interpreted as the area of the region lying under the graph of $y = \tan x$ on the interval $[0, \frac{\pi}{8}]$, since for $y = \tan x$ on $[0, \frac{\pi}{8}]$ with $\Delta x = \frac{\pi/8 - 0}{n} = \frac{\pi}{8n}$, $x_i = 0 + i \Delta x = \frac{i\pi}{8n}$, and $x_i^* = x_i$, the expression for the area is

$$A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \tan\left(\frac{i\pi}{8n}\right) \frac{\pi}{8n}.$$

Note that this answer is not unique, since the expression for the area is the same for the function $y = \tan(x - k\pi)$ on the interval $[k\pi, k\pi + \frac{\pi}{8}]$, where k is any integer.