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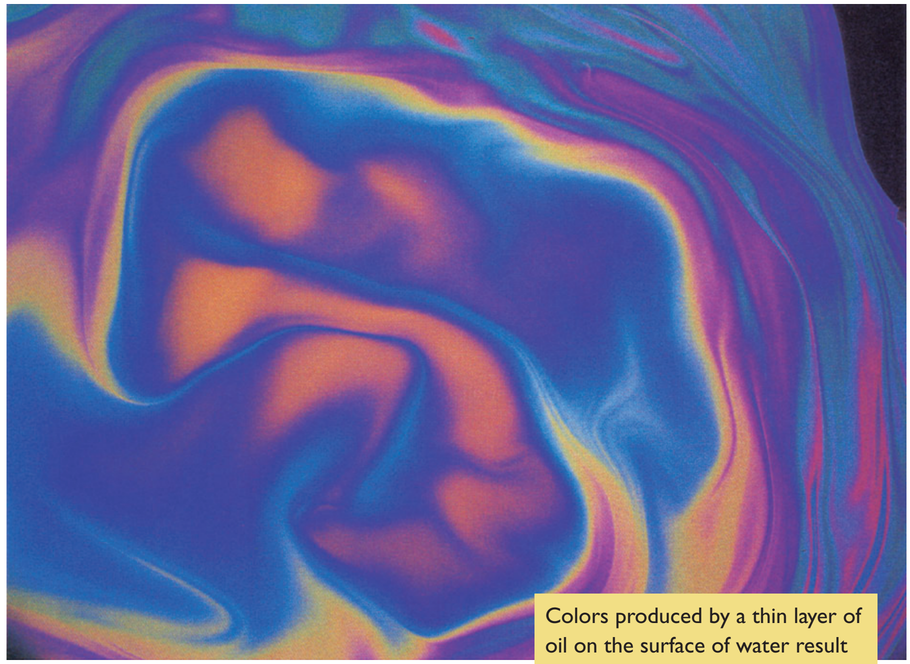
CHAPTER 26 Wave Optics



Colors produced by a thin layer of oil on the surface of water result from constructive and destructive interference of light.

Why is the sky blue? What causes the beautiful colors in a soap bubble or an oil film? Why are clouds and ocean surf white, though both are formed by tiny drops of clear, colorless water? Why do Polaroid sunglasses reduce reflected glare? In this chapter we shall answer these questions. Additionally, we shall see how two light beams can combine to produce darkness, we shall show how to measure the wavelength of light using a meter stick, and we shall see why the magnification of any optical microscope is limited by the wave properties of light.

We begin by giving a brief qualitative introduction to the wave phenomena of polarization, diffraction, and interference before returning to a more quantitative discussion of each. In our description of various experiments we shall often use a laser as our light source because of its wonderfully simple properties.

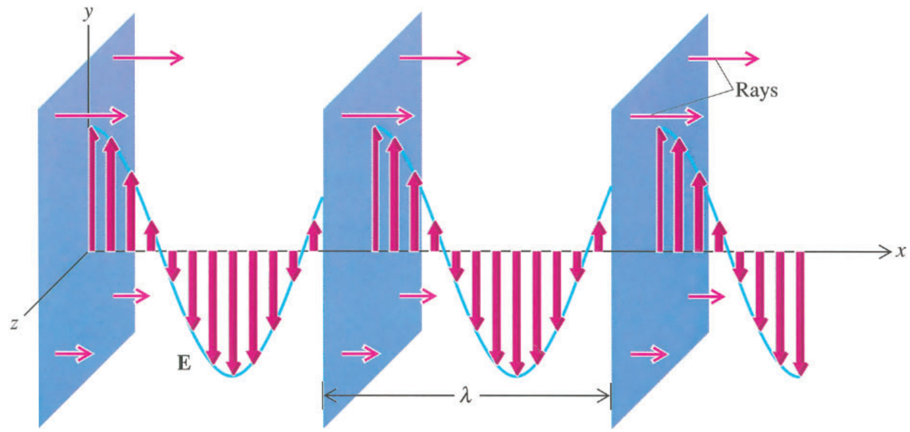


Fig. 26–1 The electric field in a linearly polarized, plane, monochromatic wave.

26–1 Wave Properties of Light

Polarization

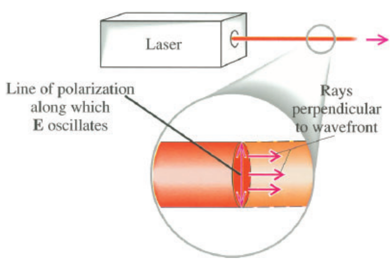


Fig. 26–2 A polarized laser beam.

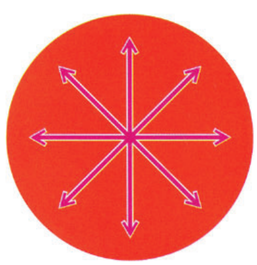


Fig. 26–3 Cross section of an unpolarized light beam.

The simplest kind of light wave is a plane, monochromatic wave, which is linearly polarized (Fig. 26–1). **Linear polarization means that the electric field vector is always directed parallel to a single line** (the y-axis in the figure). Fig. 26–1 shows only the electric field. An associated magnetic field is parallel to the z-axis and oscillates in phase with the electric field. Values of the electric field are shown at a particular instant of time for various points along the x-axis. In a plane wave the value of the electric field is the same along any plane perpendicular to the direction of the wave's motion. The figure shows plane wavefronts, along which the electric field is maximum. Rays show the direction of motion of the wave—along the x-axis.

The plane wave described in Fig. 26–1 is approximated by a section of a polarized laser beam (Fig. 26–2). Within the beam, rays are approximately parallel, and wavefronts are approximately cross sections of the beam.* By turning the laser on its side, rotating it 90°, we can produce a wave linearly polarized in the horizontal direction, rather than in the vertical direction. By rotating the laser through some other angle we can get polarization in any direction perpendicular to the beam.

Most natural light sources and many lasers have random polarization. This means that at a given instant the electric field at any point in the wave is just as likely to be directed along any line perpendicular to the direction of motion. The light is then said to be unpolarized, and we indicate this state as shown in Fig. 26–3.

Frequency Bandwidths

Any real source of light is not exactly monochromatic; that is, there is never just one precise value of frequency. Instead there is a range or band of frequencies, which may be wide or narrow. The narrower the band, the more nearly the wave approximates a monochromatic wave. Laser light is nearly monochromatic. A common helium-neon laser emits light at a frequency of 4.74×10^{14} Hz with a bandwidth of about 10^8 Hz. This means that the frequency range is less than 1 part in 10^6. More expensive, frequency-stabilized lasers have bandwidths as low as 10^4 Hz. Some lasers have even achieved a stabilized frequency range of less than 100 Hz.

*There is some slight spreading of the beam, and hence the beam is not exactly a plane wave. A plane wave is also approximated by a small section of a spherical wave from a distant point source.

By way of comparison, spectral lines emitted by various gas discharge tubes typically have bandwidths of roughly 10^9 Hz, and white light, ranging in frequency from 4×10^{14} Hz to 7×10^{14} Hz, has a bandwidth of 3×10^{14} Hz. The order of magnitude of these bandwidths is summarized in Table 26–1.

Table 26–1 Frequency bandwidths typical of various kinds of light

Light	Δf (Hz)
Stabilized He-Ne laser	10^4
Common He-Ne laser	10^8
Spectral line	10^9
White light	10^{14}

Coherence

It is often important to be able to predict the relationship between the phase of a light wave at two different times at the same point in space. For example, suppose that at some instant t_0, at one point in a laser beam, the electric field vector has its maximum value; that is, you are at a peak in the wave. At a time Δt later, will the wave again have its peak value or will it have some other value (Fig. 26–4)? If the laser light were truly monochromatic, the solution would be easy. We could simply determine the exact number of cycles elapsed in a given time interval Δt by multiplying the frequency (the number of cycles per second) by the time interval Δt. (If the result were a whole number, the wave would again be at a peak. Or if the result were a whole number plus $\frac{1}{4}$, the electric field would be zero at that instant.) However, even laser light is not exactly monochromatic. There is always some frequency range f to $f + \Delta f$. The number of cycles per second during any particular time interval can be anywhere in this range. So the number of cycles completed during a time interval Δt is somewhere in the range $f\,\Delta t$ to $(f + \Delta f)\,\Delta t$. If the time interval Δt is small enough, the product $\Delta f\,\Delta t$ will be much less than 1 cycle, and there will be little uncertainty in the number of cycles completed, or in the final phase of the cycle. We can then predict the final phase from the initial phase, and we say that the wave is coherent over the time interval Δt. This means that there is a definite, predictable phase relationship. The condition for coherence then is that the time interval be small enough that

$$\Delta f\,\Delta t << 1$$

or that

$$\Delta t << \frac{1}{\Delta f} \qquad \text{(condition for coherence)} \qquad (26\text{–}1)$$

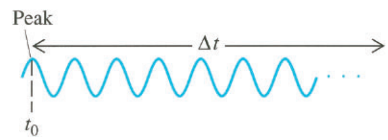


Fig. 26–4 What phase of the cycle occurs at a time Δt after t_0?

EXAMPLE 1 Coherence of Light Sources

Will you have coherence over a time interval of 10^{-6} s for light from (a) a gas discharge tube; (b) a stabilized He-Ne laser?

SOLUTION **(a)** For a single line from a gas discharge tube, we see from Table 26–1 that even for a single spectral line $\Delta f = 10^9$ Hz. Thus $1/\Delta f = 10^{-9}$ s, and the time interval $\Delta t = 10^{-6}$ s is much too long to satisfy Eq. 26–1, since 10^{-6} s is certainly not less than 10^{-9} s. Thus this kind of light is not coherent over such a time interval. The number of cycles completed is uncertain by $\Delta f\,\Delta t = (10^9\ \text{s}^{-1})(10^{-6}\ \text{s}) = 10^3$ cycles, and so there is no ability to predict the phase over such a time interval.

(b) For light from a stabilized He-Ne laser, $\Delta f = 10^4$ Hz. Thus the condition that must be satisfied for such light is

$$\Delta t << \frac{1}{\Delta f}$$

or

$$\Delta t << 10^{-4}\ \text{s}$$

A time interval of 10^{-6} s satisfies this condition. Thus the laser light is coherent over this time interval. Notice that $\Delta f \Delta t = (10^4\ \text{s}^{-1})(10^{-6}\ \text{s}) = 10^{-2}$, and so we know the number of elapsed cycles with an uncertainty of only one hundredth of a cycle.

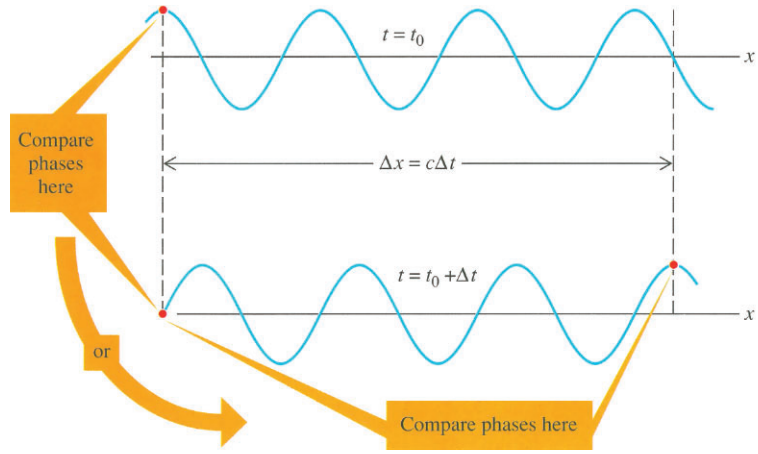


Fig. 26–5 During a time interval Δt a wave peak advances a distance $\Delta x = c\,\Delta t$. Thus comparing the phases at two points a distance Δx apart at a fixed time is the same as comparing the phases at the same point in space over a time interval Δt.

