

$h(-x) = f(g(-x)) = f(-g(x))$ . At this point, we can't simplify the expression, so we might try to find a counterexample to show that  $h$  is not an odd function. Let  $g(x) = x$ , an odd function, and  $f(x) = x^2 + x$ . Then  $h(x) = x^2 + x$ , which is neither even nor odd.

Now suppose  $f$  is an odd function. Then  $f(-g(x)) = -f(g(x)) = -h(x)$ . Hence,  $h(-x) = -h(x)$ , and so  $h$  is odd if both  $f$  and  $g$  are odd.

Now suppose  $f$  is an even function. Then  $f(-g(x)) = f(g(x)) = h(x)$ . Hence,  $h(-x) = h(x)$ , and so  $h$  is even if  $g$  is odd and  $f$  is even.