number, m_ℓ, which can take on the values $m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$. The z-component of the orbital angular momentum is given by:

$$L_z = \frac{m_\ell h}{2\pi},$$ (Eq. 28.5: ***z*-component of the orbital angular momentum**)

Finally, the spin angular momentum can take on one of only two values, conventionally referred to as "spin up" and "spin down." The spin angular momentum is characterized by the spin quantum number, which can take on values of +1/2 or –1/2.

Understanding the periodic table of elements

One key to understanding the periodic table is the Pauli exclusion principle – no two electrons in an atom can have the same set of four quantum numbers.

The layout of the periodic table of the elements has to do with the highest-energy filled or partly filled subshell in the ground-state electron configuration of an atom. Elements that have similar ground-state configurations (such as four electrons in a p subshell) are grouped in a column in the periodic table, and generally have similar chemical properties.

Complete ground-state configurations are written with triplets of numbers and letters, in the form $3p^4$. The first number (3, in this case) represents the value of n, the principal quantum number, for the orbital. The letter (p, in this case) stands for the value of the orbital quantum number, ℓ. The second number (4, in this case) is the number of electrons in the subshell in this configuration.

Applications of atomic physics

Two applications of atomic physics include some lasers (such as the helium-neon laser) and the properties of fluorescence and phosphorescence. These applications involve the emission of photons from electrons in atoms dropping down from one energy level to another.

End-of-Chapter Exercises

Exercises 1 – 12 are conceptual questions that are designed to see if you have understood the main concepts of the chapter.

1. In a particular line spectrum, photons are observed that have an energy of 2.5 eV. Choose the phrase below that best completes the following sentence. The element producing the light definitely has …
   (i)… an electron energy level at an energy of 2.5 eV.
   (ii)… an electron energy level at an energy of –2.5 eV.
   (iii)… two electron energy levels that are 2.5 eV apart in energy.

2. Imagine that there is an atom with electron energy levels at the following energies: –88 eV, –78 eV, and –60 eV. Confining ourselves to electron transitions between these levels only, would we expect to see photons emitted from this atom with the following energies? Explain why or why not. (a) 60 eV, (b) –60 eV, (c) 18 eV, (d) 138 eV.

3. To be visible to the human eye, photons must have an energy between about 1.8 eV and 3.1 eV. In a particular atom, one of the electron energy levels is at an energy of –60.0 eV. Can electron transitions associated with this energy level produce photons that are visible to the human eye? Explain why or why not.

4. In Exploration 28.1, we determined that the energies of the three lowest energy levels in hydrogen have energies of $E_1 = -13.6$ eV, $E_2 = -3.40$ eV, and $E_3 = -1.51$ eV. Photons are produced by electrons that transition from the $n = 3$ level to the $n = 1$ level, as well as from the $n = 2$ level to the $n = 1$ level. Which of these photons have the larger (a) energy? (b) frequency? (c) wavelength?

5. Figure 28.13 shows a graph of the probability, per unit distance, of finding the electron in the *3s* orbital of hydrogen, at various distances from the nucleus. (a) At approximately what distance from the nucleus is the electron most likely to be found? Compare this to the value of 9 Bohr radii that is predicted by the Bohr model for the $n = 3$ state. (b) If we determined the area under the curve for this graph, for the region from $r = 0$ to $r =$ infinity, what value would that area turn out to be?

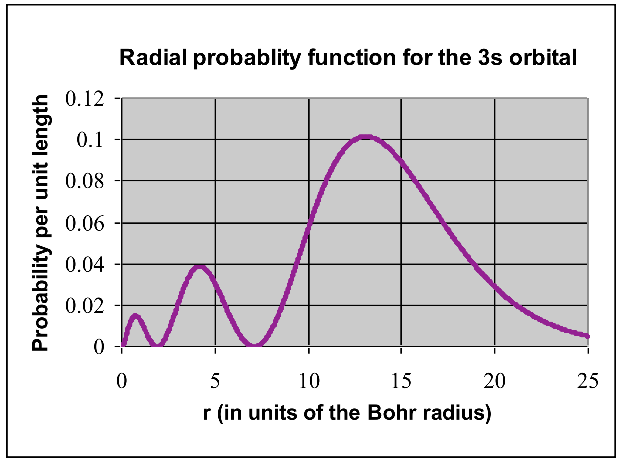

Figure 28.13: A graph of the probability, per unit length, of finding the electron in the *3s* orbital of hydrogen at various distances from the nucleus. For Exercise 5.

