$$f(t) = \sqrt[4]{1+\tan t} = (1+\tan t)^{1/4} \quad \Rightarrow \quad f'(t) = \tfrac{1}{4}(1+\tan t)^{-3/4}\sec^2 t$$
$$= \frac{\sec^2 t}{4\sqrt[4]{(1+\tan t)^3}}$$