Each wavefront in a light wave advances at the speed of light. Therefore, we can relate the coherence of light at a fixed point in a plane wave at two different times to coherence at two different points in a plane wave at the same time. As illustrated in Fig. 26–5, during a time interval Δt a wavefront advances a distance $\Delta x = c\,\Delta t$, and so comparison of phases at two points Δx apart is equivalent to comparing the phases at a fixed point over a time interval Δt. Since $\Delta t << 1/\Delta f$ is the condition for coherence, two points in a wavefront will be coherent if

$$\Delta x << \frac{c}{\Delta f}$$

The distance $c/\Delta f$ is called the **coherence length,** denoted by x_c.

$$x_c = \frac{c}{\Delta f} \qquad (26\text{–}2)$$

The condition for coherence may be expressed in terms of x_c:

$$\Delta x << x_c \qquad \text{(coherence condition)} \qquad (26\text{–}3)$$

For a stabilized He-Ne laser with a frequency range of 10^4 Hz, we find

$$x_c = \frac{c}{\Delta f} = \frac{3 \times 10^8 \text{ m/s}}{10^4 \text{ Hz}} = 3 \times 10^4 \text{ m}$$

Two points in the laser beam have a predictable phase relationship, as long as they are much less than 30,000 m apart! For a common laboratory He-Ne laser, the bandwidth is of the order of 10^8 Hz. Thus

$$x_c = \frac{c}{\Delta f} = \frac{3 \times 10^8 \text{ m/s}}{10^8 \text{ Hz}} = 3 \text{ m}$$

The two points must be much closer than 3 m. Certainly two points a few cm apart are coherent. For white light,

$$x_c = \frac{c}{\Delta f} = \frac{3 \times 10^8 \text{ m/s}}{10^{14} \text{ Hz}} = 3 \times 10^{-6} \text{ m}$$

Thus points in a beam of white light must be considerably less than a thousandth of a millimeter apart to be coherent.

Diffraction

Suppose you are standing behind an open doorway, listening to a conversation in the next room. You can easily hear the voices from the room because the sound waves bend around the doorway. This phenomenon is called **diffraction.** It is a property common to all waves to bend or diffract around an obstacle. However, the amount of bending depends on the wavelength of the wave and the dimensions of the obstacle. In general, the longer the wavelength, the greater is the diffraction. Light, with its relatively short wavelength, bends or diffracts very little around an open doorway, but sound waves with their much longer wavelengths, diffract a great deal. Thus you can hear the conversation though you cannot see those who are talking.

Seeing diffraction of light requires careful observation. Suppose we pass an intense beam of light through a narrow slit in an opaque screen and project it onto a white screen (Fig. 26–6). If the slit is relatively wide, (say, at least a millimeter), we get an image of the slit on the screen (Fig. 26–6a). As predicted by geometrical optics, the rays passing through the slit travel straight to the screen. But if we make the slit very narrow (say, less than about 0.1 mm), the image on the screen actually gets wider (Fig. 26–6b), violating the prediction of geometrical optics. We find that the narrower we make the slit, the more the light bends outward. Of course if we make the slit much less than 0.1 mm, there will be too little light to be seen, even if the light source illuminating the slit is very intense. But if we could make a slit with a width much less than the wavelength of light and still have enough light intensity to see the small amount of light passing through, we would see the light spread out in all directions, forming a cylindrical wave. Or if we replaced the slit by a tiny circular hole, with a diameter much less than the wavelength of light, we would produce a spherical wave. This result, illustrated in Fig. 26–7, is the essence of the **Huygens-Fresnel principle,** according to which **each section of wavefront in the diffracting aperture is the source of a spherical wave.** Fig. 26–8 shows a photograph, illustrating this principle for water waves. The waves in Fig. 26–8 are incident on an aperture much smaller than the wavelength.

Diffraction of light was first observed and recorded by the Jesuit priest Francesco Grimaldi, a contemporary of Newton. Grimaldi observed the spreading of a narrow beam of sunlight entering a darkened room. Geometrical optics, which was then based on a picture of light consisting of particles,* could not account for this phenomenon, and so Grimaldi proposed that light is a wave. Grimaldi's idea was rejected by Newton, who believed that if light were a wave, the diffraction effect would be much greater than observed. It must have seemed unlikely to Newton that light could have the incredibly small wavelength necessary to explain such a small amount of diffraction. Newton's authority was so great that the wave theory was not accepted for another 200 years.

Interference

Like sound waves or waves on a string, light waves can interfere constructively or destructively. Constructive interference occurs when two light waves are in phase, and destructive interference occurs when two light waves are 180° out of phase. The colors in the photograph of the oil slick at the beginning of this chapter result from interference of the light reflected from the top and bottom surfaces of the thin film of oil. The thickness of the film varies, and, as a result, different colors of light interfere constructively.

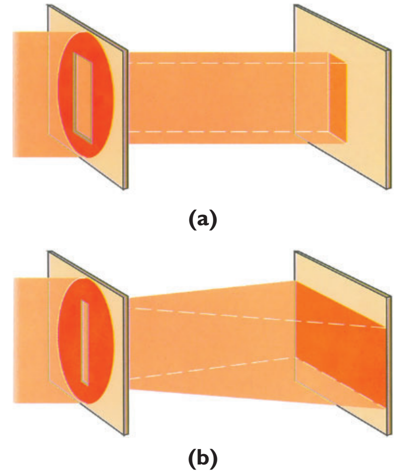


Fig. 26–6 **(a)** Little diffraction is produced by a slit wider than 1 mm. **(b)** Considerable diffraction is produced by a slit less than 0.1 mm wide.

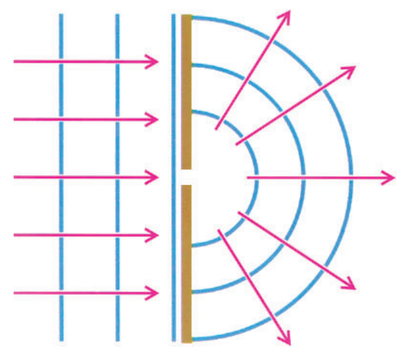


Fig. 26–7 Cross section of a spherical wave, resulting from diffraction of light by a circular hole with a diameter much smaller than the wavelength of the light.

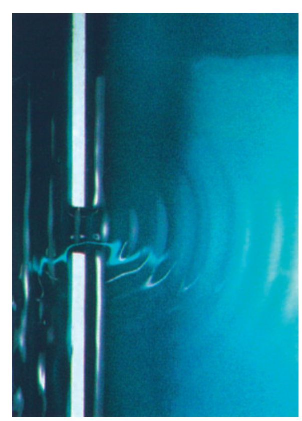
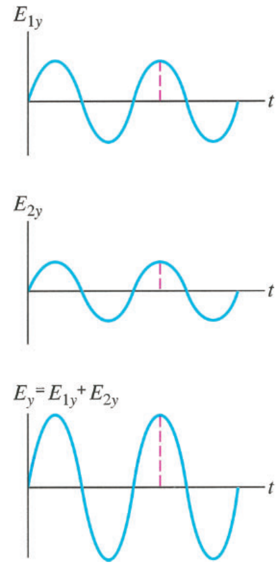


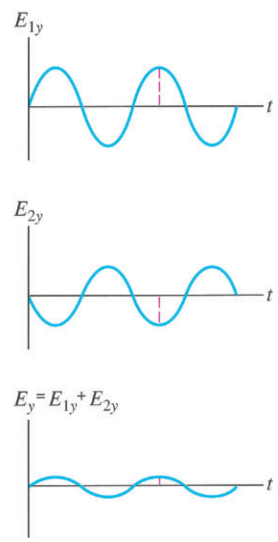
Fig. 26–8 Diffraction of water waves.

*In the twentieth century it was discovered that light does indeed have some particle-like properties. However this modern idea of a photon as a "particle" of light refers to emission or absorption of light, not to the way it propagates. Light travels as a wave, not as a bunch of particles, contrary to Newton's belief. The dual character of light as wavelike (in transmission) and particle-like (in absorption and emission) is at the heart of modern quantum physics, to be discussed in Chapter 28.

For example, where the film appears blue the thickness is such that blue light reflected from the two surfaces interferes constructively, while red light interferes destructively. In the sections that follow we shall investigate interference and diffraction phenomena quantitatively.



(a) Constructive interference



(b) Destructive interference

Fig. 26–9 Two light waves can interfere either **(a)** constructively or **(b)** destructively.

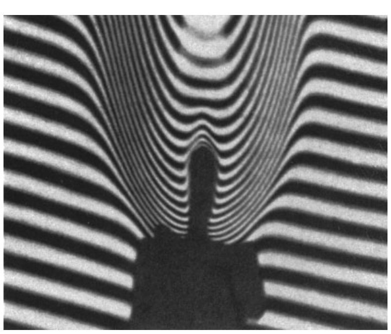
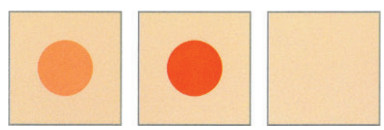


Fig. 26–11 An interference pattern.

26–2 Interference

Fig. 26–9a illustrates **constructive interference** of light, which occurs when two light waves are in phase at a certain point in space over a period of time. Fig. 26–9b shows **destructive interference,** which occurs at a point in space where the waves are 180° out of phase over a period of time. If the two waves are of equal amplitude and 180° out of phase, the presence of two sources of light actually produces darkness! Fig. 26–10 illustrates the intensity of light that is seen at a given point in space where two waves of equal amplitude interfere either constructively or destructively. Since the intensity of a wave is proportional to the square of its amplitude, constructive interference of two equal-amplitude waves produces in the resultant wave 2 times the amplitude or 4 times the intensity of the individual waves. Whether the interference is constructive or destructive at a given point in space depends on the position of the point relative to the sources of light. Interference of light typically produces a pattern of light and dark areas (Fig. 26–11).



(a) (b) (c)

Fig. 26–10 Light seen on a screen at a point in space where: **(a)** There is one light wave of intensity I; **(b)** There are two light waves, each of intensity I, interfering constructively and producing a total intensity $4I$; **(c)** There are two light waves, each of intensity I, interfering destructively and producing no light.

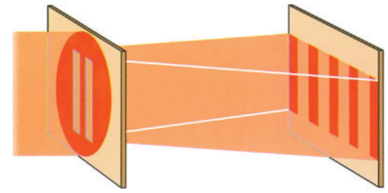
In order for the eye to perceive interference of light, there must be a definite phase relationship between the two waves over a time interval that the eye can detect. The eye has a response time on the order of $\frac{1}{20}$ of a second. Thus interference effects must be stable for at least this long to be visible. This is longer than the coherence time of even the most monochromatic sources available today. The relative phases of two independent light sources (say, two different lasers) will vary randomly over time intervals greater than the coherence time. Thus, if we illuminate an area with two different sources, the interference of their light waves at a given point will rapidly oscillate from constructive to destructive, and so no interference pattern is visible. All one sees is a uniform illumination equal to the sum of the two intensities. For example, two equal-amplitude waves from separate sources produce instantaneous intensities rapidly oscillating between 0 and 4 times each wave's intensity I, at each point in space. Thus one sees only the time average of the instantaneous intensity, which at all points is the same: twice the intensity of each source's wave, since the average of 0 and $4I$ equals $2I$. With the present state of technology, it is impossible to see interference of light from two independent light sources.*

*It is possible to detect electronically interference of two independent sources, as demonstrated by Brown and Twiss in 1952.

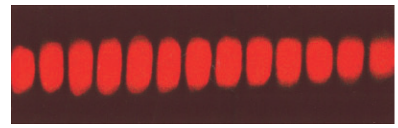
Interference effects are easily observable when a single wavefront is divided into two separate parts, which then follow separate paths to the point where interference is observed. Since the two waves arise from a common wavefront, changes in phase are common to each, and the interference pattern remains constant.

