Most of your calculations in chemistry are likely to be done using a calculator, and calculators often provide more digits in the answer than you would be justified in reporting as scientific data. This section shows you how to round off an answer to reflect the approximate range of certainty warranted by the data.

**Measurements, Calculations, and Uncertainty**

In Section 1.5, you read about the issue of uncertainty in measurement and learned to report measured values to reflect this uncertainty. For example, an inexpensive letter scale might show you that the mass of a nickel is 5 grams, but this is not an exact measurement. It is reasonable to assume that the letter scale measures mass with a precision of ±1 g and that the nickel therefore has a mass between 4 grams and 6 grams. You could use a more sophisticated instrument with a precision of ±0.01 g and report the mass of the nickel as 5.00 g. The purpose of the zeros in this value is to show that this measurement of the nickel's mass has an uncertainty of plus or minus 0.01 g. With this instrument, we can assume that the mass of the nickel is between 4.99 g and 5.01 g. *Unless we are told otherwise, we assume that values from measurements have an uncertainty of plus or minus one in the last decimal place reported.* Using a far more precise balance found in a chemistry laboratory, you could determine the mass to be 4.9800 g, but this measurement still has an uncertainty of ±0.0001 g. *Measurements never give exact values.*

![Image of balances showing measurements of nickel mass](image)

**Figure 2.1**

**Measurement Precision**

Even highly precise measurements have some uncertainty. Each of these balances yields a different precision for the mass of a nickel.

<table>
<thead>
<tr>
<th>Balance</th>
<th>Mass</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.0 g</td>
<td>meaning 4.9 g to 5.1 g</td>
</tr>
<tr>
<td></td>
<td>4.98 g</td>
<td>meaning 4.97 g to 4.99 g</td>
</tr>
<tr>
<td></td>
<td>4.9800 g</td>
<td>meaning 4.9799 g to 4.9801 g</td>
</tr>
</tbody>
</table>
If a calculation is performed using all exact values and if the answer is not rounded off, the answer is exact, but this is a rare occurrence. The values used in calculations are usually not exact, and the answers should be expressed in a way that reflects the proper degree of uncertainty. Consider the conversion of the mass of our nickel from grams to pounds. (There are 453.6 g per pound.)

\[
? \text{ lb} = 4.9800 \text{ g} \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right) = 0.01098 \text{ lb (or } 1.098 \times 10^{-2} \text{ lb)}
\]

The number 4.9800 is somewhat uncertain because it comes from a measurement. The number 453.6 was derived from a calculation, and the answer to that calculation was rounded off to four digits. Therefore, the number 453.6 is also uncertain. Thus any answer we obtain using these numbers is inevitably going to be uncertain as well.

Different calculators or computers report different numbers of decimal places in their answers. For example, perhaps a computer reports the answer to 4.9800 divided by 453.6 as 0.0109783597884. If we were to report this result as the mass of our nickel, we would be suggesting that we were certain of the mass to a precision of ±0.00000000000001, which is not the case. Instead, we report 0.01098 lb (or 1.098 \times 10^{-2} \text{ lb}), which is a better reflection of the uncertainty in the numbers we used to calculate our answer.

**Rounding Off Answers Derived from Multiplication and Division**

There are three general steps to rounding off answers so that they reflect the uncertainty of the values used in a calculation. Consider the example below, which shows how the mass of a hydrogen atom in micrograms can be converted into the equivalent mass in pounds.

\[
? \text{ lb} = 1.67 \times 10^{-18} \text{ µg} \left( \frac{1 \text{ g}}{10^6 \text{ µg}} \right) \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right)
\]

The first step in rounding off is to decide which of the numbers in the calculation affect the uncertainty of the answer. We can assume that \(1.67 \times 10^{-18} \text{ µg}\) comes from a measurement, and all measurements are uncertain to some extent. Thus \(1.67 \times 10^{-18}\) affects the uncertainty of our answer. The \(10^6\) number comes from the definition of the metric prefix micro-, so it is exact. Because it has no effect on the uncertainty of our answer, we will not consider it when we are deciding how to round off our answer. The 453.6 comes from a calculation that was rounded off, so it is not exact. It affects the uncertainty of our answer and must be considered when we round our answer.

