$$y = \frac{\cos x}{2+\sin x} \quad \Rightarrow \quad y' = \frac{(2+\sin x)(-\sin x) - \cos x\ \cos x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2} = \frac{-2\sin x - 1}{(2+\sin x)^2} = 0$$

when $-2\sin x - 1 = 0 \quad \Leftrightarrow \quad \sin x = -\frac{1}{2} \quad \Leftrightarrow \quad x = \frac{11\pi}{6} + 2\pi n$ or $x = \frac{7\pi}{6} + 2\pi n$, n an integer. So $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$ and the points on the curve with horizontal tangents are: $\left(\frac{11\pi}{6} + 2\pi n, \frac{1}{\sqrt{3}}\right)$, $\left(\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}}\right)$, n an integer.