6. Figure 28.14 shows the relative probability per unit length as a function of position for a particle confined to a line. The particle is definitely located somewhere between $x = 0$ and $x = d$. (a) At what location(s) between $x = 0$ and $x = d$ is the particle most likely to be found? (b) At what location(s) between $x = 0$ and $x = d$ is the particle least likely to be found?

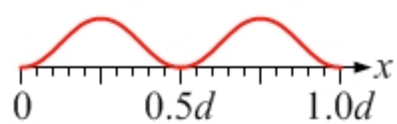

Figure 28.14: A graph of the relative probability per unit length of finding a particle confined to line at a particular position, as a function of position. For Exercise 6.

7. The Bohr model of the atom was an important stage along the path to our understanding of the atom, but it is important to recognize that the Bohr model is quite different from our modern view of the atom. Describe at least two ways in which the Bohr model differs from the modern view.

8. Each of the four quantum numbers is associated with a physical property of the electron. What physical property is associated with (a) the principal quantum number, n? (b) the orbital quantum number, ℓ ? (c) the magnetic quantum number, m_ℓ? (d) the spin quantum number, m_s?

9. If the value of the principal quantum number is $n = 2$, what are the allowed values of (a) ℓ, the orbital quantum number, (b) m_ℓ, the magnetic quantum number, and (c) m_s, the spin quantum number? (d) How many different states are there that have $n = 2$?

10. Explain why electrons tend to fill the $4s^2$ orbital before the $3d^{10}$ orbital.

11. Are the following electron configurations valid or invalid? Note that the configurations do not have to be ground-state configurations. If a configuration is valid, state which element it represents. If a configuration is invalid, explain why. (a) $1s^2\,2s^3\,2p^6$. (b) $1s^2\,2s^1\,2p^6\,3s^2\,3p^3$. (c) $1s^2\,2s^2\,2p^4\,3s^2\,3p^3$. (d) $1s^2\,2s^2\,2p^8\,3s^1$.

12. Choose the phrase below that best completes the following sentence. A mixture of helium and neon was chosen to create a laser because …
    (i) … helium and neon are from the same column in the periodic table.
    (ii) … there is an energy level in helium that has almost the same energy as an energy level in neon.
    (iii) … the difference between two energy levels in helium is almost the same as the difference between two energy levels in neon.

Exercises 13 – 18 involve line spectra.

13. Two energy levels in a particular atom are at energies of –23.4 eV and –25.6 eV. When an electron makes a transition from one of these levels to the other, a photon is emitted. For the photon, find the (a) energy, (b) frequency, and (c) wavelength. (d) For the transition described here, which level does the electron transition to?

14. In Exploration 28.1, we calculated the $n = 2$ energy level for hydrogen to have an energy of –3.40 eV. (a) What are the energies of the $n = 4$ and $n = 5$ levels for hydrogen? (b) If the electron in the hydrogen atom makes a transition from the $n = 4$ level to the $n = 2$ level, what is the energy of the emitted photon? (c) Repeat part (b), with the electron transitioning from $n = 5$ to $n = 2$, instead.

15. In Exploration 28.1, we sketched an energy-level diagram with the three lowest energy levels of hydrogen. The lines in the visible region of the spectrum that are emitted by hydrogen correspond to electron transitions that come from a higher level to the $n = 2$ level. Because of this, let's focus on the part of the energy-level diagram between an energy of 0 and the $n = 2$ level. (a) Determine the energies of the $n = 4$, $n = 5$, $n = 6$, and $n = 7$ levels for hydrogen. (b) Sketch an energy-level diagram to show the energy levels for $n = 2$ through $n = 7$ for hydrogen. (c) Determine the energies of the photons emitted when electrons transition from the $n = 3$ through $n = 7$ levels down to the $n = 2$ level. (d) Calculate the wavelengths for these photons.

16. A hypothetical atom has electron energy levels at the following energies: –14 eV, –30 eV, –52 eV, and –80 eV. Assume that electron transitions occur between these levels only. (a) How many different photon energies are possible? (b) List the photon energies, from smallest to largest.

