

Answer to Essential Question 6.2: At $t = 15$ s. The graph in Figure 6.4 is helpful in determining when Suki reaches her minimum speed. As long as the net force is negative, Suki slows down (unless her velocity becomes negative, which never happens in this case). Suki continues to slow down until $t = 15$ s. After that time, the net force is positive, so Suki speeds up after $t = 15$ s.

6-3 Implication of Newton's Third Law: Momentum is Conserved

EXPLORATION 6.3A – Two carts collide

Let's do an experiment in which two carts, cart 1 and cart 2, collide with one another on a horizontal track, as shown in Figure 6.6. How does the momentum of each cart change? What happens to the momentum of the two-cart system? The upward normal force applied by the track on each cart is balanced by the downward force of gravity, so the net force experienced by each cart during the collision is that applied by the other cart.

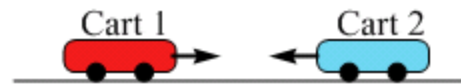


Figure 6.6: Two carts colliding.

Let's use the subscripts i for the initial situation (before the collision), and f for the final situation (after the collision).

The collision changes the momentum of cart 1 from \vec{p}_{1i} to $\vec{p}_{1f} = \vec{p}_{1i} + \Delta\vec{p}_1$.

Similarly, the collision changes the momentum of cart 2 from \vec{p}_{2i} to $\vec{p}_{2f} = \vec{p}_{2i} + \Delta\vec{p}_2$.

The total momentum of the system beforehand is $\vec{p}_{1i} + \vec{p}_{2i}$.

The total momentum of the system afterwards is $\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \Delta\vec{p}_1 + \vec{p}_{2i} + \Delta\vec{p}_2$.

Consider $\Delta\vec{p}_1$, the change in momentum experienced by cart 1 in the collision. This change in momentum comes from the force applied to cart 1 by cart 2 during the collision. Similarly, $\Delta\vec{p}_2$, cart 2's change in momentum, comes from the force applied to cart 2 by cart 1 during the collision. Newton's third law tells us that, no matter what, the force applied to cart 1 by cart 2 is equal and opposite to that applied to cart 2 by cart 1. Keeping in mind that the change in momentum is directly proportional to the net force, and that we're talking about vectors, this means:

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1.$$

Substituting this result into our expression for the total momentum of the system after the collision shows that momentum is conserved (momentum remains constant):

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}.$$

Key idea: The total momentum of the system after the collision equals the total momentum of the system before the collision. This **Law of Conservation of Momentum** applies to any system where there is no net external force. **Related End-of-Chapter Exercise: 4.**

We'll spend more time on the law of conservation of momentum in Chapter 7 but, for now, consider the following exploration.

EXPLORATION 6.3B – An explosive situation

Two carts are placed back-to-back on a horizontal track. One cart contains a spring-loaded piston. When the spring is released, the piston pushes against the other cart and the two carts move in opposite directions along the track, as shown in Figure 6.7. Assume the carts are initially at rest in the center of the track and that friction is negligible.

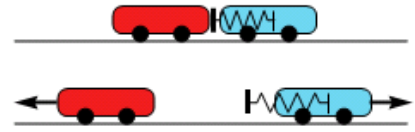


Figure 6.7: A diagram showing the initial situation (top), and the situation after the carts have moved apart (bottom).

Step 1 - If the two carts have equal masses, is momentum conserved in this process? A good answer to this question is “it depends.” The momentum of each cart individually is not conserved, because each cart starts with no momentum and ends up with a non-zero momentum. This is because each cart experiences a net force (applied by the other cart), so its momentum changes according to the impulse equation (Equation 6.3).

On the other hand, the law of conservation of momentum tells us that *the momentum of the two-cart system is conserved because no net external force acts on this system*. The upward normal force, exerted by the track on this system, balances the downward force of gravity. Cart 1 acquires some momentum because of the force applied by cart 2, but cart 2 acquires an equal-and-opposite momentum because of the equal-and-opposite force applied to it by cart 1. The net momentum of the two-cart system is zero, even when the carts are in motion. Momentum is a vector, so the momentum of one cart is cancelled by the momentum of the other cart.

Step 2 - If we double the mass of one of the carts and repeat the experiment, is momentum conserved? Yes, the momentum of this system is conserved because no net external force acts on the system. Changing the mass of one cart will change the magnitude of the momentum acquired by each cart, but the momentum of the two-cart system is always zero, both before and after the spring is released. To conserve momentum, the force applied on cart 1 by cart 2 must be equal and opposite to the force applied on cart 2 by cart 1. Newton’s third law tells us that these forces are equal and opposite, no matter how the masses compare.

Step 3 - If we make the experiment more interesting, by balancing the track on a brick before releasing the spring, will the track tip over after the spring is released? If we tried this experiment when the masses are equal, what would happen? The track would remain balanced, even when the carts are in motion, because of the symmetry. The tendency of cart 1 to tip the track one way is balanced by the tendency of cart 2 to tip it the opposite way. We don’t have the same symmetry in step 2, but the track still remains balanced. The cart with half the mass of the other cart is always twice as far from the balance point. That maintains the balance, as shown in Figure 6.8.

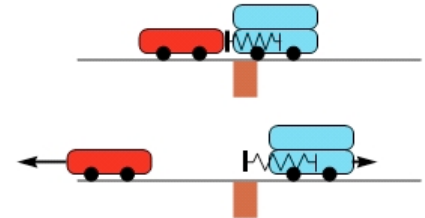


Figure 6.8: As the carts move apart, the track remains balanced on the brick even if the carts have different masses.

Key idea for momentum conservation: Even if the momenta of individual parts of a system are not conserved, the momentum of the entire system is conserved (constant), as long as no net external force acts on the system. Conservation of momentum is a consequence of Newton’s third law. **Related End-of-Chapter Exercises: 44, 45.**

Essential Question 6.3: In Exploration 6.3B, the momentum of the system is always zero. Is there anything about the two-cart system that remains at rest and that shows clearly why the track doesn’t tip over when balanced on the brick?