

Answer to Essential Question 27.5: The interference pattern produced depends on the de Broglie wavelength of the particles incident. To produce the same pattern as the electrons, the protons must have the same wavelength. Examining Equation 27.8, we see that wavelength is determined by momentum, so the protons need to have the same momentum as the electrons. To have the same momentum with a factor of 1800 in the mass, the protons must have a speed of $v/1800$.

27-6 Heisenberg's Uncertainty Principle

The interplay between the uncertainty in an object's position, Δx , and the uncertainty in its momentum, Δp , was first quantified by the German physicist Werner Heisenberg (1901 – 1976). Heisenberg won the Nobel Prize in Physics in 1932 for his contributions to quantum theory. Heisenberg's uncertainty principle states that you cannot know something to infinite accuracy. More specifically, the uncertainty principle states that for two linked quantities, such as the position and momentum of an object, the more accurately you know one of those quantities, the less accurately you can know the other.

$$\Delta x \Delta p \geq \frac{h}{4\pi} = 5.273 \times 10^{-35} \text{ J s.} \quad (\text{Equation 27.9: Heisenberg's uncertainty principle})$$

The uncertainty principle also applies to other linked quantities, such as energy and time, or to two components of angular momentum.

For any object that we are used to dealing with on a daily basis, Heisenberg's uncertainty principle is essentially irrelevant. For instance, if you could measure the position of a 1 kg water bottle, which you see as being at rest on a table, with an uncertainty of 1 nm, Heisenberg's uncertainty principle tells us that the uncertainty in the water bottle's momentum must be at least 5.273×10^{-26} kg m/s. With its mass being 1 kg, the water bottle's velocity must therefore be uncertain by at least 5.273×10^{-26} m/s, a number so small that it is essentially meaningless.

For tiny objects like electrons, however, the limitations associated with Heisenberg's uncertainty principle are quite important. For the electron bound to the nucleus of a hydrogen atom, for instance, making a reasonable assumption about the uncertainty of the electron's momentum leads to an uncertainty in position that is similar in size to the atom itself. Thus, for the hydrogen atom, we can say that the electron is in the atom, but we cannot say exactly where in the atom it is at any point in time. We will investigate this idea further in Chapter 28.

Applying the Uncertainty Principle to the single-slit experiment

As we discussed earlier in this chapter, electrons interact with single and double slits in much the same way that light does. In both cases, the resulting diffraction pattern or interference pattern can be understood in terms of the wave nature of the electrons or of the light. However, applying the uncertainty principle can also give us some insight into the experiment.

Imagine a beam of electrons traveling toward a wide slit. We have a good idea of the speed and direction of the electrons in the beam – in other words, the momentum of the electrons is well known. The slit is so wide that, while we know that the electrons pass through the slit, we do not have much information about their position. With a large uncertainty in position, according to the Uncertainty Principle, the uncertainty in momentum can be small. Thus, the momentum of each electron is essentially unchanged from what it was before encountering the slit, and the electrons pass through the slit in a straight line.

The narrower we make the slit, however, the more knowledge we have about the position of an electron when it passes through the slit. By the Uncertainty Principle, the more accurately

we know an object's position, the less accurately we can know its momentum. In the experiment, this uncertainty in position manifests itself as a larger spread in the beam after the beam passes through the slit. As we discussed with light in Chapter 25, the smaller the width of the slit, the more spread there is in the beam. This concept is shown in Figure 27.10.

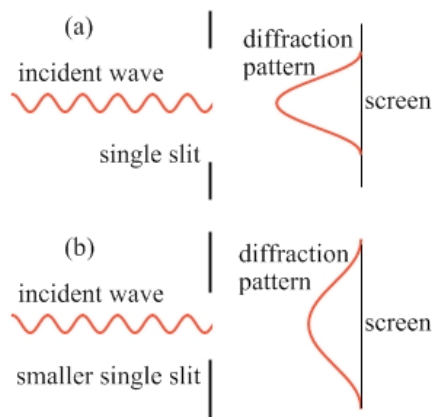


Figure 27.10: The single slit that an electron beam passes through in (a) is larger than that in (b), but the spread in the beam is smaller in (a). This is a consequence of the uncertainty principle.

EXAMPLE 27.6 – Investigating the Uncertainty Principle

Table 27.1 shows the mass, momentum, uncertainty in position, and uncertainty in momentum (assuming we are minimizing this uncertainty, according to the Uncertainty Principle) for three objects, a baseball, a virus, and an electron. In each case, the object's velocity is 10 m/s, and the uncertainty in position is 1 angstrom (Å). $1 \text{ \AA} = 1 \times 10^{-10} \text{ m}$. (a) Some of the values in Table 27.1 are missing. Complete the table to fill in the missing data. (b) Comment on the size of the uncertainty in momentum in the three cases.

Object	Mass (kg)	Momentum (kg m/s)	Δx (m)	Δp (kg m/s)
Baseball	0.15	1.5	1×10^{-10}	5.27×10^{-25}
Virus	2.0×10^{-17}		1×10^{-10}	
Electron	9.11×10^{-31}		1×10^{-10}	

Table 27.1: The mass and the uncertainty in position for a particular situation involving three objects of very different mass. In each case, the velocity is 10 m/s. The momentum and momentum uncertainty are not shown for the virus and the electron.

SOLUTION

(a) The missing data are shown in Table 27.2. Note that, with the uncertainty in position being the same in each case, the uncertainty in momentum is also the same in each case.

Object	Mass (kg)	Momentum (kg m/s)	Δx (m)	Δp (kg m/s)
Baseball	0.15	1.5	1×10^{-10}	5.27×10^{-25}
Virus	2.0×10^{-17}	2.0×10^{-16}	1×10^{-10}	5.27×10^{-25}
Electron	9.11×10^{-31}	9.11×10^{-30}	1×10^{-10}	5.27×10^{-25}

Table 27.2: The mass, momentum, position uncertainty, and momentum uncertainty in a particular situation involving three objects of very different mass.

(b) For a baseball, the limitations imposed by the uncertainty principle are so small as to be meaningless. Even for a tiny object like a virus, the momentum uncertainty is much smaller than the momentum. However, for the electron, the momentum uncertainty is orders of magnitude larger than its momentum, giving us little confidence in the stated value of the electron's velocity.

Related End-of-Chapter Exercises: 26 – 30.

Essential Question 27.6: What if Planck's constant, h , had a value around 1 J s instead of its actual value of $6.6 \times 10^{-34} \text{ J s}$? Would this change how we interact with the world?