17. Figure 28.15 shows an energy-level diagram for a hypothetical atom. Sketch the corresponding emission spectrum for this atom, showing the energies of the photons associated with electron transitions between the four levels shown in the figure.

18. Figure 28.15 shows an energy-level diagram for a hypothetical atom. Sketch the corresponding emission spectrum for this atom, showing the wavelengths of the photons associated with electron transitions between the four levels shown in the figure.

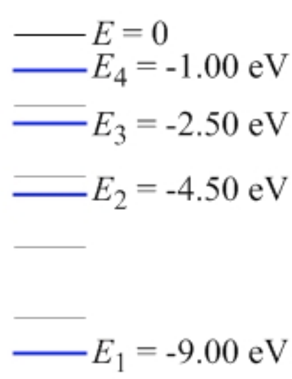

Figure 28.15: An energy-level diagram for a hypothetical atom. For Exercises 17 and 18.

Exercises 19 – 24 involve the four quantum numbers that are associated with the quantum-mechanical view of the atom.

19. If the value of the orbital quantum number is $\ell = 3$, what are the allowed values of (a) m_ℓ, the magnetic quantum number, and (b) m_s, the spin quantum number? (d) How many different states are there that have $\ell = 3$?

20. If the value of the principal quantum number is $n = 4$, what are the allowed values of (a) ℓ, the orbital quantum number, (b) m_ℓ, the magnetic quantum number, and (c) m_s, the spin quantum number? (d) How many different states are there that have $n = 4$?

21. If the value of the orbital quantum number is $\ell = 4$, what are the allowed values of the principal quantum number, n?

22. Figure 28.16 shows the allowed directions of the orbital angular momentum when the orbital quantum number, ℓ, has a particular value. (a) What is the value of ℓ in this case? (b) Re-draw the picture, and label each of the vectors with its corresponding m_ℓ value. (c) In units of $h/(2\pi)$, what is the length of each of the vectors?

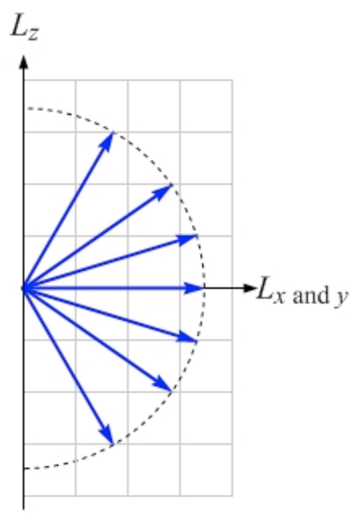

Figure 28.16: The allowed directions of the orbital angular momentum, when ℓ has a particular value. The units on both axes are angular momentum units of $h/(2\pi)$.

For Exercise 22.

23. The value of the magnetic quantum number for a particular electron in the 3d orbital happens to be $m_\ell = +1$. What, if anything, can we say about the values of the other three quantum numbers for this electron?

Exercises 24 – 28 involve applications of atomic physics.

24. Astronomers, particularly astronomers who use radio telescopes, have been able to examine a number of interesting features of the universe through measurements related to what is known as the "21-centimeter line" of hydrogen. The 21-centimeter line refers to photons that have a wavelength of about 21 cm, associated with a transition between two different states of hydrogen. (a) How does the energy of a photon with a wavelength of 21 cm compare to that of a photon in the visible spectrum? (b) Do some research regarding the transition in hydrogen that produces these 21-cm photons, and write a couple of paragraphs explaining the transition, and how the 21-cm line is used in astronomy.

25. One way to view emission spectra is to excite a tube of gas with high voltage, and view the emitted light through a diffraction grating that is held so that the plane of the grating is perpendicular to the direction in which the light travels from the tube to the grating. Recall that we discussed diffraction gratings in Chapter 25. The spectrum of helium gas includes lines at the following wavelengths: 447.1 nm, 501.6 nm, 587.6 nm, and 667.8 nm. When viewed through a diffraction grating in which the openings in the grating are separated by 1500 nm, at what angle (relative to the light coming from the tube) is the first-order line for the (a) 447.1 nm wavelength? (b) 667.8 nm wavelength? (c) How many complete spectra (confining ourselves to the four lines listed here) can be seen when viewing the tube through the diffraction grating?

