2.1 Unit Analysis

Many of the questions asked in chemistry and in everyday life can be answered by converting one unit of measure into another. For example, suppose you are taking care of your nephew for the weekend, and he breaks his arm. The doctor sets the arm, puts it in a cast, and prescribes an analgesic to help control the pain. Back at home, after filling the prescription, you realize that the label calls for 2 teaspoons of medicine every six hours, but the measuring device that the pharmacy gave you is calibrated in milliliters. You can’t find a measuring teaspoon in your kitchen, so you’ve got to figure out how many milliliters are equivalent to 2 tsp. It’s been a rough day, there’s a crying boy on the couch, and you’re really tired. Is there a technique for doing the necessary calculation that is simple and reliable? Unit analysis to the rescue!

The main purpose of this chapter is to show you how to make many different types of unit conversions, such as the one required above. You will find that the stepwise thought process associated with the procedure called **unit analysis** not only guides you in figuring out how to set up unit conversion problems but also gives you confidence that your answers are correct.

### An Overview of the General Procedure

*In every affair, consider what precedes and follows, and then undertake it.*  
---  
Epictetus (c. 55-c. 135) Greek Philosopher

You will see many different types of unit conversions in this chapter, but they can all be worked using the same general procedure. To illustrate the process, we will convert 2 teaspoons to milliliters and solve the problem of how much medicine to give the little boy described above.

The first step in the procedure is to identify the unit for the value we want to calculate. We write this on the left side of an equals sign. Next, we identify the value that we will convert into the desired value, and we write it on the right side of the equals sign. (Remember that a value constitutes both a number and a unit.) We want to know how many milliliters are equivalent to 2 tsp. We express this question as

\[
\text{? mL} = 2 \text{ tsp}
\]

Desired unit \quad Given unit

Next, we multiply by one or more conversion factors that enable us to cancel the unwanted units and generate the desired units. A **conversion factor** is a ratio that describes the relationship between two units. To create a conversion factor for converting teaspoons to milliliters we can look in any modern cookbook (check its index under “metric conversions”) and discover that the relationship between teaspoons and milliliters is

\[
1 \text{ tsp} = 5 \text{ mL}
\]

---

1Unit analysis has other names, including the factor-label method, the conversion factor method, and dimensional analysis.
This relationship can be used to produce two ratios, or conversion factors:

\[
\left( \frac{5 \text{ mL}}{1 \text{ tsp}} \right) \quad \text{or} \quad \left( \frac{1 \text{ tsp}}{5 \text{ mL}} \right)
\]

The first of these can be used to convert teaspoons to milliliters, and the second can be used to convert milliliters to teaspoons.

The final step in the procedure is to multiply the known unit (2 tsp) by the proper conversion factor, the one that converts teaspoons to milliliters.

\[
? \text{ mL} = 2 \text{ tsp} \times \left( \frac{5 \text{ mL}}{1 \text{ tsp}} \right) = 10 \text{ mL}
\]

Because 1 teaspoon is equivalent to 5 milliliters, multiplying by 5 mL/1 tsp is the same as multiplying by 1. The volume associated with 2 tsp does not change when we multiply by the conversion factor, but the value (number and unit) does. Because one milliliter is one-fifth the volume of one teaspoon, there are five times as many milliliters for a given volume. Therefore, 2 tsp and 10 mL represent the same volume.

Note that the units in a unit analysis setup cancel just like variables in an algebraic equation. Therefore, when we want to convert tsp to mL, we choose the ratio that has tsp on the bottom to cancel the tsp unit in our original value and leave us with the desired unit of mL. If you have used correct conversion factors, and if your units cancel to yield the desired unit or units, you can be confident that you will arrive at the correct answer.

**Metric-Metric Conversions**

As you saw in Chapter 1, one of the convenient features of the metric system is that the relationships between metric units can be derived from the metric prefixes. These relationships can easily be translated into conversion factors. For example, milli- means \(10^{-3}\) (or 0.001 or 1/1000), so a milliliter (mL) is \(10^{-3}\) liters (L). Thus there are 1000 or \(10^3\) milliliters in a liter. (A complete list of the prefixes that you need to know to solve the problems in this text is in Table 1.2.) Two possible sets of conversion factors for relating milliliters to liters can be obtained from these relationships.

\[
10^3 \text{ mL} = 1 \text{ L} \quad \text{leads to} \quad \frac{10^3 \text{ mL}}{1 \text{ L}} \quad \text{or} \quad \frac{1 \text{ L}}{10^3 \text{ mL}}
\]

\[
1 \text{ mL} = 10^{-3} \text{ L} \quad \text{leads to} \quad \frac{1 \text{ mL}}{10^{-3} \text{ L}} \quad \text{or} \quad \frac{10^{-3} \text{ L}}{1 \text{ mL}}
\]

In the remainder of this text, metric-metric conversion factors will have positive exponents like those found in the first set of conversion factors above.
**Example 2.1 - Conversion Factors**

**Objective 2**

Write two conversion factors that relate nanometers and meters. Use positive exponents in each.