Young's Double Slit

One of the simplest ways to produce interfering light is to use a double-slit arrangement like the one studied by the English physician Thomas Young in 1801.* If a monochromatic plane wave is incident on a pair of thin, closely spaced slits, the two slits serve as sources of coherent light. The slits must be narrow enough and close enough that there is a significant amount of diffraction and overlap of the two wavefronts. As illustrated in Fig. 26–12, what is seen on a screen in front of the slits is a pattern of alternating light and dark fringes. Fig. 26–13 shows how the location of the fringes relates to the distance to each of the slits. Point P is equidistant from the two slits, and so the two waves are exactly in phase at this point, and therefore point P is at the center of an interference maximum—a bright fringe. The first dark fringe above the central bright fringe is at a point Q, which is one-half wavelength farther from slit 2 than from slit 1. The next bright fringe is at point R, which is 1 wavelength farther from slit 2 than from slit 1.



(a)



(b)

Fig. 26–12 **(a)** Overlapping wavefronts interfere constructively at certain points and destructively at others. (Figure is not drawn to scale.) **(b)** Photograph of interference fringes from double slits illuminated with a He-Ne laser.

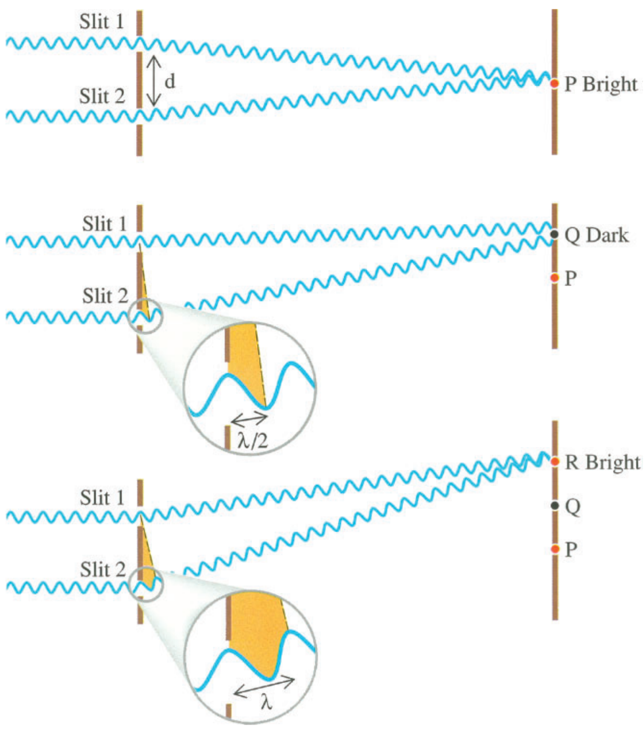


Fig. 26–13 Constructive interference occurs at P and R. Destructive interference occurs at Q. (Figures are not drawn to scale.)

*In 1801 Young used his double-slit experiment to measure the wavelength of light and to provide support for his belief that light is a wave. His ideas gained wide acceptance only after many years.

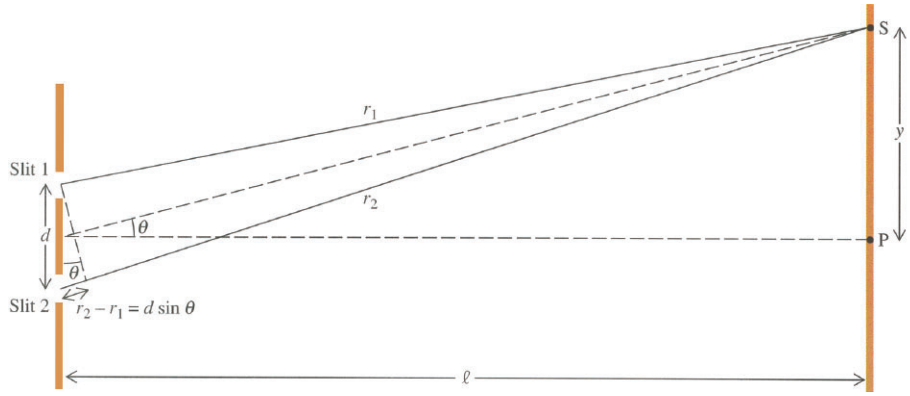


Fig. 26–14 A point S on a screen is located a distance r_1 from slit 1 and a distance r_2 from slit 2. (This figure is not drawn to scale.)

The location of the fringes can be determined with the aid of Fig. 26–14. The figure shows an arbitrary point S some distance y from the center of the interference pattern at point P. The angular displacement of point S is measured by the angle θ. The difference in the path lengths from S to each of the slits is related to the same angle θ. As shown in the figure, this path-length difference is $d \sin \theta$. Constructive interference occurs when this distance equals a whole number of wavelengths:

$$d \sin \theta = m\lambda \quad m = 0, 1, 2, \ldots \quad \text{(constructive interference)} \quad (26\text{–}4)$$

Destructive interference occurs when the difference in path lengths equals a whole number of wavelengths plus $\frac{1}{2}$ wavelength:

$$d \sin \theta = (m + \tfrac{1}{2})\lambda \quad m = 0, 1, 2, \ldots \quad \text{(destructive interference)} \quad (26\text{–}5)$$

EXAMPLE 2 Measuring Light's Wavelength With a Meter Stick

Light from a He-Ne laser illuminates two narrow slits, 0.20 mm apart, producing interference fringes on a wall 6.67 m from the slits (Fig. 26–12). The centers of the bright fringes are 2.1 cm apart. (a) Determine the wavelength of the laser light. (b) What would the fringe separation be if the slits were illuminated with violet light of wavelength 400 nm?

SOLUTION **(a)** From Fig. 26–14, we see that the distance y from the center of the interference pattern to any point S is related to the angle θ and the distance ℓ from slits to screen:

$$\tan \theta = \frac{y}{\ell}$$

Any point in the interference pattern is at a very small angle θ, for which $\sin \theta$ and $\tan \theta$ are very nearly identical. Thus

$$\sin \theta \approx \frac{y}{\ell}$$

Inserting this equation into Eq. 26–4 yields an expression for the distance y_m to the mth fringe

$$d\frac{y_m}{\ell} = m\lambda \quad m = 0, 1, 2, \ldots$$

or

$$y_m = \frac{m\lambda\ell}{d}$$

EXAMPLE 2 Measuring Light's Wavelength With a Meter Stick—Continued

Thus

$$y_0 = 0, \quad y_1 = \frac{\lambda \ell}{d} \quad y_2 = \frac{2\lambda \ell}{d}, \ldots$$

The fringes are equally spaced, separated by a distance

$$\Delta y = \frac{\lambda \ell}{d} \tag{26–6}$$

Solving for λ, we find

$$\lambda = \frac{d\,\Delta y}{\ell} = \frac{(2.0 \times 10^{-4}\text{ m})(2.1 \times 10^{-2}\text{ m})}{6.67\text{ m}}$$

$$= 6.3 \times 10^{-7}\text{ m}$$

$$= 630\text{ nm}$$

Thus, using measurements in cm and m, we indirectly measure the wavelength of the laser light, a length less than a thousandth of a mm.

(b) Applying Eq. 26–6, we find that for violet light of wavelength 400 nm, the fringe spacing changes to

$$\Delta y = \frac{\lambda \ell}{d} = \frac{(400 \times 10^{-9}\text{ m})(6.67\text{ m})}{2.0 \times 10^{-4}\text{ m}} = 1.3 \times 10^{-2}\text{ m}$$

$$= 1.3\text{ cm}$$

Because of its shorter wavelength, violet light produces interference fringes that are closer together.

Thin Films

There is another simple way to split a single light wave into separate, coherent waves, which can then interfere. When light is incident on a partially reflecting surface (for example, a glass surface), part of the incident light is reflected and part is transmitted into the second medium. If the two light waves again come together, after having followed paths of somewhat different length, they will interfere. The difference in path lengths, however, must be less than the coherence length of the light, or the two waves will be totally incoherent. The Michelson interferometer, described in Problem 21, uses a half-silvered mirror to equally divide the incident wavefront from a monochromatic source.

A thin transparent film can also serve to produce two coherent light waves, one reflected from the top and one from the bottom of the film (Fig. 26–15). If the film is very thin, you can see the interference, even with a white light source, as indicated by the photo of the colored oil film at the beginning of this chapter.

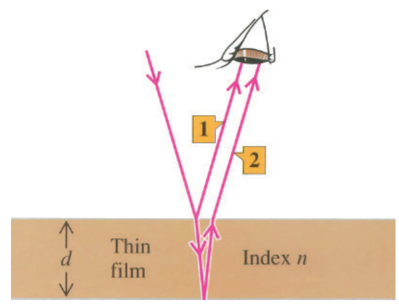


Fig. 26–15 The top and bottom surfaces of a thin film reflect light upward.

In Fig. 26–15 the second light wave travels a distance greater than the first. The difference in the path lengths may cause a phase difference between the two waves. In addition, there may also be phase changes produced by the reflections. **When light is incident on the surface of a medium with a higher refractive index than that of the incident medium, reflected light experiences a 180° phase change.** If the second medium has a lower index than the first, reflection causes no phase change. The situation is the same as that of a wave pulse on a rope, partially reflected because of a change in density of the rope. As illustrated in Chapter 16, Fig. 16–11, the reflected pulse is inverted if the second section of rope has higher density than the first. If the second section is of lower density than the first, there is no inversion of the reflected pulse, that is, no change in phase.

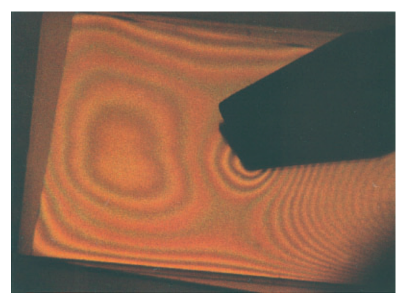


Fig. 26–16 An interference pattern is formed by reflection from the thin film of air between two microscope slides.

The microscope slides in Fig. 26–16 show a pattern of interference fringes produced by light reflected on either side of the thin film of air trapped between the slides. The air film varies in thickness, and hence the interference alternates between constructive (bright) and destructive (dark).

EXAMPLE 3 Colors On An Oil Slick

What color will be brightest when white light is reflected at normal incidence from a film of oil 250 nm thick on the surface of a puddle of water? The oil has a refractive index of 1.4.

SOLUTION The light reflected from the upper surface of the oil film undergoes a 180° phase change, since oil's refractive index is higher than air's (Fig. 26–17). The light reflected from the lower surface of the oil experiences no phase change, since water's index is lower than oil's. Thus, if light reflected from the lower surface is to arrive at the upper surface in phase with the light reflected from the top surface, the extra distance traveled must give it a net 180° phase change. This means that the additional path length, equal to twice the film's thickness t for normal incidence, must equal a whole number of wavelengths plus $\frac{1}{2}$ wavelength.

$$2t = (m + \tfrac{1}{2})\lambda_n \quad \text{where } m = 0, 1, 2, \ldots \qquad (26\text{–}7)$$

The wavelength λ_n is of course the wavelength in the oil. This is related to the vacuum wavelength λ_0 by Eq. 23–10:

$$\lambda_n = \frac{\lambda_0}{n}$$

where n is the refractive index of the oil. Inserting this expression into Eq. 26–7 and solving for λ_0, we find

$$\lambda_0 = \frac{2tn}{\mathrm{m} + \frac{1}{2}} \quad \text{for } m = 0, 1, 2, \ldots \qquad (26\text{–}8)$$

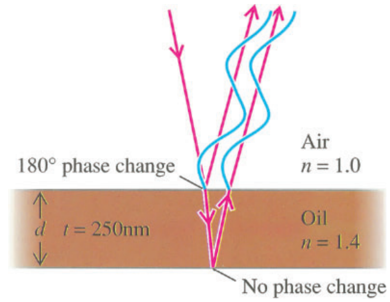


Fig. 26–17

Trying possible values of m, we find

$$m = 0: \quad \lambda_0 = \frac{2tn}{\frac{1}{2}} = \frac{2(250\text{ nm})(1.4)}{\frac{1}{2}} = 1400\text{ nm}$$

$$m = 1: \quad \lambda_0 = \frac{2tn}{\frac{3}{2}} = \frac{1400\text{ nm}}{3} = 470\text{ nm}$$

$$m = 2: \quad \lambda_0 = \frac{2tn}{\frac{5}{2}} = \frac{1400\text{ nm}}{3} = 280\text{ nm}$$

Only for $m = 1$ do we find a wavelength in the visible range (400 to 700 nm). Blue light of wavelength 470 nm will interfere constructively, and therefore blue will be the color most strongly reflected by the film. The film will appear blue. One can show that red light will experience destructive interference (Problem 14).

