$$
\begin{aligned}
f(\theta) &= \frac{\sec\theta}{2+\sec\theta} \quad \Rightarrow \\
f'(\theta) &= \frac{(2+\sec\theta)(\sec\theta\tan\theta) - (\sec\theta)(\sec\theta\tan\theta)}{(2+\sec\theta)^2} \\
&= \frac{(\sec\theta\tan\theta)\left[(2+\sec\theta) - \sec\theta\right]}{(2+\sec\theta)^2} = \frac{2\sec\theta\tan\theta}{(2+\sec\theta)^2}
\end{aligned}
$$