Notice that

$\cos 4x = \sin 8x = 2\sin 4x \cos 4x \quad \Leftrightarrow$

$2\sin 4x \cos 4x - \cos 4x = 0 \quad \Leftrightarrow$

$\cos 4x\,(2\sin 4x - 1) = 0 \quad \Leftrightarrow$

$2\sin 4x = 1$ or $\cos 4x = 0 \quad \Leftrightarrow$

$x = \frac{\pi}{24}$ or $\frac{\pi}{8}$.

$$\begin{aligned} A &= \int_0^{\pi/24} (2\cos 4x - 2\sin 8x)\,dx + \int_{\pi/24}^{\pi/8} (2\sin 8x - 2\cos 4x)\,dx \\ &= 2\left[\tfrac{1}{4}\sin 4x + \tfrac{1}{8}\cos 8x\right]_0^{\pi/24} + 2\left[-\tfrac{1}{8}\cos 8x - \tfrac{1}{4}\sin 4x\right]_{\pi/24}^{\pi/8} \\ &= 2\left(\tfrac{1}{4}\cdot\tfrac{1}{2} + \tfrac{1}{8}\cdot\tfrac{1}{2}\right) - 2\left(0 + \tfrac{1}{8}\cdot 1\right) + 2\left(\tfrac{1}{8} - \tfrac{1}{4}\right) - 2\left(-\tfrac{1}{8}\cdot\tfrac{1}{2} - \tfrac{1}{4}\cdot\tfrac{1}{2}\right) = \tfrac{1}{4} \end{aligned}$$