Answer to Essential Question 9.8: Eliminating the speed terms means that we can write Bernoulli's equation as: $\rho g y_1 + P_1 = \rho g y_2 + P_2$. Re-arranging this equation to solve for the pressure at point 2 gives: $P_2 = P_1 + \rho g y_1 - \rho g y_2 = P_1 + \rho g (y_1 - y_2)$. This equation is equivalent to Equation 9.7, the equation for pressure in a static fluid.

9-9 Examples Involving Bernoulli's Equation

EXPLORATION 9.9 – Pressure inside a pipe
Step 1 - ***Make a prediction. In the pipe shown in Figure 9.24, is the pressure higher at point 2, where the fluid flows fastest, or at point 1? The fluid in the pipe flows from left to right.***

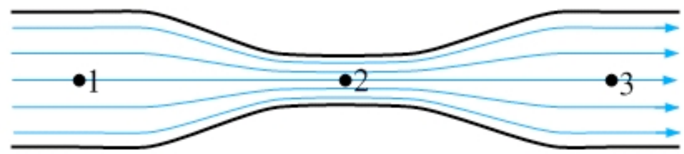

Figure 9.24: Fluid flowing through a pipe from left to right.

Many people predict that the pressure is higher at point 2, where the fluid is moving faster.

Step 2 - ***Apply the continuity equation, and Bernoulli's equation, to rank points 1, 2, and 3 according to pressure, from largest to smallest.*** Let's see if the common prediction, that the pressure is highest at point 2, is correct. First, apply the continuity equation: $A_1 v_1 = A_2 v_2 = A_3 v_3$. Looking at the tube, we know that $A_1 = A_3 > A_2$, which tells us that $v_2 > v_1 = v_3$.

Now, let's apply Bernoulli's Equation. Comparing points 1 and 2, we start with:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 .$$

The vertical positions of these two points are equal so the $\rho g y$ terms cancel out:

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2 .$$

Let's re-write this as: $$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 .$$

The continuity equation told us that $v_2 > v_1$, so the right-hand side of the above equation is positive. This means the left-hand side must also be positive, implying that $P_1 > P_2$. Thus, the pressure at point 2, where the fluid speed is highest, is less than the pressure at point 1. For points at the same height, higher speed corresponds to lower pressure. We can make sense of this by considering a parcel of fluid that moves from point 1 to point 2. Because this parcel of fluid speeds up as it travels from point 1 to point 2, there must be a net force acting on it that is directed right. This force must come from a difference in pressure between points 1 and 2. For the force to be directed right, the pressure must be larger on the left, at point 1.

We can also use Bernoulli's equation to show that the pressure at point 3 is equal to that at point 1. Thus we can conclude that $P_1 = P_3 > P_2$.

Key idea for an enclosed fluid: In general, in an enclosed fluid the pressure decreases as the speed of the fluid flow increases. **Related End-of-Chapter Exercises: 52, 53.**

EXAMPLE 9.9 – How fast?

A Styrofoam cylinder, filled with water, sits on a table. You then poke a small hole through the side of the cylinder, 20 cm below the top of the water surface. What is the speed of the fluid emerging from the hole?

SOLUTION

As usual, begin by drawing a diagram of the situation, as shown in Figure 9.25.

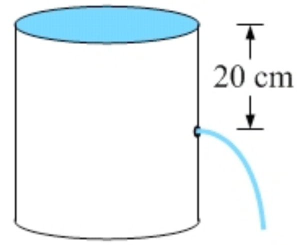

Figure 9.25: The Styrofoam container of water, with a small hole 20 cm from the top.

We're going to apply Bernoulli's equation, which means identifying two points that we can relate via the equation. Point 2 is outside the container where the hole is, because that is the place where we're trying to find the speed. Point 1 needs to be somewhere inside the container. Any point inside will do, although the most sensible places are either at the top of the container, where we know the pressure, or inside the container at the level of the hole. Let's choose a point at the very top, and apply Bernoulli's equation:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 + P_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2 + P_2 .$$

First, we should recognize that, because both of our points are exposed to the atmosphere, we have $P_1 = P_2 = P_{atm}$. The pressure terms cancel in the equation, leaving:

$$\rho g y_1 + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2 .$$

We can cancel factors of density. We are also free to define a zero for our y positions to be anywhere we find convenient. If we say $y = 0$ at the level of the hole we get $y_1 = +20 \text{ cm}$ and $y_2 = 0$, so the equation reduces to:

$$g y_1 + \frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 .$$

If we knew the fluid speed at point 1 we could solve for the speed at point 2. This is a good time to bring in the continuity equation, which relates the speeds at the two points: $A_1 v_1 = A_2 v_2$. In this case we can say that, because A_1, the cross-sectional area of the cylinder, is so much larger than A_2, the cross-sectional area of the hole, then v_1 is much smaller than v_2. Thus, the $(1/2)v_1^2$ term is negligible compared to the $(1/2)v_2^2$ term. Our equation thus reduces to:

$$g y_1 = \frac{1}{2} v_2^2 .$$

Solving for the speed at which the fluid emerges from the hole gives:

$$v_2 = \sqrt{2 g y_1} = \sqrt{2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 0.20 \text{ m}} = 2.0 \text{ m/s} .$$

Related End-of-Chapter Exercises: 27, 28.

Essential Question 9.9: If you dropped an object from rest, what would its speed be after it had fallen through a distance of 20 cm? How does this compare to the result of the Example 9.9, where we found the speed of water emerging from a hole 20 cm below the water surface?