EXAMPLE 4 Nonreflective Glass Coating

Uncoated glass reflects 4% of the light incident on its surface at normal incidence. Sometimes glass is coated with a thin layer of a transparent material so that the intensity of the reflected light is reduced. Find the minimum thickness of a coating of magnesium fluoride, MgF_2 ($n = 1.38$), which will produce destructive interference at a wavelength in the middle of the visible spectrum (550 nm).

SOLUTION Both reflected waves experience a 180° phase change, since both are reflected from a medium with a higher index than that of the incident medium (Fig. 26–18). The only relative change in phase results from a difference in path length. Destructive interference occurs for a minimum path difference of $\frac{1}{2}$ wavelength.

$$2t = \tfrac{1}{2}\lambda_n = \frac{\lambda_0}{2n}$$

or

$$t = \frac{\lambda_0}{4n} = \frac{550\text{ nm}}{4(1.38)} = 99.6\text{ nm}$$

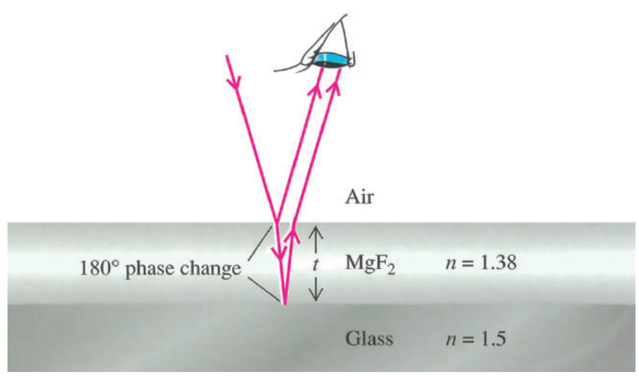


Fig. 26–18

EXAMPLE 4 Nonreflective Glass Coating—Continued

This is the minimum thickness of MgF_2 that will produce destructive interference for $\lambda_0 = 550$ nm. Such a layer will not completely eliminate reflection at this wavelength, since the amplitudes of the two interfering waves are not equal, and so there is only partial cancellation of the waves, as in Fig. 26–9b.

One might wonder whether this coating will produce a reduction in reflected intensity at other wavelengths or whether it could perhaps enhance reflection by constructive interference at some wavelengths. The condition for constructive interference here is that the path-length difference equals a whole number of wavelengths.

$$2t = m\lambda_n = m\frac{\lambda_0}{n}$$

or

$$\lambda_0 = \frac{2tn}{m} \quad \text{where } m = 1, 2, 3, \ldots$$

$$m = 1: \quad \lambda_0 = 2tn = 2(99.6\text{ nm})(1.38) = 275\text{ nm}$$

$$m = 2: \quad \lambda_0 = \frac{2tn}{2} = 138\text{ nm}$$

Other values of m yield smaller wavelengths. Thus no value of m gives a wavelength in the visible range. No visible light interferes constructively. A more detailed analysis, which takes into account the intensities of the interfering waves, shows that the effect of the coating is to reduce reflection fairly uniformly across the visible spectrum to an average of about 1% of incident intensity, compared to a 4% reflection for uncoated glass. However, there is a slight enhancement of reflected intensity in the blue part of the spectrum. Coating glass with several thin layers of different materials, carefully selected for index and thickness, can provide a further reduction in intensity. One of the most important applications of such coatings is for lenses in optical instruments, which use a large number of lenses that would otherwise produce much unwanted reflected light. Fig. 26–19 shows eyeglasses that have a nonreflective coating on one lens.

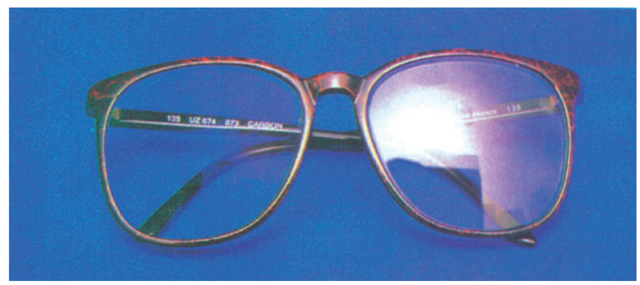


Fig. 26–19

26–3 Diffraction

Historical Background

The quantitative study of diffraction was of great historical importance in establishing the wave nature of light. Although Young's double-slit experiment supported the wave theory, many nineteenth-century scientists clung to Newton's particle theory of light. Full acceptance of the wave theory followed careful quantitative studies of diffraction by various scientists, especially Fresnel and Arago. In evaluating the wave theory of diffraction proposed by Fresnel, Poisson objected that it led to a rather strange and, to Poisson, an obviously false prediction: that at the center of the shadow of a round object would be a bright spot. Poisson argued that waves diffracted around the edges would travel an equal distance to the center and therefore interfere constructively there, if the wave theory were correct. Poisson presented this argument as a proof that the wave theory was wrong. Arago promptly performed the crucial experiment and found the predicted bright spot at the center of the shadow (Fig. 26–20). Based on such results, the wave theory of light was strongly established by 1820.

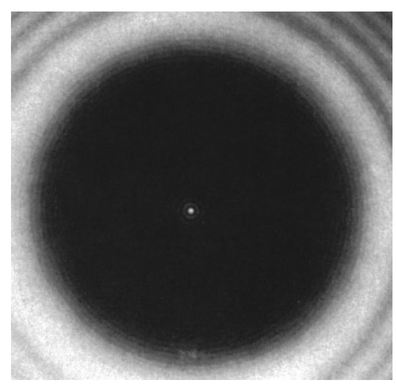


Fig. 26–20 Diffraction pattern of a penny. Constructive interference of light diffracted around the edge of the penny produces a bright spot at the center of the shadow.

Fraunhofer and Fresnel Diffraction

When an object producing diffraction is illuminated by a plane wave and the resulting diffraction pattern is viewed on a screen at a large enough distance from the object, detailed analysis of the diffraction pattern is greatly simplified. This case is referred to as "Fraunhofer diffraction." When the illuminating source is not a plane wave or the screen is not far enough away, the diffraction is called "Fresnel diffraction." Analysis of Fresnel diffraction is more complicated than analysis of Fraunhofer diffraction, and the Fresnel diffraction pattern itself looks quite different from the Fraunhofer diffraction pattern of the same object. Figs. 26–20 and 26–21 are examples of Fresnel diffraction. We shall analyze only Fraunhofer diffraction because of its relative simplicity.

Single Slit

A particularly simple case, Fraunhofer diffraction by a single slit, is shown in Fig. 26–22. In Section 26–1 we described diffraction of light around the edges of the slit (Fig. 26–6b). Actually the phenomenon is somewhat more complicated because light coming from various parts of the slit interferes to form a series of light and dark bands, as shown in Fig. 26–22.

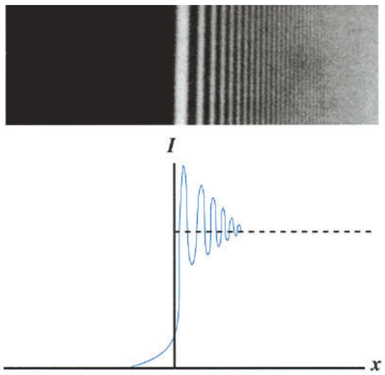


Fig. 26–21 Diffraction by a straight edge and the corresponding graph of intensity versus position. The dashed line indicates the intensity predicted by geometrical optics, implying a sharp-edged shadow.

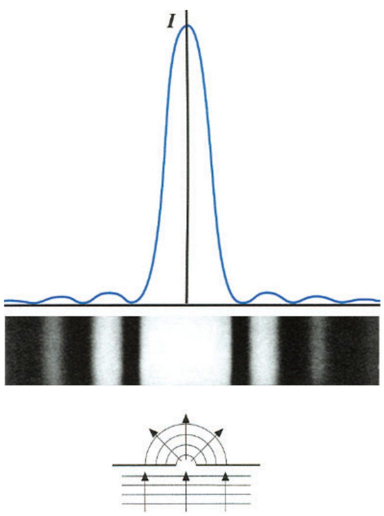
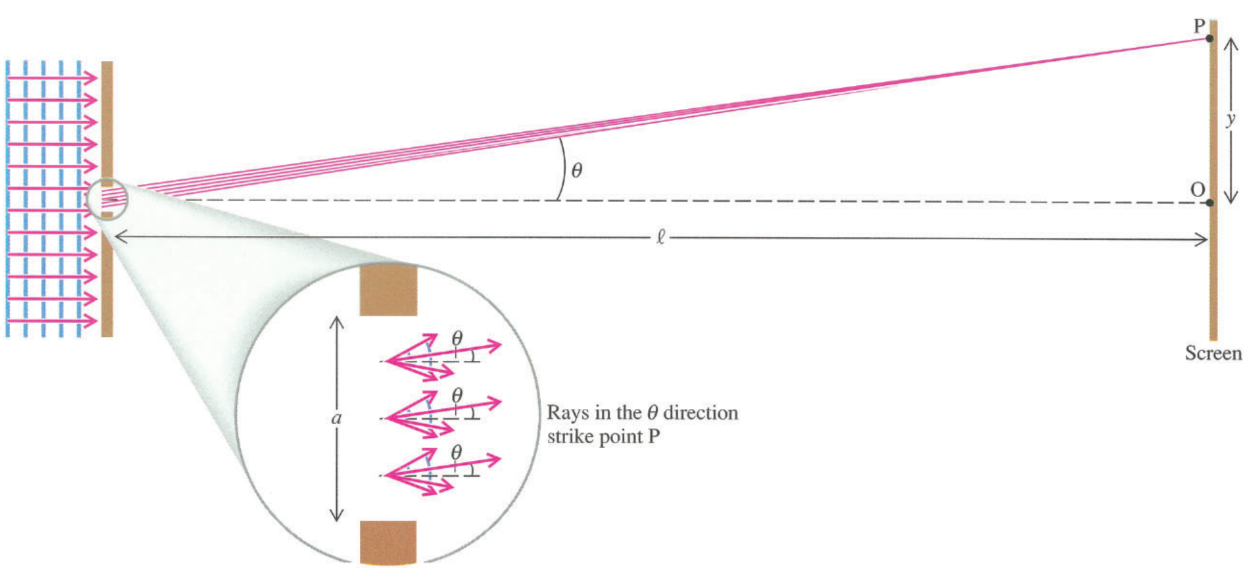


Fig. 26–22 Single-slit diffraction. Light diffracted by the slit forms a series of light and dark bands on a distant screen.

Fig. 26–23 Rays emanate from points in the opening, a thin slit of width a. All rays in the θ direction strike the screen at point P. (This figure is not drawn to scale.)



We can use the **Huygens-Fresnel principle** to analyze single-slit diffraction. According to this principle, **each section of a wavefront in the diffracting aperture is the source of a spherical wavelet. The amplitude of the light wave at any point beyond the aperture is the superposition of all these wavelets.** Fig. 26–23 shows a plane wave incident on a narrow slit and a distant screen for viewing the resulting diffraction pattern. An enlarged section of the figure shows the spherical wavelets and associated rays emanating from several points in the opening. Rays striking the screen at point P interfere constructively or destructively, depending on the relative phases of the waves. Since P is at a great distance, the rays reaching P are nearly parallel. These rays form an angle θ with a line drawn to point O, which is directly opposite the slit.

