

**Answer to Essential Question 15.3:** In a constant-volume process, for a monatomic ideal gas:

$$Q = \Delta E_{\text{int}} = \frac{3}{2} n R \Delta T = \frac{3}{2} (0.19 \text{ moles}) (8.31 \text{ J/mol K}) (400 \text{ K} - 200 \text{ K}) = +480 \text{ J}.$$

Solving for the temperature change when 800 J of heat is added, we get:

$$\Delta T = \frac{2Q}{3nR} = \frac{2 \times 800 \text{ J}}{3(0.19 \text{ moles})(8.31 \text{ J/mol K})} = +333 \text{ K}.$$

Because the initial temperature is 200 K, the final temperature is  $200 \text{ K} + 333 \text{ K} = 533 \text{ K}$ . Applying the ideal gas law, gives the corresponding pressure:

$$P = \frac{nRT}{V} = \frac{(0.19 \text{ moles})(8.31 \text{ J/mol K})(533 \text{ K})}{4.0 \text{ L}} = 213 \text{ kPa}.$$

## 15-4 Constant Temperature and Adiabatic Processes

Let's now consider two more thermodynamic processes, the constant temperature (also known as isothermal) process and the adiabatic process.

**A constant-temperature (isothermal process:** Because the temperature is constant there is no change in internal energy. The First Law of Thermodynamics tells us that, in this case,  $Q = W$ , and it can be shown (using calculus is the most straightforward way to prove this) that:

$$Q = W = nRT \ln \left( \frac{V_f}{V_i} \right). \quad (\text{Eq. 15.8: Heat and work for an isothermal process})$$

### EXAMPLE 15.4A – Add heat at constant temperature

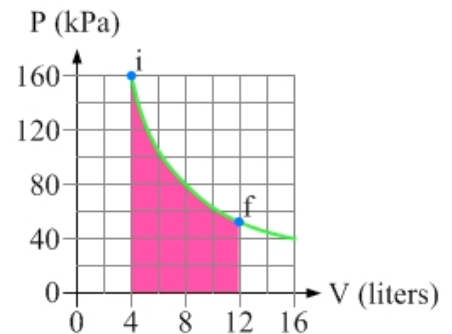
700 J of heat is added to a system of ideal gas, while the temperature is kept constant at 400 K. The system initially has a pressure of 160 kPa and occupies a volume of 4.0 liters.

- Is this possible? Can temperature remain constant while heat is added? Explain.
- Sketch this process on a P-V diagram, keeping in mind the following question: When heat is added at constant temperature does the gas pressure increase, as in a constant-volume process, or does the volume increase, as in a constant-pressure process?
- What are the final values of the gas pressure and the volume of the gas?

### SOLUTION

(a) This is possible. The first law of thermodynamics tells us that heat  $Q$  is converted into some combination of internal energy and/or work. When the temperature is constant we have the special case of no change in internal energy, so all the heat is converted into work.

(b) The temperature is constant so the process proceeds along an isotherm, shown in Figure 15.10. Because  $Q$  is positive, and  $Q = W$ , the work is also positive. Positive work means that the volume must increase, so the pressure drops to keep the temperature constant.



**Figure 15.10:** A P-V diagram corresponding to heat being added to a system at constant temperature. The final pressure and volume are calculated in part (c). The area corresponding to the work done by the gas is shown shaded.

(c) First, let's re-write Eq. 15.8 using the ideal gas law:  $Q = nRT \ln\left(\frac{V_f}{V_i}\right) = P_i V_i \ln\left(\frac{V_f}{V_i}\right)$ .

Isolating the logarithm on the right side gives:  $\frac{Q}{P_i V_i} = \ln\left(\frac{V_f}{V_i}\right)$ .

Taking the exponential of both sides:  $e^{Q/(P_i V_i)} = e^{\ln(V_f/V_i)} = \frac{V_f}{V_i}$ .

The final volume is thus:  $V_f = V_i e^{Q/(P_i V_i)} = (4.0 \text{ L}) e^{700 \text{ J}/(160 \text{ kPa} \times 4.0 \text{ L})} = 11.9 \text{ L}$ .

Solving for the final pressure gives:  $P_f = \frac{nRT}{V_f} = \frac{P_i V_i}{V_f} = \frac{(160 \text{ kPa})(4.0 \text{ L})}{11.94 \text{ L}} = 54 \text{ kPa}$ .

**An adiabatic process:** In an adiabatic process no heat is added to or removed from the gas (i.e.,  $Q = 0$ ). Examples include systems insulated so no heat is exchanged with the surroundings, and systems in which processes happen so fast that there is no time to add or remove heat. Because  $Q = 0$  for an adiabatic process the First Law of Thermodynamics tells us that  $\Delta E_{\text{int}} = -W$ . The energy for any work done comes from the change in the system's internal energy.

$P V^\gamma = \text{constant}$ , (Equation 15.9: **Equation for an adiabatic process on the P-V diagram**)

where  $\gamma$  is the ratio of the heat capacity at constant pressure to the heat capacity at constant volume:

$\gamma = \frac{C_P}{C_V}$  (Equation 15.10: The constant  $\gamma$  for an adiabatic process.

#### EXAMPLE 15.4B – Analyzing an adiabatic process

A system of monatomic ideal gas experiences an adiabatic expansion that moves it from an initial state, at 400 K, to a final state at a temperature of 200 K. The process is shown on the P-V diagram in Figure 15.11. Calculate the values of the final pressure and volume.

#### SOLUTION

Because the gas is monatomic, we have:  $\gamma = \frac{C_P}{C_V} = \frac{5R/2}{3R/2} = \frac{5}{3}$ .

From Equation 15.9, we know that:  $P_f V_f^\gamma = P_i V_i^\gamma$

From the ideal gas law, we have  $P_f = nRT_f/V_f$ , as well as  $n = P_i V_i/(RT_i)$ .

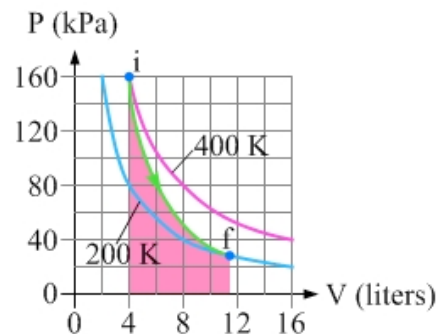
Combining these results leads to  $T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$ . (Equation 15.11)

Solving for the final volume gives:  $V_f = V_i (T_i/T_f)^{1/(\gamma-1)} = (4.0 \text{ L})(2)^{3/2} = 11.3 \text{ L}$ .

The final pressure is thus:  $P_f = P_i (V_i/V_f)^\gamma = (160 \text{ kPa}) \left(\frac{4.0 \text{ L}}{11.31 \text{ L}}\right)^{5/3} = 28 \text{ kPa}$ .

**Related End-of-Chapter Exercises for this section: 4, 22 – 27.**

**Essential Question 15.4:** How much work is done by the gas in Example 15.4B?



**Figure 15.11:** The P-V diagram for the adiabatic expansion.