26. One half of the 1986 Nobel Prize in Physics was awarded to Binning and Rohrer. Do some research about these two scientists, and write a couple of paragraphs regarding what they won the Nobel Prize for, and how it relates to the material covered in this chapter.

27. A neon sign, which is a glass tube filed with excited neon gas, gives off a red-orange glow, as illustrated by the photograph in Figure 28.17. If the sign is a color other than red-orange, the tube is generally not filled with neon. Explain qualitatively what the observation about the color of the neon sign tells us about the energy levels and electron transitions for neon atoms.

Figure 28.17: A neon sign, for Exercise 27. Photo credit: PhotoDisc, Inc.

28. The second (the unit of time) is defined as the duration of 9 192 631 770 cycles of the radiation emitted by an electron making a transition between two ground-state energy levels of the cesium-133 atom. What is the difference between these two energy levels, in electron volts?

General problems and conceptual questions

29. Niels Bohr, for whom the Bohr model is named, had an interesting life. Do some research about him, and write a couple of paragraphs about him. Include some descriptions about Bohr as a person as well as about his contributions to science.

30. The spectrum emitted by excited sodium gas includes two very closely spaced lines in the yellow part of the visual spectrum, at wavelengths of 588.995 nm and 589.592 nm. The photons associated with this light come from electron transitions that start from one of two closely-spaced 3p levels, and which end at the same 3s level. What is the energy difference of these two 3p levels?

31. When an electron in a particular atom transitions from one energy level to another, the atom emits a photon that has a wavelength of 623 nm. One of the energy levels associated with this transition has an energy of –30.28 eV. What is the energy of the other level?

32. Start with the electron in the ground state in the hydrogen atom, and use Equation 28.2 to calculate energy. How much energy is required to (a) ionize the atom (removing the electron completely)? (b) excite the electron from the ground state to the $n = 9$ level?

33. Which requires more energy, to excite the electron in hydrogen from the ground state to the $n = 2$ level, or to excite the electron from the $n = 2$ level to the $n = 100$ level? Explain.

34. A particular atom emits photons with wavelengths of 485 nm and 607 nm. Which wavelength is associated with the larger difference in energy between two energy levels in that atom? Briefly justify your answer.

35. The spectrum of helium gas includes lines at the following wavelengths: 447.1 nm, 501.6 nm, 587.6 nm, and 667.8 nm. Find the corresponding energy for these photons.

36. If you have the energy-level diagram for a particular atom, you can predict the emission spectrum for that atom (as we did in Exercises 17 and 18). If you have the emission spectrum for an atom, can you determine the atom's energy-level diagram? Explain.

37. When a spectral tube containing gas from a particular element is excited by applying high voltage, the spectrum shown in Figure 28.18 is observed. (a) What is the minimum number of energy levels that can be used to obtain this three-line spectrum? (b) Using the minimum number of levels, sketch an energy-level diagram that is consistent with Figure 28.18.

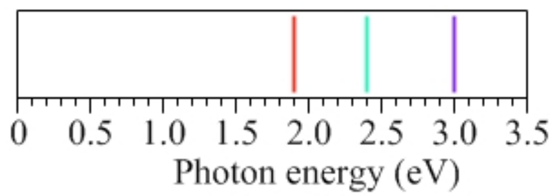

Figure 28.18: The lines in the visible spectrum that are seen when a particular gas is excited with high voltage, for Exercise 37.

38. When a spectral tube containing gas from a particular element is excited by applying high voltage, photons with the following energies are emitted: 3 eV, 13 eV, 16 eV, 25 eV, 38 eV, and 41 eV. No other photons in this range are observed. One of the electron energy levels in the corresponding energy-level diagram is at an energy of –20 eV. (a) What is the minimum number of energy levels that can be used to obtain the given six-line spectrum? (b) Using the minimum number of levels, sketch an energy-level diagram that is consistent with the information given here. (c) Is there only one possible answer to (b)? Explain.

