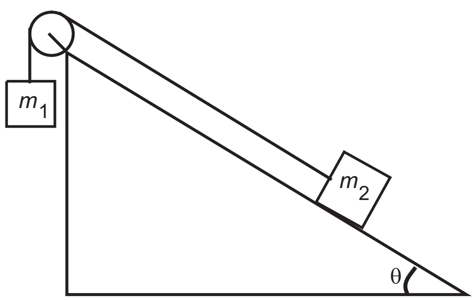
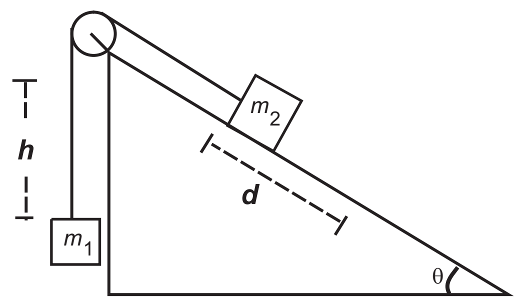
Solving the Inclined Plane Problem Using Newton's Second Law of Motion

The set up in the diagram below is the classic inclined plane problem. You may have solved this problem using the Conservation of Energy Principle. Here, we will solve it using Newton's Second Law of Motion.

The two masses are connected by an inelastic string that does not stretch. Therefore, the tension will be the same at all points along the string and the two masses will move with the same acceleration. The distance d that m_2 moves along the inclined plane will be the same as the vertical distance h through which mass m_1 falls. However, the vertical distance through which m_2 rises will be $d\sin\theta$.



Initial position of the two masses



Final position of the two masses

Figure 1: Two-mass system on a ramp

Outlined below are the steps involved in solving this problem using Newton's Second Law of Motion, which is $\sum_i F_i = ma$.

1. Draw the free body diagram for mass m_1, label the forces, and indicate the direction of the acceleration vector (see Fig. 2 below).

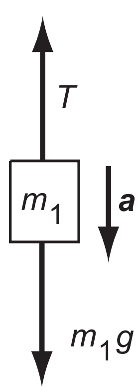


Figure 2: Free-body diagram for m_1

2. Write Newton's Second Law for the vertical motionmass m_1 in terms of m_1, tension T in the string, and acceleration a of the mass.

$$T - m_1 g = m_1 a \tag{1}$$

3. Draw the free body diagram for mass m_2, label the forces, and indicate the direction of the acceleration vector.

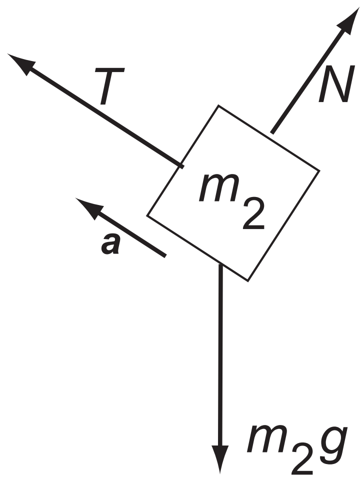


Figure 3: Free-body diagram for m_2

4. Since the motion of m_2 is parallel to the inclined plane, it is useful to consider components parallel and perpendicular to the inclined plane for $m_2 g$ (see Fig. 4 below).

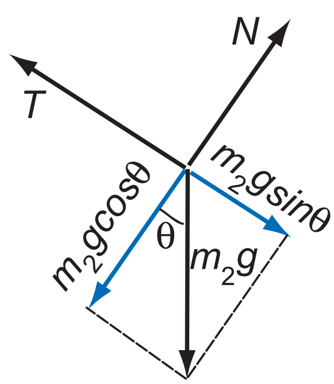


Figure 4: Components of $m_2 g$ parallel and perpendicular to the inclined plane

5. Write Newton's Second Law of motion for m_2 for motion parallel and perpendicular to the plane.

$$T - m_2 g \sin\theta = m_2 a \tag{2}$$

$$N - m_2 g \cos\theta = 0 \tag{3}$$

6. Solve for acceleration a using Eq. (1) and Eq. (2).

a. Subtract Eq. (1) from Eq. (2) to eliminate T.

$$m_2 g \mathtt{sin}\theta - m_1 g = (m_2 - m_1)a \tag{4}$$

b. Divide both sides of Eq. (4) by $(m_2 - m_1)$ to obtain a.

$$a = \frac{m_2 g \mathtt{sin}\theta - m_1 g}{m_2 - m_1} \tag{5}$$

7. Use the kinematic equation $v_{\mathtt{f}}^2 = v_0^2 + 2ay$ to find the velocity of m_1.

Note: The initial velocity of m_1 is zero and it moves through a vertical distance h in the time that m_2 moves a distance d along the plane. Both masses move with the same acceleration a.

$$\begin{aligned} v_f &= \sqrt{2ah} \\ &= \sqrt{2h \frac{m_2 g \mathtt{sin}\theta - m_1 g}{m_2 - m_1}} \end{aligned} \tag{5}$$