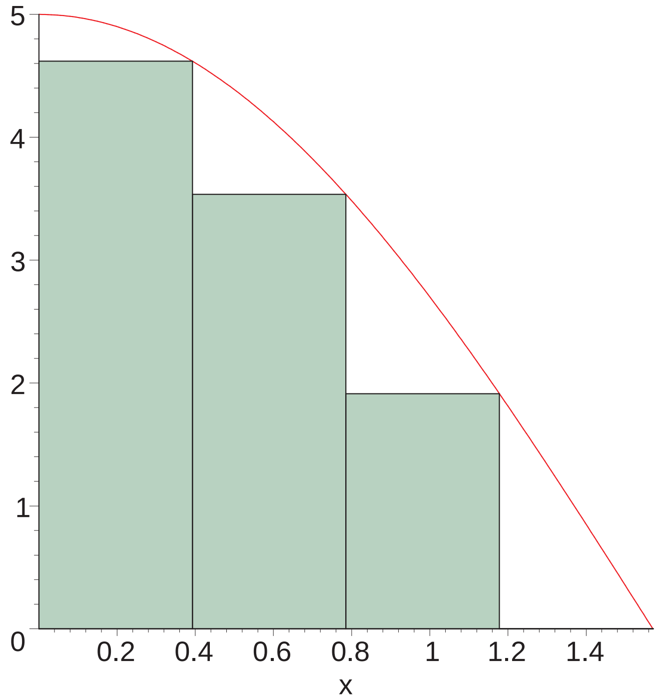
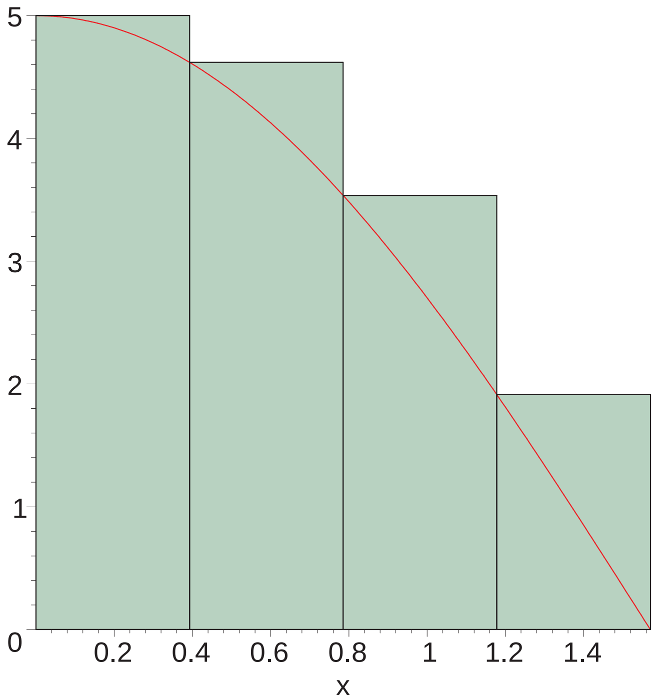
(a) $$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x \quad \left[\ \Delta x = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}\ \right] \quad = \left[\sum_{i=1}^{4} f(x_i)\right]\Delta x$$
$$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\Delta x$$
$$= 5\left[\cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{4\pi}{8}\right]\frac{\pi}{8}$$
$$\approx 5(0.9239 + 0.7071 + 0.3827 + 0)\frac{\pi}{8} \approx 3.953831301$$

Since f is *decreasing* on $[0, \pi/2]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4.



(b) $$L_4 = \sum_{i=1}^{4} f(x_{i-1})\Delta x = \left[\sum_{i=1}^{4} f\left(x_{i-1}\right)\right]\Delta x$$
$$= [f(x_0) + f(x_1) + f(x_2) + f(x_3)]\,\Delta x$$
$$= 5\left[\cos 0 + \cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8}\right]\frac{\pi}{8}$$
$$\approx 5(1 + 0.9239 + 0.7071 + 0.3827)\frac{\pi}{8} \approx 5.91732671$$

L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(0) \cdot \frac{\pi}{8} - f\left(\frac{\pi}{2}\right) \cdot \frac{\pi}{8}$.



5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x