**Solution**

Nano- means $10^{-9}$, so nanometer means $10^{-9}$ meters.

\[ 1 \text{ nm} = 10^{-9} \text{ m} \quad \text{and} \quad 10^9 \text{ nm} = 1 \text{ m} \]

Because we want our conversion factors to have positive exponents, we will build our ratios from the equation on the right ($10^9 \text{ nm} = 1 \text{ m}$):

\[ \frac{10^9 \text{ nm}}{1 \text{ m}} \quad \text{or} \quad \frac{1 \text{ m}}{10^9 \text{ nm}} \]

**Exercise 2.1 - Conversion Factors**

**Objective 2**

Write two conversion factors that relate the following pairs of metric units. Use positive exponents for each.

- a. meter and kilometer
- b. meter and centimeter
- c. liter and gigaliter
- d. gram and microgram
- e. gram and megagram

**Example 2.2 - Unit Conversions**

**Objective 3**

Convert 365 nanometers to kilometers.

**Solution**

We want the answer in kilometers (km), and the units we are given are nanometers (nm), so, we are converting from one metric length unit to another metric length unit. We begin by writing

\[ ? \text{ km} = 365 \text{ nm} \]

We continue constructing the unit analysis setup by writing the “skeleton” of a conversion factor: the parentheses, the line dividing the numerator and the denominator, and the unit that we know we want to cancel. This step helps to organize our thoughts by showing us that our first conversion factor must have the nm unit on the bottom to cancel the nm unit associated with 365 nm.

\[
\text{Desired unit} \quad \text{Skeleton} \\
? \text{ km} = 365 \text{ nm} \left( \frac{\text{Unit to be cancelled}}{\text{Given value}} \right) 
\]
Note that in this problem both the desired metric unit and the known one have prefixes. One way to make this type of conversion is to change the given unit to the corresponding metric base unit, and then change that metric base unit to the desired unit. We write two conversion factors, one for each of these changes.

Sometimes it is useful to write a simple description of your plan first. Our plan in this instance is

\[ \text{nm} \to \text{m} \to \text{km} \]

The unit analysis setup would therefore be

\[
? \text{ km} = 365 \text{ nm} \left( \frac{1 \text{ m}}{10^9 \text{ nm}} \right) \left( \frac{1 \text{ km}}{10^3 \text{ m}} \right) = 3.65 \times 10^{-10} \text{ km}
\]

The \( \text{nm} \) and \( \text{m} \) units cancel, leaving the answer in \( \text{km} \).

**EXERCISE 2.2 - Unit Conversions**

Convert 4.352 micrograms to megagrams.

**English-Metric Conversions**

English units\(^2\) are still common in some countries, while people in other countries (and the scientific community everywhere), use metric units almost exclusively. Unit analysis provides a convenient method for converting between English and metric units. Several of the most commonly needed English-metric conversion factors are listed in Table 2.1 on the next page. Because the English inch is defined as 2.54 cm, the number 2.54 in this value is exact. The numbers in the other conversion factors in Table 2.1 are not exact.

\(^2\)Table A.2 in Appendix A shows some useful English-English conversion factors.
Table 2.1
English-Metric Unit Conversion Factors

<table>
<thead>
<tr>
<th>Type of measurement</th>
<th>Probably most useful to know</th>
<th>Also useful to know</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>2.54 cm 1 in. (exact)</td>
<td>1.609 km 1 mi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>39.37 in. 1 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.094 yd 1 m</td>
</tr>
<tr>
<td>mass</td>
<td>453.6 g 1 lb</td>
<td>2.205 lb 1 kg</td>
</tr>
<tr>
<td>volume</td>
<td>3.785 L 1 gal</td>
<td>1.057 qt 1 L</td>
</tr>
</tbody>
</table>

Example 2.3 - Unit Conversions

The mass of a hydrogen atom is $1.67 \times 10^{-18}$ micrograms. Convert this mass into pounds.

Solution
We start with

$$\text{? lb} = 1.67 \times 10^{-18} \, \mu g$$

We then add the skeleton of the first conversion factor.

$$\text{? lb} = 1.67 \times 10^{-18} \, \mu g \left( \frac{1 \, \text{g}}{10^6 \, \mu g} \right)$$

We are converting from metric mass to English mass. Table 2.1 contains a conversion factor that is convenient for most English-metric mass conversions, the one relating grams to pounds. Our given unit is micrograms. If we convert the micrograms into grams, we can then convert the gram unit into pounds.

$$\mu g \rightarrow g \rightarrow \text{lb}$$

Converts metric mass unit to English mass unit

$$\text{? lb} = 1.67 \times 10^{-18} \, \mu g \left( \frac{1 \, \text{g}}{10^6 \, \mu g} \right) \left( \frac{1 \, \text{lb}}{453.6 \, \text{g}} \right) = 3.68 \times 10^{-27} \, \text{lb}$$

Exercise 2.3 - Conversion Factors

The volume of the Earth's oceans is estimated to be $1.5 \times 10^{18}$ kiloliters. What is this volume in gallons?

Sometimes the British are obstinate about the change from English to metric units. Greengrocer Steve Thoburn went to jail for refusing to switch from pounds to kilograms.