Fig. 26–24 shows rays directed at the center of the diffraction pattern ($\theta = 0$). Since these parallel rays travel equal distances to the screen, interference is constructive, and the center of the pattern is therefore a diffraction maximum.

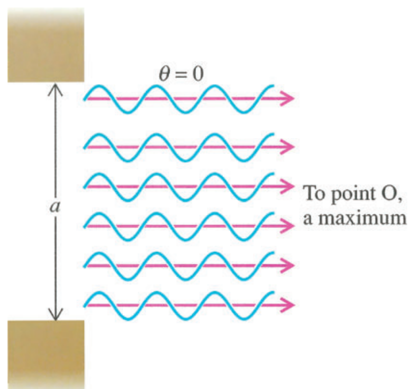


Fig. 26–24 Rays in the $\theta = 0$ direction strike the screen at point O, directly in front of the slit.

Next we shall locate the diffraction minima produced by a single slit. We can use Fig. 26–25 to find the values of θ corresponding to diffraction minima. The first minimum is indicated in Fig. 26–25a, where the slit of width a has been divided into two halves. We compare *any* ray from the top half with a ray a distance $a/2$ below it. Let the difference in path length of two such rays be $\lambda/2$, so that they interfere destructively. As shown in the figure this occurs for rays at an angle θ, where

$$a \sin \theta = \lambda$$

Since *all* rays from the top half of the slit can be paired with a cancelling ray from the bottom half, this angle corresponds to a diffraction minimum. A similar argument can be applied to find other diffraction minima. In Fig. 26–25b the slit is divided into 4 segments, and it is shown that another diffraction minimum occurs at an angle θ, such that

$$a \sin \theta = 2\lambda$$

In Fig. 26–25c, the slit is divided into 6 segments, and a diffraction minimum is found at an angle θ, where

$$a \sin \theta = 3\lambda$$

In general, single-slit diffraction minima occur at any angle θ satisfying the equation

$$a \sin \theta = m\lambda \qquad m = 1, 2, 3, \ldots \qquad \text{(diffraction minima)} \qquad (26\text{–}9)$$

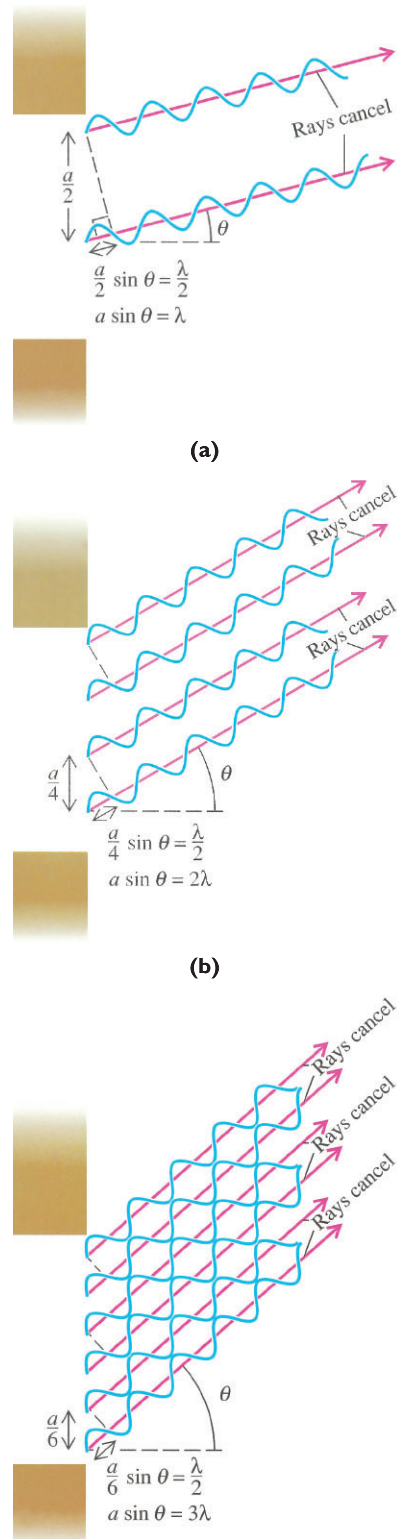


Fig. 26–25 Locating the **(a)** first, **(b)** second, and **(c)** third diffraction minima for a single slit of width *a*.

EXAMPLE 5 A Wide Image of a Thin Slit

Find the width of the central diffraction maximum of a slit of width 0.100 mm, as seen on a screen 2.00 m from the slit. The slit is illuminated by a He-Ne laser beam ($\lambda = 633$ nm).

SOLUTION The edge of the central diffraction maximum will correspond to the first minimum ($m = 1$). Applying Eq. 26–9, we find the angle θ corresponding to this point.

$$a \sin \theta = m\lambda = \lambda$$

or

$$\sin \theta = \frac{\lambda}{a} = \frac{633 \text{ nm}}{1.00 \times 10^{-4} \text{ m}} = 6.33 \times 10^{-3}$$

$$\theta = 6.33 \times 10^{-3} \text{ rad} \quad (\text{or } 0.363°)$$

Next we find the linear distance y from the center of the pattern to the point on the screen corresponding to this angle. Since the angle is quite small, we may approximate $\sin \theta$ by y/ℓ, where y and ℓ are shown in Fig. 26–23. Thus

$$\frac{y}{\ell} \approx \sin \theta = 6.33 \times 10^{-3}$$

or

$$y = (6.33 \times 10^{-3})\ell = (6.33 \times 10^{-3})(2.00 \text{ m})$$

$$= 1.27 \times 10^{-2} \text{ m} = 1.27 \text{ cm}$$

Since the central maximum extends an equal distance below the midpoint, the width of the maximum is double this value, or 2.54 cm. Thus the central maximum has a width 254 times the slit width (0.0100 cm). In other words, the width of the slit's central image is 254 times greater than the width of the image predicted by geometrical optics.

Circular Aperture

Fraunhofer diffraction by a circular aperture is of particular importance because of its application to the eye and to optical instruments, which generally have circular apertures. However, quantitative analysis of the circular aperture is considerably more complicated than analysis of the slit. Therefore we shall simply state without proof the one most important result of that analysis: the first diffraction minimum of an aperture of diameter D is at an angle θ, where

$$D \sin \theta = 1.22\lambda$$

Notice that this equation is similar to the equation for the first minimum of a slit of width a (Eq. 26–9 with $m = 1$: $a \sin \theta = \lambda$). Since $\sin \theta$ is very nearly equal to θ in radians, for the angles we normally encounter, we can express this result

$$\theta \approx 1.22\frac{\lambda}{D} \qquad \text{(first minimum; circular aperture)} \qquad (26\text{–}10)$$

Fig. 26–26 shows the Fraunhofer diffraction pattern of a circular aperture and the corresponding graph of intensity versus θ. The circular aperture was illuminated by a plane wave. One can think of this wave as originating from a distant point source. Since most of the light energy is concentrated in the central disk, called the "Airy disk," for simplicity we can regard this disk as the image of the point.

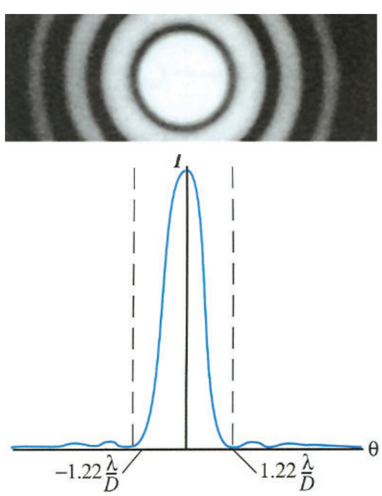


Fig. 26–26 Fraunhofer diffraction by a circular aperture. The central maximum is called the "Airy disk," which contains 84% of the light in the pattern. Since the diameter of the Airy disk is inversely proportional to the aperture diameter D, as the diameter of the aperture decreases, the disk gets bigger.

Rayleigh Criterion for Resolution

When a point source of light is imaged by an optical system with a circular aperture, the image is an Airy disk. For example, the image of a star formed by a telescope is such a disk. If two points are very close, their Airy disks will overlap, and you may not be able to distinguish separate images. Fig. 26–27 shows the image of two points that are (a) clearly resolved, (b) barely resolved, or (c) unresolved. As a quantitative measure of the resolution of two points, Lord Rayleigh proposed the following criterion—**the Rayleigh criterion: two points are barely resolved when the center of one's Airy disk is at the edge of the other's Airy disk.**

Fig. 26–28 illustrates the formation of the image of two points that are barely resolved according to Rayleigh's criterion. Notice that the angular separation θ_{min} of the two points P and Q is the angle from the center of an Airy disk to the first minimum, expressed by Eq. 26–10. Thus two points are resolved only if they subtend an angle at least as big as this minimum value θ_{min}, where

$$\theta_{min} = 1.22\frac{\lambda}{D} \qquad \text{(Rayleigh's criterion for resolution)} \qquad (26\text{–}11)$$

Any image formed by an optical system consists of a set of Airy disks, each of which is the image of a single point on the object. The size of these disks determines the resolution.

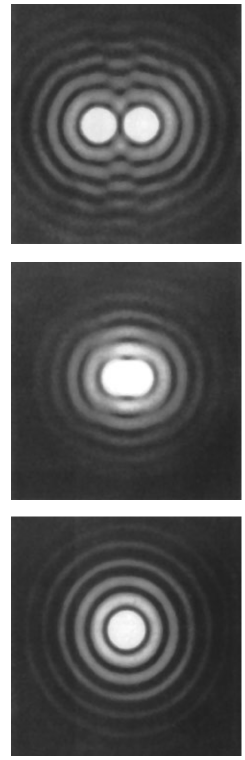


Fig. 26–27 Resolution of two point sources of light diffracted by a circular aperture.

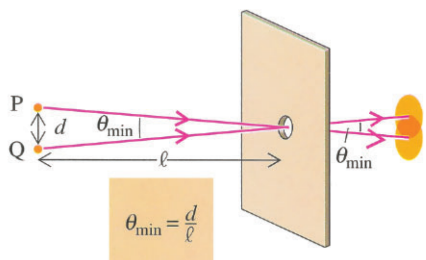


Fig. 26–28 Rayleigh's criterion for resolution.

EXAMPLE 6 Diffraction of Light by the Eye

(a) Find the prediction of diffraction theory for the minimum angle subtended by two points that are barely resolved by the eye. Assume a pupil diameter of 2.0 mm, and use a wavelength at the center of the visible spectrum. (b) Find the distance between the two points if they are 25 cm from the eye, at its near point.

SOLUTION **(a)** We apply Eq. 26–11, using for λ the wavelength inside the eye, where the refractive index $n = 1.34$. At the center of the visible spectrum the vacuum wavelength $\lambda_0 = 550$ nm, and

$$\lambda = \frac{\lambda_0}{n}$$

Thus

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22\frac{\lambda_0}{nD} = 1.22\frac{5.5 \times 10^{-7}\text{ m}}{(1.34)(2.0 \times 10^{-3}\text{ m})}$$

$$= 2.5 \times 10^{-4}\text{ rad}$$

(b) From Fig. 26–28 we see that this angle equals the separation d between the points divided by the distance of 25 cm.

$$\frac{d}{25\text{ cm}} = 2.5 \times 10^{-4}\text{ rad}$$

$$d = (25\text{ cm})(2.5 \times 10^{-4}\text{ rad}) = 6.3 \times 10^{-3}\text{ cm}$$

The eye should be unable to resolve points closer than about 0.06 mm.

The minimum angle we have calculated is fairly close to the measured minimum angle between points barely resolved by the normal eye. The factors other than diffraction that affect this minimum angle are discussed in Chapter 25, Section 25–5 (Factors Limiting Visual Acuity).

EXAMPLE 7 Diffraction Limit of a Microscope

Find an expression for the minimum separation between two points that are barely resolved by a microscope with an objective of diameter D and focal length f.

