

Answer to Essential Question 10.10: We solved for the force of tension in Exploration 10.10, so we can go on solve for the components of the hinge force. To find the y -component of the hinge force we could set up another torque equation, relative to an axis through the middle of the rod, for instance, or we could now make use of the force equation we worked out in Step 2. To find the x -component of the hinge force we can only use the force equation because, no matter which axis we choose, the torque from the x -component of the hinge force always exactly balances the torque from the x -component of the force of tension.

Making use of our x -component force equation, $+F_{Hx} - F_T \cos(30^\circ) = 0$, we find that:

$$F_{Hx} = F_T \cos(30^\circ) = (20 \text{ N}) \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ N}.$$

Using the y -component force equation, $+F_{Hy} - mg + F_T \sin(30^\circ) = 0$, we get:

$$+F_{Hy} = mg - F_T \sin(30^\circ) = 20 \text{ N} - 20 \text{ N} \left(\frac{1}{2} \right) = 10 \text{ N}.$$

Note that, if we combine the two components of the hinge force, we find that the hinge force is 20 N, with an angle of 30° between the hinge force and the rod. In other words, the hinge force is a mirror image of the force of tension, because of the symmetry of the situation (both forces are applied at the ends of the rod, while the force of gravity is applied at the exact center).

10-11 A General Method for Solving Static Equilibrium Problems

Now that we have explored the idea of applying the concept of torque to solve a static equilibrium problem, let's list the basic steps in the process.

A General Method for Solving a Static Equilibrium Problem

Objects in static equilibrium remain at rest, so both the acceleration and the angular acceleration are zero. This allows us to use special-case of Newton's second law and Newton's second law for rotation.

1. Draw a diagram of the situation.
2. Draw a free-body diagram showing all the forces acting on the object.
3. Choose a rotational coordinate system. Pick an appropriate axis to take torques about, and then apply Newton's second law for rotation ($\sum \tau = 0$) to obtain one or more torque equations.
4. If necessary, choose an appropriate x - y coordinate system for forces. Apply Newton's second law ($\sum \vec{F} = 0$) to obtain one or more force equations.
5. Combine the resulting equations to solve the problem.

Let's apply the method in the following example.

EXAMPLE 10.11 – Supporting the board

A uniform board with a weight of 240 N and a length of 2.0 m rests horizontally on two supports. Support A is under the left end of the board, while Support B is 50 cm from the right end (150 cm from the left end, in other words).

(a) Which support exerts more force on the board? Without doing the calculations to find the two support forces, come up with a conceptual argument to justify your answer.

(b) Find the two support forces.

SOLUTION

As usual, let's begin by drawing a diagram of the situation. We should also sketch a free-body diagram to show all the forces acting on the board. The diagram and free-body diagram are shown in Figure 10.27.

(a) Support B exerts a larger force on the board than support A. One way to see this is to sum torques about an axis through the center of the board, and perpendicular to the page. Taking counterclockwise to be the positive direction for torque, applying Newton's second law for rotation gives:

$$-(1.0 \text{ m})F_A + (0.5 \text{ m})F_B = 0.$$

The force of gravity does not appear in this torque equation because the force of gravity passes through the axis, and thus does not give rise to a torque about that axis. Because the torques from the two support forces must balance one another, and the distance from support B to the axis is half that of the distance from support A to the axis, the force exerted on the board by support B must be twice as large as that exerted by support A.

(b) To solve for the support forces, we could combine the torque equation above with the force equation we get by applying Newton's second law, or we could set up another torque equation by taking an axis perpendicular to the page through one of the supports. Let's do the latter, using Figure 10.28 to help us set up the new torque equation, summing torques about an axis through support A.

Applying Newton's second law for rotation, $\Sigma \tau = I\alpha = 0$, taking counterclockwise to be positive, gives:

$$-(1.0 \text{ m})mg + (1.5 \text{ m})F_B = 0.$$

$$\text{Thus, } F_B = \frac{2}{3}mg = \frac{2}{3}(240 \text{ N}) = +160 \text{ N}.$$

There are several ways to solve for the force applied by support A. Let's apply Newton's second law, $\Sigma \vec{F} = m\vec{a} = 0$, taking up to be the positive direction:

$$+F_A - mg + F_B = 0.$$

$$\text{This gives: } +F_A = mg - F_B = mg - \frac{2}{3}mg = \frac{1}{3}mg = \frac{1}{3}(240 \text{ N}) = +80 \text{ N}.$$

The fact that both support forces work out to be positive means they are in the direction shown in the diagrams, up.

Related End-of-Chapter Exercises: 33, 36.

Essential Question 10.11: In Example 10.11, would the support forces change if support B was moved a short distance to the right of its original position? If so, how would the forces change?

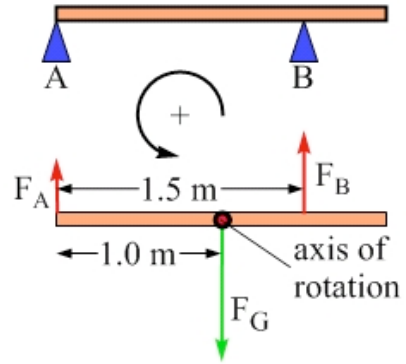


Figure 10.27: A diagram and free-body diagram for the board on two supports.

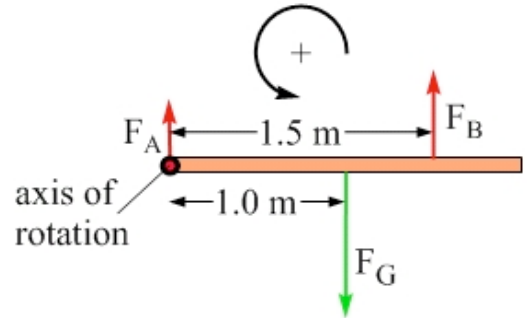


Figure 10.28: If we take torques about an axis through support A, the force applied to the board from support A does not give rise to a torque, because that force passes through the axis.