The second step in rounding off is to consider the degree of uncertainty in each of our inexact values. We can determine their relative uncertainties by counting the numbers of significant figures: three in \(1.67 \times 10^{-18}\) and four in 453.6. The number of **significant figures**, which is equal to the number of meaningful digits in a value, reflects the degree of uncertainty in the value (this is discussed more specifically in Study Sheet 2.1). A larger number of significant figures indicates a smaller uncertainty.

The final step is to round off our answer to reflect the most uncertain value used in our calculation. **When an answer is calculated by multiplying or dividing, we round it off to the same number of significant figures as the inexact value with the fewest significant figures.** For our example, that value is \(1.67 \times 10^{-18} \text{ µg}\), with three significant figures, so
we round off the calculated result, \(3.681657848325 \times 10^{-27}\), to \(3.68 \times 10^{-27}\).  

The following sample study sheet provides a detailed guide to rounding off numbers calculated using multiplication and division. (Addition and subtraction will be covered in the subsequent discussion.) Examples 2.4 and 2.5 demonstrate these steps.

**Tip-off**  After calculating a number using multiplication and division, you need to round it off to the correct number of significant figures.

**General Steps**

**Step 1** Determine whether each value is exact or not, and ignore exact values.
- Numbers that come from definitions are exact.
  - Numbers in metric-metric conversion factors that are derived from the metric prefixes are exact, such as
    \[
    \frac{10^3 \text{ g}}{1 \text{ kg}}
    \]
    Numbers in English-English conversion factors with the same type of unit (for example, both length units) top and bottom are exact, such as
    \[
    \frac{12 \text{ in.}}{1 \text{ ft}}
    \]
    The number 2.54 in the following conversion factor is exact.
    \[
    \frac{2.54 \text{ cm}}{1 \text{ in.}}
    \]
  - Numbers derived from counting are exact. For example, there are exactly five toes in the normal foot.
    \[
    \frac{5 \text{ toes}}{1 \text{ foot}}
    \]
  - Values that come from measurements are never exact.
  - We will assume that values derived from calculations are not exact unless otherwise indicated. (With one exception, the numbers relating English to metric units that you will see in this text have been calculated and rounded, so they are not exact. The exception is 2.54 cm/1 in. The 2.54 comes from a definition.)

**Step 2** Determine the number of significant figures in each value that is not exact.
- *All non-zero digits are significant.*
  - \(1.35\) 11.275 g ——— Five significant figures
Zeros between nonzero digits are significant.

A zero between nonzero digits

\[10.275 \text{ g} \] Five significant figures

Zeros to the left of nonzero digits are not significant.

Not significant figures

0.000102 kg which can be described as \(1.02 \times 10^{-4}\) kg

Both have three significant figures.

Zeros to the right of nonzero digits in numbers that include decimal points are significant.

Five significant figures — 10.200 g

20.0 mL — Three significant figures

Unnecessary for reporting size of value, but do reflect degree of uncertainty.

Zeros to the right of nonzero digits in numbers without decimal points are ambiguous for significant figures.

Precise to ±1 kg or ±10 kg?

Two or three significant figures?

Important for reporting size of value, but unclear about degree of uncertainty

Use scientific notation to remove ambiguity.

OBJECTIVE 8

**STEP 3** When multiplying and dividing, round your answer off to the same number of significant figures as the value containing the fewest significant figures.

If the digit to the right of the final digit you want to retain is less than 5, round down (the last digit remains the same).

26.221 rounded to three significant figures is 26.2

First digit dropped is less than 5
2.2 Rounding Off and Significant Figures

Objective 8

Example 2.4 - Rounding Off Answers Derived from Multiplication and Division

The average human body contains 5.2 L of blood. What is this volume in quarts? The unit analysis setup for this conversion is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[ \frac{? \text{ qt}}{? \text{ gal}} = \frac{5.2 \text{ L}}{3.785 \text{ L}} \left( \frac{1 \text{ gal}}{4 \text{ qt}} \right) \]

Solution

A typical calculator shows the answer to this calculation to be 5.4953765, a number with far too many decimal places, considering the uncertainty of the values used in the calculation. It needs to be rounded to the correct significant figures.

Step 1 The 5.2 L is based on measurement, so it is not exact. The 3.785 L is part of an English-metric conversion factor, and we assume those factors are not exact except for 2.54 cm/in. On the other hand, 4 qt/gal is an English-English conversion factor based on the definition of quart and gallon; thus the 4 is exact.