SOLUTION Using Fig. 26–29 and applying Eq. 26–11, we find

$$d = f\theta_{\text{min}} = 1.22\frac{f\lambda}{D} \qquad (26\text{–}12)$$

To make d as small as possible we need to minimize the ratio f/D. But it is not possible to make f less than about $D/2$, the radius of the lens.* Setting $f = D/2$ in the expression above, we obtain

$$d = 0.61\lambda$$

*For a simple symmetrical lens made of glass with a refractive index of 1.5, the focal length equals the radius of curvature of the first surface, as shown in Problem 24–37. But the radius of curvature can be no smaller than the radius of the lens itself. Therefore the focal length is always less than the radius $D/2$.

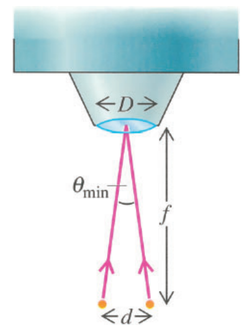


Fig. 26–29

Thus **the minimum distance between two points that can be resolved by any microscope equals roughly half the wavelength of the light used to illuminate the points.** For example, using the value $\lambda = 550$ nm for the center of the visible spectrum, we find

$$d = (0.61)(550 \times 10^{-9}\text{ m})$$
$$= 3.4 \times 10^{-7}\text{ m}$$

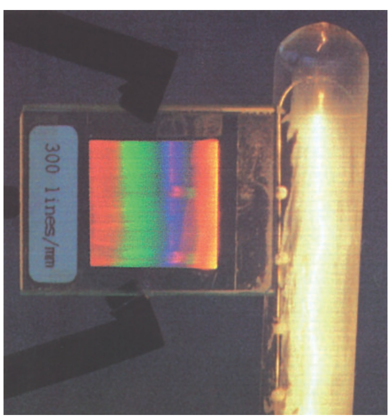


Fig. 26–30 A diffraction grating consists of thousands of narrow, closely spaced slits.

Diffraction Gratings

A diffraction grating consists of thousands of very narrow, closely spaced slits, made by etching precisely spaced grooves on a glass plate (Fig. 26–30). The slits are the transparent spaces between the grooves. Typically a grating consists of thousands or tens of thousands of transparent lines per cm. It follows from our discussion of single-slit diffraction that each line, because of its extremely narrow width, produces diffracted light spread over a considerable angle—perhaps 20° or 30°. Of course, the diffraction pattern of one such line alone would not produce enough intensity to be seen by itself. But when diffracted light from thousands of lines interfere, bright, sharp diffraction maxima are produced, (Figs. 26–31 and 26–32).

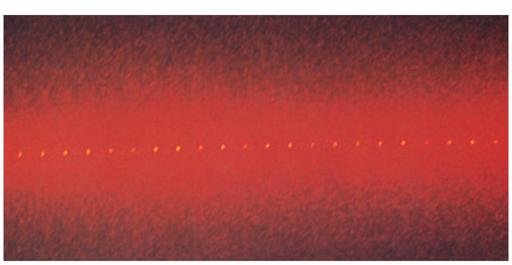


Fig. 26–31 The diffraction pattern produced by a grating illuminated by a He-Ne laser.

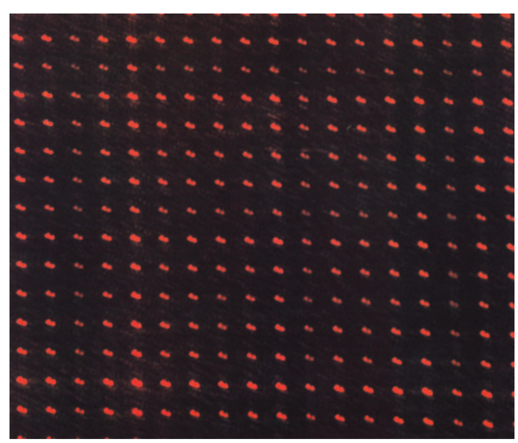


Fig. 26–32 The diffraction pattern produced by two perpendicular diffraction gratings illuminated by a He-Ne laser.

Fig. 26–33 can be used to locate the diffraction maxima of a grating with spacing d between adjacent slits. As indicated in the figure, for light in the θ direction, the difference in path length of rays from adjacent slits is $d \sin \theta$. These adjacent rays will interfere constructively if the difference in path length is an integral multiple of λ, that is, 0, λ, 2λ, 3λ, and so forth. Indeed rays from all the slits will interfere constructively if they are directed at an angle θ, such that

$$d \sin \theta = m\lambda \qquad m = 0, 1, 2, \ldots \qquad (26\text{–}13)$$

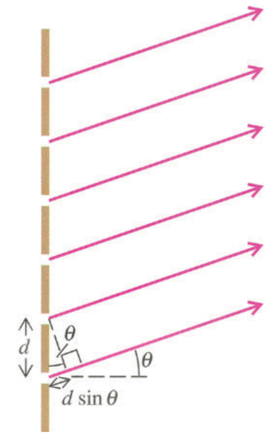


Fig. 26–33 Rays from a diffraction grating.

Unlike the maxima produced by a double slit, diffraction grating maxima are very narrow and sharp, as shown in Fig. 26–31. This can be understood when we consider how a slight change in the angle θ away from a value satisfying Eq. 26–13 will affect the intensity. Suppose that the change in angle is so slight that light from adjacent slits is still nearly in phase. If there were only two slits interfering, such an angle would still give an intensity close to the maximum value. However, with the thousands of slits in a grating, there can be destructive interference in many ways. For example, light from slits 100 spacings apart might interfere destructively. If all pairs of slits 100 spacings apart interfere destructively, there will be no light in that particular direction.

Diffraction gratings can be used to measure the wavelength of light, as illustrated in the following example.

EXAMPLE 8 Separating the Sodium Doublet

Find the first-order ($m = 1$) diffraction angles for the sodium doublet, using a grating with 10^6 lines/m. The sodium doublet consists of two yellow lines in the spectrum of sodium, with nearly identical wavelengths: 589.00 nm and 589.59 nm.


Fig. 26–34

SOLUTION Applying Eq. 26–13 to each of the wavelengths, with $d = 10^{-6}$ m and $m = 1$, we find

$$\sin \theta = \frac{m\lambda}{d}$$

$$\sin \theta_1 = \frac{(1)(589.00 \times 10^{-9}\text{ m})}{10^{-6}\text{ m}} = 0.58900$$

$$\theta_1 = 36.09°$$

$$\sin \theta_2 = \frac{(1)(589.59 \times 10^{-9}\text{ m})}{10^{-6}\text{ m}} = 0.58959$$

$$\theta_2 = 36.13°$$

Thus the angular separation between the lines is 0.04°, or 7×10^{-4} rad. Viewed at a distance of 1 m, the lines are 0.7 mm apart.

X-ray diffraction is a technique that utilizes the small spacing between the atoms in a crystal as a three-dimensional diffraction grating. The atomic spacing is on the same order as wavelengths in the X-ray portion of the electromagnetic spectrum. So X rays, rather than visible light, are diffracted by a crystal. And the resulting diffraction pattern can be used to discover the crystal structure. X-ray diffraction of DNA was used by Watson and Crick in discovering the structure of DNA in 1951 (Fig. 26–35).

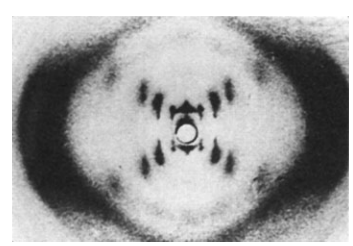


Fig. 26–35 X-ray diffraction pattern of DNA. The double helix structure of DNA was revealed by this historic photograph taken by Rosalind Franklin.

26–4 Polarization

Polarization by Absorption

Most light sources produce unpolarized light (Fig. 26–3), as opposed to the polarized light produced by some lasers (Figs. 26–1 and 26–2). However, there are ways to polarize light that is initially unpolarized, or to change the direction of polarization of polarized light. One way is to pass the light through a Polaroid sheet, a synthetic material first produced by Edwin Land in 1928 when he was an undergraduate.

There is a direction along each Polaroid sheet called its "transmission axis." Light linearly polarized along this axis passes through the sheet (Fig. 26–36a), whereas light polarized in the perpendicular direction is completely absorbed (Fig. 26–36b). If the incident light is linearly polarized at some angle θ relative to the transmission axis, the light will be partially absorbed and partially transmitted. As illustrated in Fig. 26–36c, the component of the electric field parallel to the axis is transmitted. The light that emerges is thus polarized along the direction of the transmission axis and has an amplitude E related to the incident amplitude E_0 by the equation

$$E = E_0 \cos\theta$$

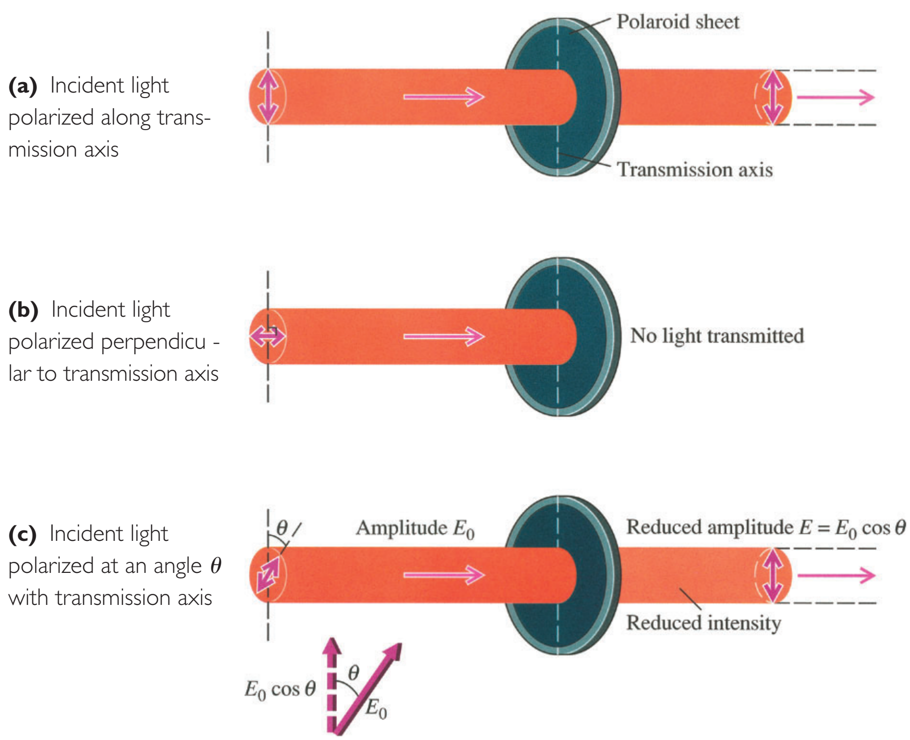


Fig. 26–36 The effect of a Polaroid sheet on initially polarized light depends on the direction of initial polarization relative to the direction of the sheet's transmission axis.

The intensity of light is proportional to the square of its amplitude (Eq. 23–5: $I_{av} = \frac{1}{2}\epsilon_0 c E_0^2$). Squaring the equation above, we obtain

$$E^2 = E_0^2 \cos^2\theta$$

Multiplying both sides of this equation by the appropriate constant ($\frac{1}{2}\epsilon_0 c$), we obtain a relationship between the average transmitted intensity I and the average incident intensity I_0:

$$I = I_0 \cos^2\theta \qquad (26\text{–}14)$$

This result is known as the **law of Malus.** The intensity of the transmitted light has its maximum value, $I = I_0$, when $\theta = 0$, and has its minimum value, $I = 0$, when $\theta = 90°$.

Unpolarized light consists of a superposition of linearly polarized waves, with varying directions of polarization, as illustrated in Fig. 26–37a. The electric field associated with each of these waves can be resolved into x and y components, relative to an arbitrary coordinate system. Since the direction of polarization is random, the resultant x and y components are equal. We can replace the many randomly directed linearly polarized waves by just two linearly polarized waves of equal intensity, with mutually perpendicular polarization directions, as illustrated in Figs. 26–37b and 26–37c.

