**Answer to Essential Question 10.6:** The correct ranking is 3>1>2. In the rotational inertia equation, the distance from the axis to the ball (the length of the rod) is squared, while the mass is not. Thus, changing the length by a factor of 2 changes the rotational inertia by a factor of 4, whereas changing the mass by a factor of 2 changes the rotational inertia by only a factor of 2.

### 10-7 An Example Problem Involving Rotational Inertia

Our measure of inertia for rotational motion is somewhat more complicated than inertia for straight-line motion, which is just mass. Consider the following example.

**EXAMPLE 10.7 – Spinning the system.**

Three balls are connected by light rods. The mass and location of each ball are:
- Ball 1 has a mass \(M\) and is located at \(x = 0, y = 0\).
- Ball 2 has a mass of \(2M\) and is located at \(x = +3.0\) m, \(y = +3.0\) m.
- Ball 3 has a mass of \(3M\) and is located at \(x = +2.0\) m, \(y = -2.0\) m.
Assume the radius of each ball is much smaller than 1 meter.

(a) Find the location of the system’s center-of-mass.
(b) Find the system’s rotational inertia about an axis perpendicular to the page that passes through the system’s center-of-mass.
(c) Find the system’s rotational inertia about an axis parallel to, and 2.0 m from, the axis through the center-of-mass.

**SOLUTION**

Let’s begin, as usual, by drawing a diagram of the situation. The diagram is shown in Figure 10.19.

(a) To find the location of the system’s center-of-mass, let’s apply Equation 6.3. To find the x-coordinate of the system’s center-of-mass:

\[
X_{CM} = \frac{x_1m_1 + x_2m_2 + x_3m_3}{m_1 + m_2 + m_3} = \frac{(0)M + (+3.0\)m)(2M) + (+2.0\)m)(3M)}{M + 2M + 3M} = \frac{(12.0\)m)M}{6M} = +2.0\)m
\]

The y-coordinate of the system’s center-of-mass is given by:

\[
Y_{CM} = \frac{y_1m_1 + y_2m_2 + y_3m_3}{m_1 + m_2 + m_3} = \frac{(0)M + (+3.0\)m)(2M) + (-2.0\)m)(3M)}{M + 2M + 3M} = \frac{(0)M}{6M} = 0.
\]

(b) To find the system’s rotational inertia about an axis through the center-of-mass we can find the rotational inertia for each ball separately, using \(I = ML^2\), and then simply add them to find the total rotational inertia. Figure 10.20 is helpful for seeing where the different \(L\) values come from.

**Figure 10.19:** A diagram showing the location of the balls in the system described in Example 10.7.

**Figure 10.20:** The center-of-mass of the system is marked at \((+2\) m, 0\)). The axis of rotation passes through that point. The dark lines show how far each ball is from the axis of rotation.
For ball 1, \( L^2 = (2.0\, \text{m})^2 = 4.0\, \text{m}^2 \) so \( I_1 = M \, L^2 = (4.0\, \text{m}^2)M \).

For ball 2, \( L^2 = 10\, \text{m}^2 \) so \( I_2 = 2M \, L^2 = (20\, \text{m}^2)M \).

For ball 3, \( L^2 = 4.0\, \text{m}^2 \) so \( I_3 = 3M \, L^2 = (12\, \text{m}^2)M \).

The total rotational inertia is the sum of these three values, \((36\, \text{m}^2)M\).

(c) To find the rotational inertia through an axis parallel to the first axis and 2.0 m away from it, let’s choose a point for this second axis to pass through. A convenient point is the origin, \(x = 0, y = 0\). Figure 10.21 shows where the \( \ell \) values come from in this case.

Repeating the process we followed in part (b) gives:

For ball 1, \( L^2 = 0 \) so \( I'_1 = 0 \).

For ball 2, \( L^2 = 18\, \text{m}^2 \) so \( I'_2 = 2M \, L^2 = (36\, \text{m}^2)M \).

For ball 3, \( L^2 = 8.0\, \text{m}^2 \) so \( I'_3 = 3M \, L^2 = (24\, \text{m}^2)M \).

The total rotational inertia is the sum of these three values, \((60\, \text{m}^2)M\).


Does it matter which point the second axis passes through? What if we had used a different point, such as \(x = 2.0\, \text{m}, y = -2.0\, \text{m}\), or any other point 2.0 m from the center-of-mass? Amazingly, it turns out that it doesn’t matter. Any axis parallel to the axis through the center-of-mass and 2.0 m from it gives a rotational inertia of \((60\, \text{m}^2)M\). It turns out that the rotational inertia of a system is minimized when the axis goes through the center-of-mass, and the rotational inertia of the system about any parallel axis a distance \(h\) from the axis through the center-of-mass can be found from

\[ I = I_{CM} + mh^2, \quad \text{(Equation 10.11: The parallel-axis theorem)} \]

where \(m\) is the total mass of the system.

Let’s check the parallel-axis theorem using our results from (b) and (c). In part (b) we found that the rotational inertia about the axis through the center-of-mass is \(I_{CM} = (36\, \text{m}^2)M\). The mass of the system is \(m = 6M\) and the second axis is \(h = 2.0\, \text{m}\) from the axis through the center-of-mass. This gives \(I = (36\, \text{m}^2)M + 6M(2.0\, \text{m})^2 = (60\, \text{m}^2)M\), as we found above.

**Essential Question 10.7:** To find the total mass of a system of objects, we simply add up the masses of the individual objects. To find the total rotational inertia of a system of objects, can we follow a similar process, adding up the rotational inertias of the individual objects.