***Answer to Essential Question 15.1***: Coming to equilibrium means reaching the same temperature as the water bath. Thus, both cylinders will experience the same change in temperature, +60°C. Internal energy is directly related to temperature. Because the same number of moles of the same type of gas experiences the same temperature change, the change in internal energy is the same in both cases. The difference between the cylinders is that the gas in cylinder 1 will expand when the temperature increases, so the gas does work moving the piston. No work is done by the gas in cylinder 2. Thus $Q_1 = \Delta E_{\text{int}} + W$ is larger than $Q_2 = \Delta E_{\text{int}}$. More heat needs to be added to cylinder 1 than cylinder 2, the difference corresponding to the work done by the gas in cylinder 1.

## 15-2 Work, and Internal Energy

The first law involves three parameters. Heat ($Q$) involves a transfer of energy into or out of a system. Let's now explore the ideas of work ($W$) and change in internal energy ($\Delta E_{\text{int}}$).

For the case of the cylinder in section 15-1, the work done by the gas on the piston is the magnitude of the force the gas exerts on the piston multiplied by the magnitude of the piston's displacement, $\Delta h$. Using the fact that $F = PA$, and that the volume and height are related by area:

$$W = F\Delta h = (PA)\Delta h = P(A\Delta h) = P\Delta V .$$

Thus, for a constant pressure process we have:
$W = P\Delta V$. (Eq. 15.2: **Work done at constant pressure**)

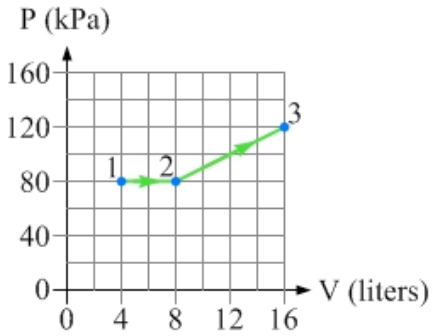


**Figure 15.6**: The P-V diagram shows an expansion from state 1 to state 2 at constant pressure, followed by another expansion that takes the system to state 3 along the path indicated.

If the pressure changes, Equation 15.2 does not apply. Is there a general method of finding work that is valid in all cases? Consider now the two processes shown in the P-V diagram in Figure 15.6.

For the constant pressure process, in which the system expands from state 1 to state 2, Equation 15.2 tells us that the work done by the gas in that process is $P\Delta V$. This is the area under the curve defining the process. For the expansion from state 2 to state 3, which is not at constant pressure, the work is still equal to the area under the curve defining the process. This gives us our general method of finding work. The two shaded areas shown in Figure 15.7 represent the work done by the gas in the two processes.

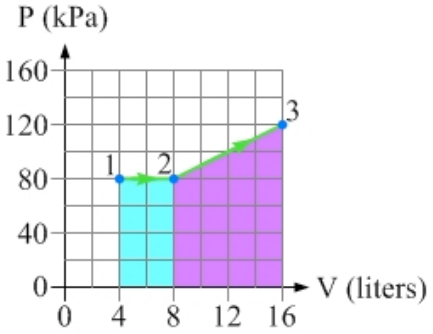


**Figure 15.7**: The area of the rectangular region under the 1to 2 process is the work done by the gas in the expansion from state 1 to state 2. The area of the region under the 2 to 3 process is the work done by the gas in moving from state 2 to state 3.

**Work:** A practical way to calculate the work done by a gas in a particular thermodynamic process is to find the area under the curve for that process on the P-V diagram.

$W$ = area under the curve on the P-V diagram.

(Equation 15.3: **Work done by a gas**)

**EXAMPLE 15.2A – Calculating the work**

Find the work done by the gas in the two processes shown in Figure 15.6.

**SOLUTION**

For the constant-pressure 1 to 2 process, the work is the area of a rectangle, as in Figure 15.7.

$$W_{1\to 2} = P\Delta V = (80\text{ kPa})(8.0\text{ L} - 4.0\text{ L}) = (80\times 10^3\text{ Pa})(4.0\times 10^{-3}\text{ m}^3) = +320\text{ J}.$$

For the expansion from state 2 to 3, the work is the area under the 2 to 3 line in Figure 15.7. This is equal to the area of a rectangle, with the top of the rectangle at the average pressure.

$$W_{2\to 3} = P_{av}\Delta V = (100\text{ kPa})(16\text{ L} - 8.0\text{ L}) = (100\times 10^3\text{ Pa})(8.0\times 10^{-3}\text{ m}^3) = +800\text{ J}.$$

Let's turn now to the change in internal energy. The change in internal energy is independent of the process that moves a system from one state to another. Thus, if we know what the change in internal energy is for one process, we can apply that to all processes.

**Change in internal energy:** If the temperature of an ideal gas changes, the change in internal energy of the gas is proportional to the change in temperature. If there is no change in temperature, there is no change in internal energy (as long as the number of moles of gas remains constant).

$$\Delta E_{\text{int}} = n C_V \Delta T \ .$$ (Equation 15.4: **Change in internal energy**)

Monatomic: $C_V = \frac{3}{2}R$ Diatomic: $C_V = \frac{5}{2}R$ Polyatomic: $C_V = 3R$

$C_V$ is the heat capacity at constant volume, which we will examine in Section 15-3.

If we just want the internal energy, we remove the deltas.

$$E_{\text{int}} = n C_V T \ .$$ (Equation 15.5: **Internal energy of an ideal gas**)

**EXAMPLE 15.2B – Calculating the change in internal energy**

Consider again the P-V diagram shown in Figure 15.6. If the gas is diatomic, find the change in internal energy associated with the two processes shown on the diagram.

**SOLUTION**

To do this, we will combine the ideal gas law with the information on the graph.

$$\Delta E_{\text{int}} = \frac{5}{2}nR\Delta T = \frac{5}{2}nR(T_2 - T_1) = \frac{5}{2}(nRT_2 - nRT_1) = \frac{5}{2}(P_2V_2 - P_1V_1).$$

Plugging in the values for the pressures and volumes shown on the P-V diagram gives:

$$\Delta E_{\text{int},1\to 2} = \frac{5}{2}\left[(80\text{ kPa})(8.0\text{ L}) - (80\text{ kPa})(4.0\text{ L})\right] = 1600\text{ J} - 800\text{ J} = +800\text{ J}.$$

Using a similar process for the expansion from state 2 to state 3 gives:

$$\Delta E_{\text{int},2\to 3} = \frac{5}{2}\left[(120\text{ kPa})(16.0\text{ L}) - (80\text{ kPa})(8.0\text{ L})\right] = 3800\text{ J} - 1600\text{ J} = +2200\text{ J}.$$

**Related End-of-Chapter Exercises: 14, 16, 46, 47.**

***Essential Question 15.2*:** If we did not know the path taken on the P-V diagram, in Figure 15.6, from state 2 to state 3, could we still find the work or the change in internal energy? Explain.