39. Equation 28.3 can be used to find the energies of the energy levels for atoms that have only one electron. Let's use Equation 28.3 to examine the emission spectrum of doubly-ionized lithium, Li^{2+} (that is, lithium atoms with just one electron). (a) The energies of the electron energy levels in hydrogen are equal to the energies of some of the electron energy levels of Li^{2+}. Which n values, for Li^{2+}, give the same energies as those of the $n = 1$, $n = 2$, and $n = 3$ levels in hydrogen? (b) With matching energy levels, the energies of the photons emitted by excited hydrogen could also be produced by excited Li^{2+} atoms. Is it possible to distinguish between spectra from hydrogen and Li^{2+}? If so, how?

40. In a hypothetical atom, the energies of the electron energy levels are given by $E_n = -(144 \text{ eV})/n^2$, where n is a positive integer. (a) What are the energies of the four lowest energy levels in this atom? (b) What are the energies of the photons emitted by electrons that transition between any two of these four levels?

41. In a hypothetical atom, the energies of the electron energy levels are given by $E_n = -(60 \text{ eV})/n$, where n is a positive integer. Consider the lowest six energy levels. For transitions between the lowest six energy levels only, determine how many different ways there are to produce photons with an energy of (a) 10 eV, and (b) 5 eV. Specify the initial and final values of n for each of these transitions.

42. The wavelengths of visible photons that are emitted by a tube of excited gas are shown in Figure 28.19. (a) Calculate the energies of these photons. (b) If these photons are emitted in electron transitions that end at an energy level of –16.5 eV, what are the energies of the energy levels that the electrons start from to produce these photons?

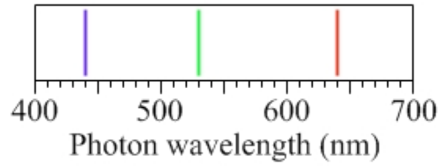

Figure 28.19: The wavelengths of photons in the visible spectrum that are seen when a particular gas is excited with high voltage, for Exercise 42.

43. The "Balmer series" is the name given to the set of photon energies (or, equivalently, wavelengths) that correspond to electrons making the transition from higher-n levels to the $n = 2$ level in hydrogen. For the Balmer series, what is the (a) smallest photon energy? (b) largest photon energy?

44. The "Lyman series" is the name given to the set of photon energies (or, equivalently, wavelengths) that correspond to electrons making the transition from higher-n levels to a specific lower level in hydrogen. (a) What is the value of n that characterizes the lower level for the Lyman series? (b) Who was Lyman? Do some research about Lyman and write a paragraph describing this scientist.

45. Return to the situation described in Exercise 44. For the Lyman series, what is the (a) largest photon wavelength? (b) smallest photon wavelength?

46. One of the wavelengths emitted by excited hydrogen gas in a tube that is at rest on the Earth is found to be 656.28 nm. When you carefully measure the corresponding wavelength emitted from the hydrogen in a distant star, however, you find the wavelength to be 658.73 nm. Having learned about the Doppler effect for electromagnetic waves, in Chapter 22, you realize that, with this data, you can determine the velocity of the distant star with respect to the Earth (at least, you can find the component of the velocity that is along the line joining the Earth and the star). Find the value of this velocity.

47. The Hubble Space Telescope is named after the American astronomer Edwin Hubble. Do some research about Edwin Hubble and write a couple of paragraphs about what he is most famous for, and how his work relates to the principles of physics discussed in this chapter. Note that an image of part of the Eagle Nebula, as seen by the Hubble Space Telescope, is shown in Figure 28.20.