Step 2 Because 5.2 contains two nonzero digits, it has two significant figures. The number 3.785 contains four nonzero digits, so it has four significant figures.

Step 3 Because the value with the fewest significant figures has two significant figures, we report two significant figures in our answer, rounding 5.4953765 to 5.5.

\[ \frac{? \text{ qt}}{? \text{ gal}} = \frac{5.2 \text{ L}}{3.785 \text{ L}} \left( \frac{1 \text{ gal}}{4 \text{ qt}} \right) = 5.5 \text{ qt} \]
Example 2.5 - Rounding Off Answers Derived from Multiplication and Division

How many minutes does it take an ant walking at 0.01 m/s to travel 6.0 feet across a picnic table? The unit analysis setup for this conversion is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[
? \text{ min} = 6.0 \text{ ft} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ s}}{0.01 \text{ m}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right)
\]

Solution

Step 1 The table's length and the ant's velocity come from measurements, so 6.0 and 0.01 are not exact. The other numbers are exact because they are derived from definitions. Thus, only 6.0 and 0.01 can limit our significant figures.

Step 2 Zeros to the right of nonzero digits in numbers that have decimal points are significant, so 6.0 contains two significant figures. Zeros to the left of nonzero digits are not significant, so 0.01 contains one significant figure.

Step 3 A typical calculator shows 3.048 for the answer. Because the value with the fewest significant figures has one significant figure, we report one significant figure in our answer. Our final answer of 3 minutes signifies that it could take 2 to 4 minutes for the ant to cross the table.

\[
? \text{ min} = 6.0 \text{ ft} \left( \frac{12 \text{ in}}{1 \text{ ft}} \right) \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) \left( \frac{1 \text{ m}}{10^2 \text{ cm}} \right) \left( \frac{1 \text{ s}}{0.01 \text{ m}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 3 \text{ min}
\]

Exercise 2.4 - Rounding Off Answers Derived from Multiplication and Division

A first-class stamp allows you to send letters weighing up to 1 oz. (There are 16 ounces per pound.) You weigh a letter and find it has a mass of 10.5 g. Can you mail this letter with one stamp? The unit analysis setup for converting 10.5 g to ounces is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[
? \text{ oz} = 10.5 \text{ g} \left( \frac{1 \text{ lb}}{453.6 \text{ g}} \right) \left( \frac{16 \text{ oz}}{1 \text{ lb}} \right)
\]
The re-entry speed of the Apollo 10 space capsule was 11.0 km/s. How many hours would it have taken for the capsule to fall through 25.0 miles of the stratosphere? The unit analysis setup for this calculation is below. Identify whether each value in the setup is exact or not. Determine the number of significant figures in each inexact value, calculate the answer, and report it to the correct number of significant figures.

\[ ? \, \text{hr} = 25.0 \, \text{mi} \left( \frac{5280 \, \text{ft}}{1 \, \text{mi}} \right) \left( \frac{12 \, \text{in.}}{1 \, \text{ft}} \right) \left( \frac{2.54 \, \text{cm}}{1 \, \text{in.}} \right) \left( \frac{1 \, \text{m}}{102 \, \text{cm}} \right) \left( \frac{1 \, \text{km}}{11.0 \, \text{km}} \right) \left( \frac{1 \, \text{min}}{60 \, \text{s}} \right) \left( \frac{1 \, \text{hr}}{60 \, \text{min}} \right) \]

Rounding Off Answers Derived from Addition and Subtraction

The following sample study sheet provides a guide to rounding off numbers calculated using addition and subtraction.

**Tip-off** After calculating a number using addition and subtraction, you need to round it off to the correct number of decimal positions.

**General Steps**

**Step 1** Determine whether each value is exact, and ignore exact values (see Study Sheet 2.1).

**Step 2** Determine the number of decimal places for each value that is not exact.

**Step 3** Round your answer to the same number of decimal places as the inexact value with the fewest decimal places.

**Example** See Example 2.6.
A laboratory procedure calls for you to determine the mass of an unknown liquid. Let’s suppose that you weigh a 100-mL beaker on a new electronic balance and record its mass as 52.3812 g. You then add 10 mL of the unknown liquid to the beaker and discover that the electronic balance has stopped working. You find a 30-year-old balance in a cupboard, use it to weigh the beaker of liquid, and record that mass as 60.2 g. What is the mass of the unknown liquid?