When unpolarized light is incident on a Polaroid sheet, only the component along the transmission axis is transmitted. Since the two components have equal intensity in unpolarized light, this means that the intensity of the transmitted light is half the intensity of the incident light (Fig. 26–38).

$$I = \tfrac{1}{2} I_0 \qquad \text{(initially unpolarized light)} \qquad (26\text{–}15)$$

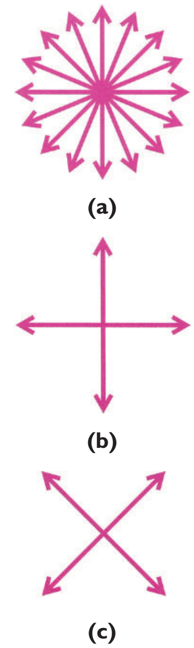


Fig. 26–37 Equivalent representations of unpolarized light.

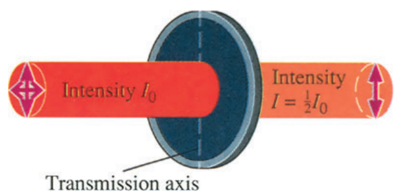


Fig. 26–38 A Polaroid sheet polarizes initially unpolarized light.

EXAMPLE 9 Light Passing Through Two Polarizers

An unpolarized laser beam of intensity 1000 W/m^2 is incident on a Polaroid sheet with a vertical transmission axis. The light passing through this sheet strikes a second Polaroid sheet, with a transmission axis at an angle of 30.0° from the vertical (Fig. 26–39). Find the polarization and the intensity of the light emerging from the second sheet.

SOLUTION The light transmitted by the first Polaroid sheet is vertically polarized and, according to Eq. 26–15, has an intensity equal to half the incident intensity.

$$I = \tfrac{1}{2} I_0 = \tfrac{1}{2}(1000\ \text{W/m}^2) = 500\ \text{W/m}^2$$

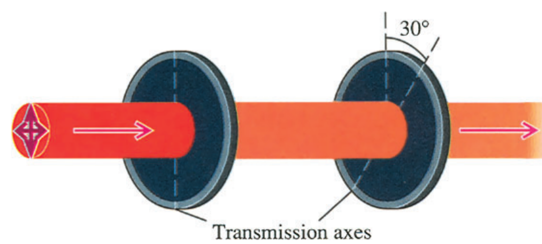


Fig. 26–39

The light incident on the second sheet is polarized at an angle of 30° relative to this sheet's transmission axis and, according to the law of Malus (Eq. 26–14), has intensity

$$I' = I_0' \cos^2\theta = (500\ \text{W/m}^2)(\cos^2 30.0°) = 375\ \text{W/m}^2$$

Polarization by Reflection

When light is reflected from the surface of a dielectric, such as water or glass, the intensity of the reflected light depends on the angle of incidence and on the polarization of the incident light. Light polarized parallel to the reflecting surface (Fig. 26–40a) is always more strongly reflected than light polarized in a perpendicular direction (Fig. 26–40b). Unpolarized light can be thought of as consisting of two equal-intensity polarized waves—one polarized parallel to the surface and a second polarized perpendicular to the first. After reflection, the component polarized parallel to the surface is more intense than the other component. Fig. 26–41 shows the intensities of the two components in a beam of initially unpolarized light reflected by water, for several angles of incidence. The intensities are predicted by Maxwell's equations. Notice that both components are more strongly reflected for very large angles of incidence. Any smooth dielectric surface becomes mirror-like as the angle of incidence approaches 90°. You can observe this effect by holding up a smooth sheet of paper so that rays from a light source are reflected at glancing incidence.

The angle of incidence at which the reflected light is 100% polarized is known as "Brewster's angle," denoted by θ_B. Brewster's angle has a value of 53° for reflection by water, as indicated in Fig. 26–41c. Maxwell's equations can be used to derive an expression for Brewster's angle, relating it to the refractive index n of the incident medium and the index n' of the reflecting medium. We present the result here without proof:

$$\tan\theta_B = \frac{n'}{n} \qquad \text{(Brewster's angle)} \qquad (26\text{–}16)$$

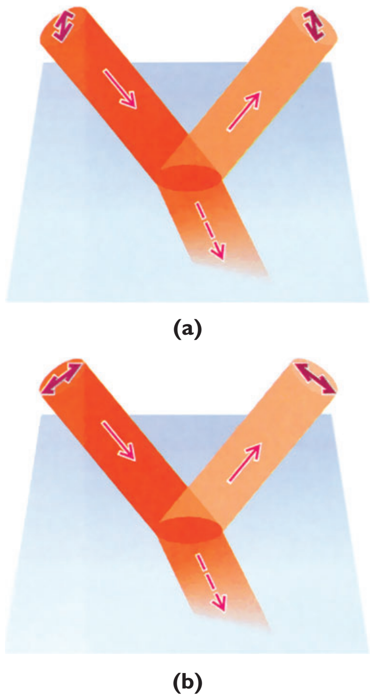
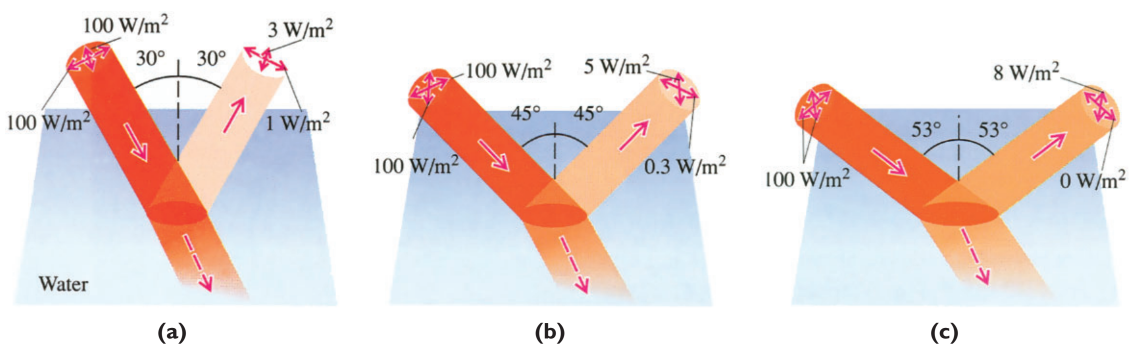


Fig. 26–40 **(a)** Light polarized parallel to the reflecting surface is more strongly reflected than **(b)** light polarized in a perpendicular direction.



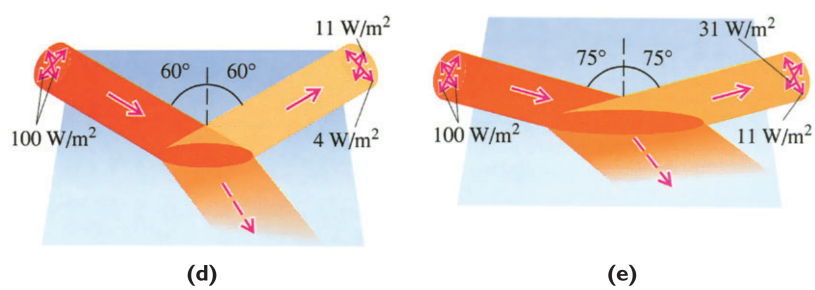


Fig. 26–41 Initially unpolarized light of total intensity 200 W/m² is reflected by water at various angles of incidence.

EXAMPLE 10 Brewster's Angle for Glass

Calculate Brewster's angle for light incident from air onto a glass surface if the glass has a refractive index of 1.5.

SOLUTION Applying Eq. 26–16, using the refractive indices for glass and air, we find

$$\tan\theta_B = \frac{n'}{n} = \frac{1.5}{1.0} = 1.5$$

$$\theta_B = 56°$$

Polaroid sunglasses are effective at reducing reflected glare from the surface of a body of water or from other surfaces (Fig. 26–42). The lenses are made of Polaroid sheets with vertical transmission axes. The reflected light consists mainly of horizontally polarized light, and such light is completely absorbed by the lenses.

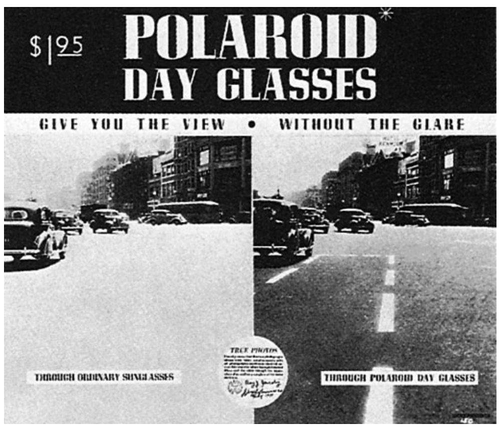


Fig. 26–42 An early ad for Polaroid sunglasses.

EXAMPLE 11 A Sunset Seen Through Polaroid Sunglasses

The setting sun is reflected from the surface of a lake at an angle of incidence of 75°. The intensity of the incident light is 200 W/m^2, as in Fig. 26–41e. Find the intensity of the reflected light reaching the eye of an observer wearing Polaroid sunglasses (Fig. 26–43).

SOLUTION From Fig. 26–41e we see that the intensity of the reflected light having a polarization along the transmission axis of the sunglasses is 11 W/m^2. Therefore this is the intensity of the reflected light reaching the eye.

Without sunglasses, the observer would see reflected light of both polarizations. From Fig. 26–41e, the intensity of reflected light seen by the observer would be 11 W/m^2 + 31 W/m^2 = 42 W/m^2.

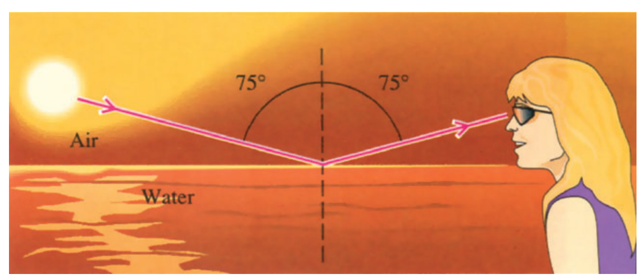


Fig. 26–43

Non-Polaroid sunglasses that produce the same darkening as these Polaroid sunglasses would absorb 50% of *all* incident light, and so they would transmit to the observer reflected light of intensity 21 W/m^2.

Polarization by Scattering

When an electromagnetic wave is incident on an atom, the atom's electrons oscillate in response to the oscillating electric field. The electrons behave like tiny antennas; they emit their own radiation with the same frequency as the incident electromagnetic wave, but scatter the radiation in various directions. The intensity of this scattered radiation depends on the light's frequency. Blue light is scattered much more effectively than red light. Fig. 26–44 shows how scattering of sunlight by the earth's atmosphere gives us blue skies and red sunsets. Observers A and B both see blue sky as a result of the blue part of the sun's spectrum being scattered toward their eyes by the atmosphere. Meanwhile, observer C sees an orange or red sun as a result of the blue part of the spectrum's having been scattered out of the beam of direct sunlight. This kind of scattering, called "Rayleigh scattering," is the result of independent, incoherent radiation by many atoms.

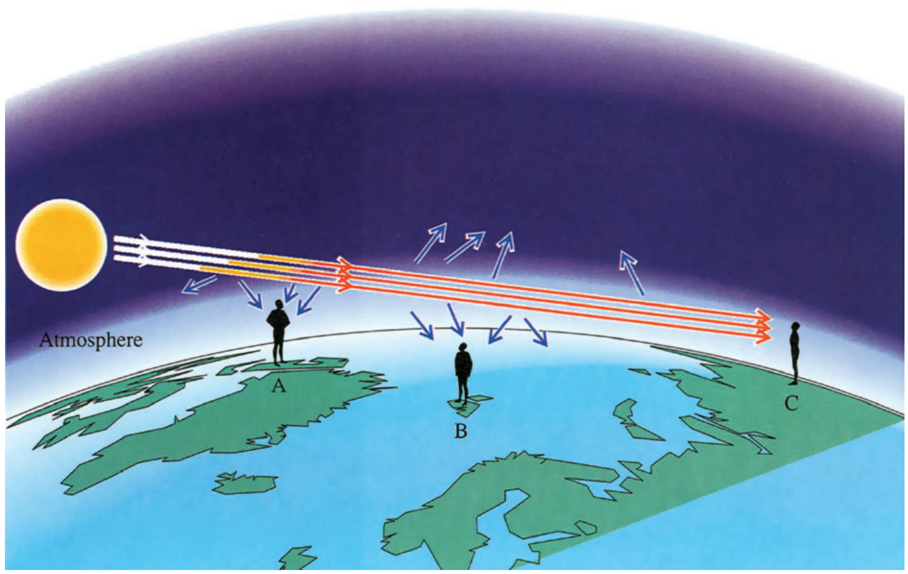


Fig. 26–44 Scattering of sunlight by the earth's atmosphere results in blue skies (seen by A and B) and red sunsets (seen by C).

