Simple Harmonic Motion

INTRODUCTION

The force that a spring exerts when it is stretched or compressed a distance x from its equilibrium length is given by Hooke's Law:

$$F_{\text{spring}} = -kx. \tag{1}$$

The quantity k is called the spring constant, or force constant. It is a property of the spring and is constant for a given spring. The minus sign indicates that the spring force is a restoring force: the force the spring exerts is always in the direction that brings the end of the spring back to its equilibrium position. If an object of mass M is suspended at rest from the lower end of a vertical spring that has unstretched length l_0, the spring will stretch enough for the upward force exerted on the object by the spring to equal the weight, Mg, of the object. If an additional mass ΔM is added at the lower end of the spring, the spring will stretch an additional amount Δl given by

$$k(\Delta l) = (\Delta M)g. \tag{2}$$

If an object of mass M is attached to the lower end of a light spring, pulled down to stretch the spring somewhat and then released, the object will oscillate up and down with a period T (the time for one complete cycle of the motion). If the mass of the spring is neglected then the period is given by

$$T = 2\pi\sqrt{\frac{M}{k}}. \tag{3}$$

SI units are kg for M, N/m for k and seconds for T. Note that 1 N/m $= \dfrac{1(\text{kg}\cdot\text{m/s}^2)}{\text{m}} = 1$ kg/s^2.

If the mass of the spring is not neglected, applying Newton's second law with the use of calculus shows that

$$T = 2\pi\sqrt{\frac{M + m_{\text{eff}}}{k}}. \tag{4}$$

m_{eff} is the "effective mass" of the spring and is given by $m_{\text{eff}} = \dfrac{m_{\text{spring}}}{3}$, where m_{spring} is the mass of the spring.

In this experiment we will measure the force constant of a spring using two different methods and we will compare the results. In the first method we measure the change in length of a vertical spring when additional mass is suspended at rest from its lower end. In the second method we will measure the period of oscillation for different total masses suspended from a vertical spring. In both procedures we will employ a graphical analysis of our data. We will also apply equation 4 to the second set of measurements in order to measure the effective mass of the spring and see if it is equal to one-third the spring mass.

OBJECTIVE

In this experiment we will study SHM (simple harmonic motion) by examining the motion of a mass that is attached to one end of a vertical spring.

APPARATUS

Stopwatch

Mass holder

Slotted mass set

Spiral spring

Support rod and base

Ruler (taped to support rod)

Balance

DISCUSSION

The experiment will use two different methods to measure the force constant, k, of a spring. In doing this, we will also test the validity of Hooke's Law and of the equation

$$T = 2\pi\sqrt{\frac{M + m_{\text{eff}}}{k}}$$

for the period of SHM.

PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

Method I: Calculating *k* From Hooke's Law

1 Hang the spring from the eye bolt at the top of the support rod. Hook the mass holder onto the bottom end of the spring and if necessary, add enough slotted masses to the mass holder to just separate the coils of the spring. Observe the position of the bottom of the mass holder on the ruler. Call this value y_0 and record its value.

2 Add *additional* mass, ΔM, to the mass hanger and let the mass hang at rest. Do this for each value of ΔM given in Table 1. Note the position, y, on the scale of the bottom of the mass holder. The change in length, Δl, of the spring in each case is $y - y_0$. Record your measured values of y for each ΔM in Table 1 and enter the quantity $\Delta l = y - y_0$. Note that in Table 1, ΔM is in grams and Δl is in cm. Convert your values of Δm to kg and your Δl to meters and record them in Table 2.

3 Open Excel and plot the data in Table 2, with Δl on the vertical axis and ΔM on the horizontal axis. Use Excel to find the slope of the straight line that best fits your data. According to the equation $k(\Delta l) = g(\Delta M)$, the vertical intercept of the line should be zero. If the vertical intercept of your line is not small or if your data does not fall close to a straight line, you have made an error in measuring, recording or plotting your data. If this is the case and you and your lab partner can't find what is wrong, ask your lab TA for help.

4 Record the numerical value of the slope of the line in your Excel graph.

5 Using $g = 9.80$ m/s^2 and the slope of the line, calculate k, the force constant of the spring. Since this value of k was determined using Method I, we will call it k_{I}. Record your result.

Method II: Calculating *k* By Measuring the Period

1 The mass of the mass holder is stamped on it. Record this value.

2 Hang the mass holder from the lower end of the spring and place a 40 gram slotted mass on the mass holder. Enter the total mass, M, the mass of the slotted mass plus the mass of the holder, in column 2 of Table 3. In column 3 enter the total mass converted to kg.

3 Displace the mass by about 3 cm downward from the equilibrium position and release to initiate simple harmonic motion. Choose a reference point (the lowest point in the motion works well) and start the stopwatch as the bottom of the loaded mass hanger moves past this point. Start counting with zero (not one) when the stopwatch starts and count each time the load passes the reference point, going in the same direction. Stop the stopwatch when 10 oscillations have been completed. Enter this time as $t_{10(1)}$ in column 4 of Table 3. Repeat the measurement with the other lab partner starting and stopping the stopwatch. Enter this time as $t_{10(2)}$. Calculate T, the period, as

$$T = \frac{t_{10(1)} + t_{10(2)}}{20} \tag{5}$$

and enter it into the table. Repeat the measurements and entries into Table 3 for each value of the slotted mass on the holder that is given for the rows of the table.

4 Now plot T^2 (in s^2) vs. M (in kg). Your data should fall close to a straight line. If it doesn't, then you have made an error in measuring, recording, or plotting your data. If this is the case and you and your lab partner can't find what is wrong, ask your lab TA for help.

5 Use Excel to calculate the slope of the straight line that is the best fit to your data and record that value.

6 Use the slope of the line in your Excel graph to calculate the force constant of the spring. Note that the calculation doesn't require knowing the value of g, the acceleration due to gravity at your location. Since this value of k was determined using Method II, we will call it k_{II}. Record your result.

7 The force constant of the spring has been measured two different ways. The average value of the two results is

$$k_{\text{av}} = \frac{k_{\text{I}} + k_{\text{II}}}{2}. \tag{6}$$

The percentage difference in the two values is given by

$$\%\ \text{diff} = \frac{|k_{\text{I}} - k_{\text{II}}|}{k_{\text{av}}} \times 100\%. \tag{7}$$

Record the percentage difference for your two values of k.

> If you have been careful with your measurements and if you have done the analysis correctly, the percentage difference between your two values of k should be less than 10%. If this is not the case, look over what you have done and ask your TA for help if you can't find where you went wrong.

Determining the Effective Mass of the Spring

1 Recall that

$$T = 2\pi\sqrt{\frac{M + m_{\text{eff}}}{k}}.$$

For the graph of your data for T^2 vs. M in Method II, what does Excel give for the y-intercept of the straight line that best fits your data? (The y-intercept is the value of T^2 when $M = 0$.) Record the value of the y-intercept.

2 From the value of the y-intercept and the value of the force constant k from Method II, calculate m_{eff} (the effective mass of the spring) and enter your result.

3 Use the balance to measure the mass m_{spring} of the spring. Calculate $\frac{m_{\text{spring}}}{3}$ and record it.

4 Recall that the percentage difference between two quantities is the magnitude of their difference divided by their average, converted to a percentage. Calculate the percentage difference between your results for m_{eff} and $\frac{m_{\text{spring}}}{3}$. Record your result.

This is a measure of how well your measured value of m_{eff} agrees with the theoretical value of $\frac{m_{\text{spring}}}{3}$.