**Solution**

You can calculate the mass of the liquid by subtracting the mass of the beaker from the mass of the beaker and the liquid.

\[
\text{60.2 g beaker with liquid} - \text{52.3812 g beaker} = 7.8188 \text{ g liquid}
\]

We can use the steps outlined in Sample Study Sheet 2.2 to decide how to round off our answer.

**Step 1** The numbers 60.2 and 52.3812 come from measurements, so they are not exact.

**Step 2** We assume that values given to us have uncertainties of ±1 in the last decimal place reported, so 60.2 has an uncertainty of ±0.1 g and 52.3812 has an uncertainty of ±0.0001 g. The first value is precise to the tenths place, and the second value is precise to four places to the right of the decimal point.

**Step 3** We round answers derived from addition and subtraction to the same number of decimal places as the value with the fewest. Therefore, we report our answer to the tenth’s place—rounding it off if necessary—to reflect this uncertainty. The answer is 7.8 g.

Be sure to remember that the guidelines for rounding answers derived from addition or subtraction are different from the guidelines for rounding answers from multiplication or division. Notice that when we are adding or subtracting, we are concerned with decimal places in the numbers used rather than with the number of significant figures. Let’s take a closer look at why. In Example 2.6, we subtracted the mass of a beaker (52.3812 g) from the mass of the beaker and an unknown liquid (60.2 g) to get the mass of the liquid. If the reading of 60.2 has an uncertainty of ±0.1 g, the actual value could be anywhere between 60.1 g and 60.3 g. The range of possible values for the mass of the beaker is 52.3811 g to 52.3813 g. This leads to a range of possible values for our answer from 7.7187 g to 7.9189 g.

\[
\begin{array}{c c c c}
60.1 \text{ g} & 60.3 \text{ g} \\
-52.3813 \text{ g} & -52.3811 \text{ g} \\
\hline
7.7187 \text{ g} & 7.9189 \text{ g}
\end{array}
\]

Note that our possible values vary from about 7.7 g to about 7.9 g, or ±0.1 of our reported answer of 7.8 g. Because our least precise value (60.2 g) has an uncertainty of ±0.1 g, our answer can be no more precise than ±0.1 g.
Use the same reasoning to prove that the following addition and subtraction problems are rounded to the correct number of decimal positions.

\[
\begin{align*}
97.40 + 31 &= 128 \\
1035.67 - 989.2 &= 46.5
\end{align*}
\]

Note that although the numbers in the addition problem have four and two significant figures, the answer is reported with three significant figures. This answer is limited to the ones place by the number 31, which we assume has an uncertainty of ±1. Note also that although the numbers in the subtraction problem have six and four significant figures, the answer has only three. The answer is limited to the tenths place by 989.2, which we assume has an uncertainty of ±0.1.

**Exercises 2.6 - Rounding Off Answers Derived from Addition and Subtraction**

Report the answers to the following calculations to the correct number of decimal positions. Assume that each number is ±1 in the last decimal position reported.

\[
\begin{align*}
a. \ 684 - 595.325 &= \\
b. \ 92.771 + 9.3 &= 
\end{align*}
\]

When people say that lead is heavier than wood, they do not mean that a pea-sized piece of lead weighs more than a truckload of pine logs. What they mean is that a sample of lead will have a greater mass than an equal volume of wood. A more concise way of putting this is that lead is more dense than wood. This type of density, formally known as mass density, is defined as mass divided by volume. It is what people usually mean by the term **density**.

\[
\text{Density} = \frac{\text{mass}}{\text{volume}}
\]

The density of lead is 11.34 g/mL, and the density of pinewood is about 0.5 g/mL. In other words, a milliliter of lead contains 11.34 g of matter, while a milliliter of pine contains only about half a gram of matter. See Table 2.2 on the next page for the densities of other common substances.

Although there are exceptions, the densities of liquids and solids generally decrease with increasing temperature\(^3\). Thus, when chemists report densities, they generally state the temperature at which the density was measured. For example, the density of ethanol is 0.806 g/mL at 0 °C but 0.789 g/mL at 20 °C.\(^4\)

\(^3\)The density of liquid water actually increases as its temperature rises from 0 °C to 4 °C. Such exceptions are very rare.

\(^4\)The temperature effect on the density of gases is more complicated, but it, too, changes with changes in temperature. This effect will be described in Chapter 13.