Let $u = 6 + \tan t$. Then $du = \sec^2 t\, dt$, so $\displaystyle\int \frac{dt}{\cos^2 t \sqrt[3]{6+\tan t}}$

$= \displaystyle\int \frac{\sec^2 t\, dt}{\sqrt[3]{6+\tan t}} = \int \frac{du}{\sqrt[3]{u}} = \int u^{-1/3}\, du = \frac{u^{2/3}}{2/3} + C = \frac{3}{2}\sqrt[3]{(6+\tan t)^2} + C.$