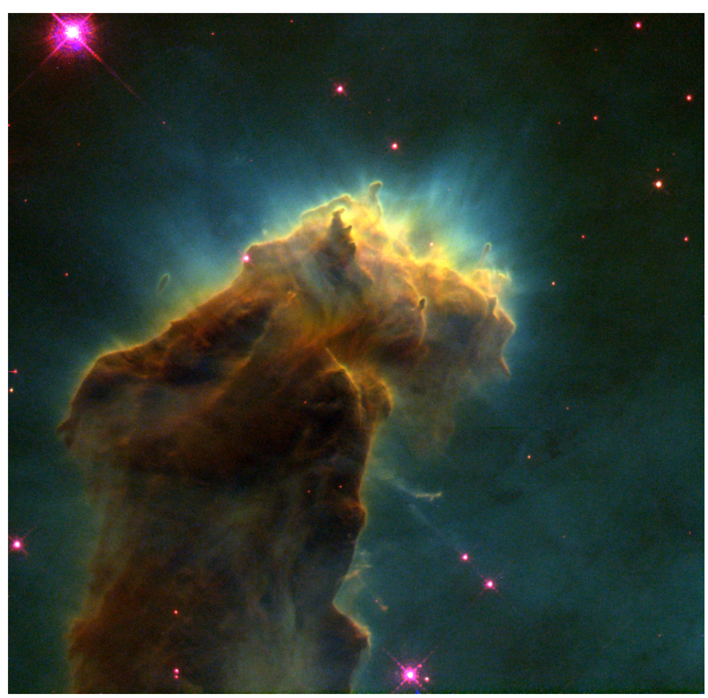

Figure 28.20: An image of a vast cloud of gas and dust that is part of the Eagle Nebula, as imaged by the Hubble Space Telescope, for Exercise 47. Image credit: Jeff Hester and Paul Scowen (Arizona State University), and NASA/ESA.

48. We discussed the photoelectric effect experiment in Chapter 27. One way the photoelectric effect experiment is often carried out is to use a mercury light source to illuminate a particular surface. The emission spectrum of mercury has strong lines at wavelengths of 365.5 nm (ultraviolet), 404.7 nm (violet), 435.8 nm (blue), and 546.074 nm (green). The light can be passed through a diffraction grating to separate the light into these different wavelengths. Assume that photons of all these wavelengths have enough energy to overcome the work function of the surface. (a) Which of these wavelengths will produce electrons with the largest kinetic energy? (b) If the maximum kinetic energy of the emitted electrons is 1.27 eV when the green light illuminates the surface, what will the maximum kinetic energy of the emitted electrons be when the violet light illuminates the surface? (c) What is the work function of this surface?

49. Figure 28.21 shows a graph of the probability, per unit distance, of finding the electron in the *2s* orbital of hydrogen, at various distances from the nucleus. (a) At approximately what distance from the nucleus is the electron most likely to be found? Compare this to the value of 4 Bohr radii that is predicted by the Bohr model for the $n = 2$ state. (b) Use the graph to estimate the probability of finding the electron at a distance of less than 2 Bohr radii from the nucleus.

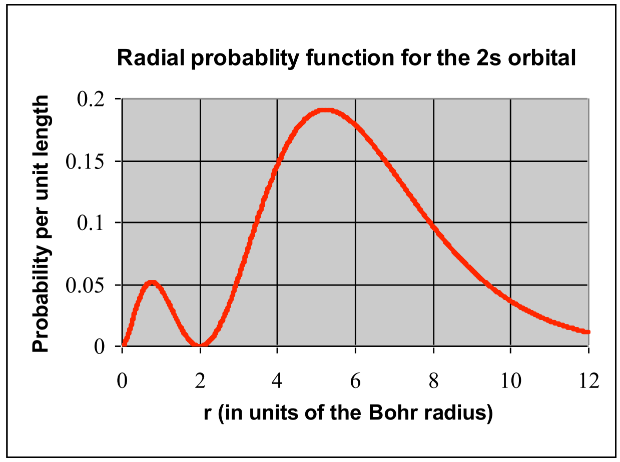

Figure 28.21: A graph of the probability, per unit length, of finding the electron in the *2s* orbital of hydrogen at various distances from the nucleus. For Exercises 49 and 50.

50. For the graph in Figure 28.21, the area under the curve for the region from $r = 0$ to $r = 5.8$ Bohr radii is equal to 0.5. (a) What is the area under the curve from the region from $r = 5.8$ Bohr radii to $r =$ infinity? (b) What does this mean, in terms of the probability of finding the electron at particular distances from the nucleus?

51. A particular electron in an atom is in a state such that its orbital quantum number is $\ell = 3$. Express your answers in terms of h (Planck's constant) and π. (a) What is the magnitude of the electron's orbital angular momentum? (b) What are the possible values of the *z*-component of the electron's orbital angular momentum?

52. A particular electron in an atom is in a state such that the *z*-component of its orbital angular momentum is $+h/2\pi$. What, if anything, does this tell us about the value of (a) the electron's orbital quantum number, ℓ ? (b) the electron's principal quantum number, n?

53. In Figures 28.5 and 28.16, we sketched vector diagrams to show the allowed directions of the orbital angular momentum vector for two specific values of ℓ . (a) Draw a similar diagram that applies when $\ell = 1$, labeling each vector with the corresponding value of m_ℓ. (b) What is the magnitude of each of the vectors on the diagram?

54. In the absence of an external magnetic field, states with the same values of n and ℓ have the same energy. For instance, with no external magnetic field, electrons in the 3p orbital in a particular atom all have the same energy. If such electrons drop from the 3p level to the 2s level, we would see a single line in the emission spectrum, corresponding to photons with an energy equal to the energy difference between the 3p and 2s levels. When an external magnetic field is applied, however, electrons within an orbital will have slightly different energies, experiencing a shift in energy from the zero-field value that is proportional to the strength of the magnetic field multiplied by the electron's quantum number. This splitting of the energy levels is known as the **Zeeman effect**. (a) For the example mentioned here, electrons transitioning from the 3p to 2s levels, explain why we only have to worry about the splitting of energy levels for the 3p states, not for the 2s states. (b) Sketch, qualitatively, an energy-level diagram showing the 3p and 2s energy levels in both the absence and presence of an external magnetic field. (c) What affect would applying an external magnetic field have on the line in the emission spectrum associated with the 3p to 2s transition?

55. Use the chart in Figure 28.9 to help you answer this question. Write out the complete ground-state configuration for the element with an atomic number of (a) 16. (b) 34. (c) Briefly explain why these elements are found in the same column in the periodic table.

56. A short-hand method for writing out the ground-state configuration of an element is to start from the configuration of a column-18 element, and then add whatever is missing. For instance, the ground-state configuration of zirconium (Zr) can be written as [Kr] $5s^2\ 4d^2$, which means that the initial terms match those of krypton (Kr), and the additional terms are $5s^2\ 4d^2$. (a) Write out the complete ground-state configuration for zirconium. (b) Why might a column-18 element be used as the starting point for the short-hand method, as opposed to, say, a column-16 element? Using this short-hand method, write out the ground-state configuration of (c) Nickel (Ni), and (d) Lead (Pb).

57. The ground-state configuration of a particular element is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^5$. (a) What is the atomic number of this element? (b) What is the name of this element?

58. Reading up on the properties and applications of different elements can be quite fascinating. Do some research about the element antimony, and write up a couple of paragraphs to tell the story of antimony.

59. Elements in column 1 of the periodic table generally make strong bonds with elements of column 17. An example of this is NaCl, table salt. To form the bond, the column-1 element essentially gives up one of its electrons to the column-17 element. Explain why this generally leads to a stable structure.

60. Two students are having a conversation. Comment on each of the statements that they make below.

Julia: I'm trying to figure out how to draw the energy-level diagram from this spectrum we were given. I converted the six wavelengths to electron volts – is the energy-level diagram just that set of six energies?

Kristin: No – those are all the photon energies. You have to think about where the photons come from – how do the photons relate to the energy levels?

Julia: OK, I remember – a photon gets released when an electron changes levels, and the photon energy equals the difference between the electron levels. So, like, this 2.5 eV photon could come from an electron dropping from a 12.5 eV level to a 10 eV level.

Kristin: That sounds right. Except, how do you know those are the right energies? You could also get 2.5 eV by dropping from 10 eV to 7.5 eV.

Julia: You're right – we could get 2.5 eV an infinite number of ways. Maybe we need to look at the other photons to narrow down the possibilities. Look, I see a pattern here for these three photons, we have 2.5 eV and 4.1 eV, and they add up to 6.6 eV, and we also have a 6.6 eV photon. So, we could have energy levels at 16.6, 12.5, and 10, and that gives us all those photons.

Kristin: Don't we need 14.1 eV, too, to get the 4.1 eV photon?

Julia: No, we've got that already, because the electron can go 16.6 to 12.5 – that's 4.1. OK, so, for those three photons we had three energy levels – if we can do three more energy levels for the other photons we'll be all set. Those other photon energies are 13.4, 17.5, and 20.0. Huh, if you add 13.4 and 17.5 you don't get 20 – how do you do those ones?