Scattering is also the basic mechanism at the heart of reflection and refraction by a solid or a liquid. But the higher density and relative immobility of the atoms in the liquid or solid mean that light scattered by neighboring atoms is coherent and can therefore interfere constructively or destructively. The result of this interference is remarkably simple. The scattered waves interfere destructively in all directions, except those corresponding to the reflected and refracted waves, for which the interference is constructive. So we see only a reflected wave and a refracted wave.

The particles of water in a cloud or in ocean surf also "scatter" sunlight, but in this case the scattered light is white, in contrast to the blue sky. The tiny water droplets simply reflect and refract incident light. The result of multiple reflections and refractions by a very large number of droplets is to redirect or scatter the incident white light in all directions.

Scattering of sunlight by the atmosphere tends to polarize the light. Fig. 26–45 shows how this polarization arises. A beam of unpolarized sunlight, incident on the atmosphere, travels along the *x*-axis. This transverse electromagnetic wave has an electric field that oscillates in the *yz* plane; there is no component along the *x*-axis, the direction of propagation of the wave. Electrons within atoms in the atmosphere oscillate in the *yz* plane, in response to the incident wave, and scatter light in various directions. The nature of radiation produced by any source of radiation is that there can be a component of the electric field in a given direction only if there is a component of motion of the radiating source parallel to that direction. Therefore there can be no *x*-component of the electric field in scattered radiation, since there is none in the incident wave. This implies that **radiation scattered** along the *yz* plane, **perpendicular to the incident beam, must be polarized.** For radiation scattered in any such direction there is only a single line perpendicular to the direction of propagation, along which the electric field vector can oscillate, as indicated in Fig. 26–45. Light scattered in other directions is partially polarized.

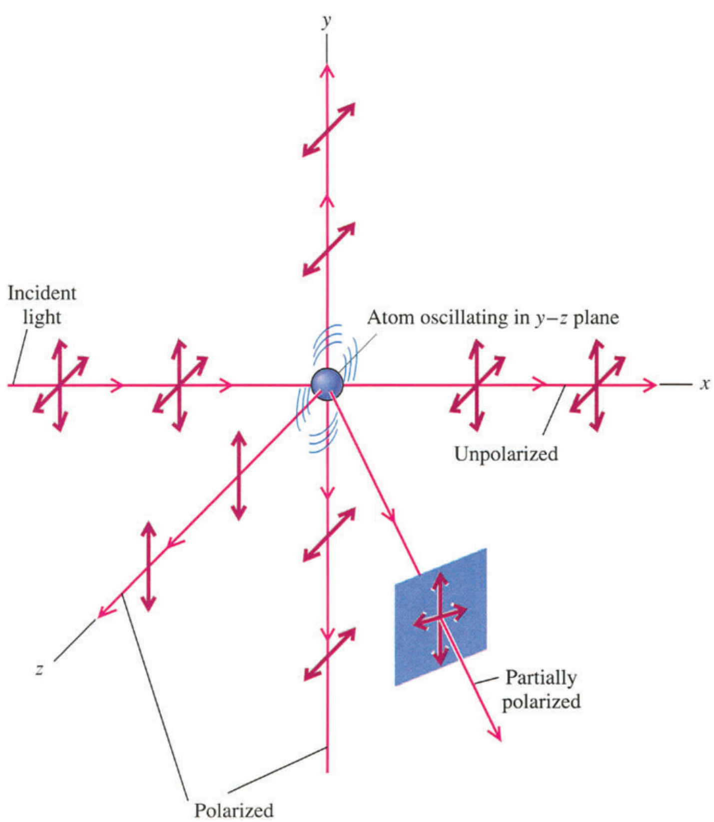


Fig. 26–45 Polarization by scattering.

Magic in the Sky

Mirage! Looming! Mountain specter! What do you think of when you hear these terms? Illusions or ghosts, perhaps. The names are suggestive of the fear and bewilderment that these phenomena have aroused over the centuries. It may surprise you to learn, however, that the names describe real optical effects in the atmosphere. You already know enough about the basic principles of optics to understand such phenomena.

Mirages

You have heard of people lost in the desert who imagined that they saw tempting pools of blue water just beyond the next sand dune and who dragged themselves forward in hopeless pursuit of them. You may think that such mirages are simply hallucinations caused by heat and thirst. Actually, they are almost always real phenomena—real light is behaving in a way that creates an illusion in which anyone, hot and thirsty or not, can share (Fig. 26–A).

Mirages are refraction phenomena. Light rays are bent by layers of air at different temperatures. Warm air has lower density than cool air and has a lower refractive index. As a ray coming from a cooler layer enters a warmer layer—which is what happens when the ray is moving downward toward a searingly hot desert surface—the ray is refracted away from the normal. The ray can be bent so much that it curves back upward (Fig. 26–B). When it reaches the viewer's eye, it is automatically traced back by the brain as if it came from a source directly in line with its final segment. That is, it is seen as if it were ahead and below, rather than ahead and above. Light coming from a clear blue sky produces the illusion of a bright blue pool on the ground ahead.

Fig. 26–A A fairly common kind of mirage: the dry surface of a road appears to be wet.

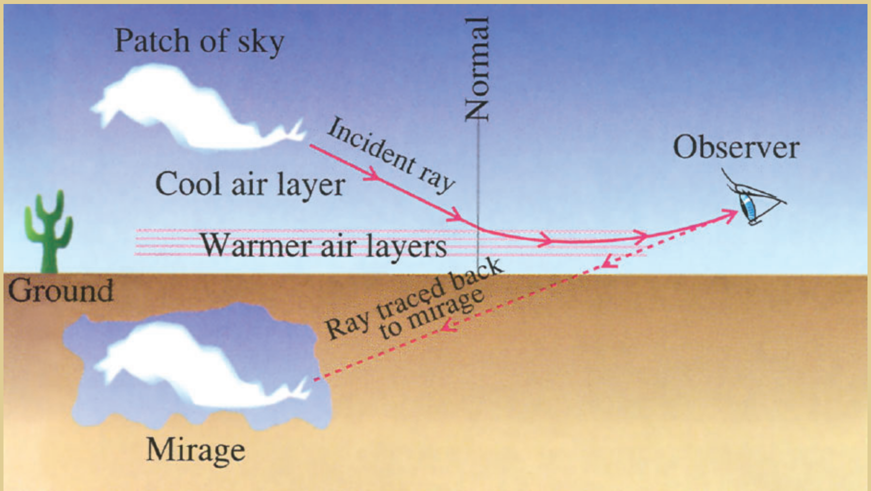


Fig. 26–B Formation of an inferior mirage.

A mirage of the sort seen in a hot desert is called an "inferior mirage" because it appears below the light source (the sky). There is another kind of mirage, called a "superior mirage," which appears above the source of light (Fig. 26–C). A superior mirage is typically caused when light moves upward from a layer of cool, dense air into warmer, less dense layers. The rays are bent away from the normal, as before, but this time they turn down rather than upward (Fig. 26–D). When they strike the eye, they are traced up to a mirage seen at their apparent point of origin. Under such conditions, a ship moving on the water below the horizon can look like a ghost ship sailing through the sky! The appearance of a superior mirage is sometimes called looming, for obvious reasons: the mirage looms above its source.

Under special conditions, looming can produce truly uncanny effects. One particular type of looming is called **fata morgana,** after Morgan le Fay, the fairy-enchantress of the King Arthur legends who lived on a magical island. Fata morgana is most often seen in the Strait of Messina, a waterway that separates Sicily from Italy and that was long dreaded for its deadly currents, rocks, and whirlpools. The mirage is caused by irregular layerings of air of various densities, which produce multiple refractions and multiple overlapping images. The result is an apparent vertical elongation of the source object, sometimes to enormous proportions. For example, when

Fig. 26–C A superior mirage of a ferry, which appears to be vertically elongated.

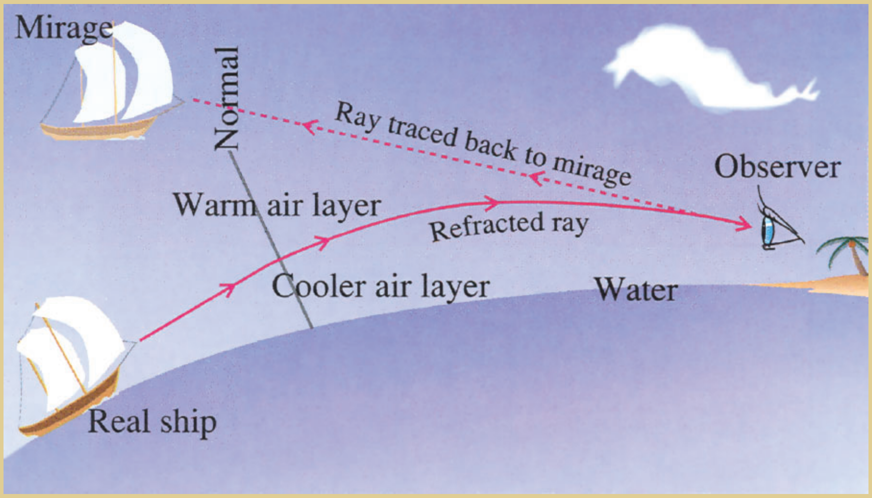


Fig. 26–D Formation of a superior mirage.

seen from a ship in the strait, objects such as trees or hills on the shore can look like huge, weirdly shaped figures that can be disorienting and dangerous to unsuspecting mariners (Fig. 26–E).

Fig. 26–E What appears in this picture to be icy castles in the sky is actually an extremely rare, mirage-like effect known as fata morgana.

Coronas and anticoronas

Interference or diffraction effects can occur when light rays pass near the edges of tiny objects in the atmosphere. If the diffraction appears around the light source, the effect is usually a ring, called a **corona.**

Coronas can appear around the sun or moon when light rays pass near the edges of water droplets in the atmosphere. Rings of different colors can be seen because different wavelengths are diffracted to varying degrees.

A Closer Look

A related phenomenon caused by the presence of hexagonal ice crystals is called a **halo** (Fig. 26–F). If the ice crystals line up in just the right way on a sunny day, they can produce two separate bright spots, one to either side of the sun. These ghost suns are called **parhelia** or more popularly, **sundogs.**

When diffraction occurs around a shadow area, the effect is called an **anticorona.** Anticoronas are also known by the names **glory** (Fig. 26–G) and **mountain specter.** These are rare and awesome phenomena.

The glory is seen as a halo surrounding the shadow of the observer's head. Because of the extreme directional dependence of this effect, no one else sees the halo around the observer's head. Two observers might each see a halo around his or her own head, but each observer will not be able to see the other's halo.

Fig. 26–F A street light blocks the sun's direct rays, allowing a halo around the sun to be clearly seen.

Fig. 26–G The bright circle around the plane's shadow is called a glory.

One particularly famous example of an anticorona occurs occasionally near the Brocken, a mountain in the Harz range of central Germany. Because it is known in German legend as the site of the Walpurgis Night witchcraft rituals, the Brocken is an appropriate location for the anticorona, which is there given the name "Brocken specter." The specter can be seen at twilight on sunny days by observers who stand near the foot of the mountain when there are misty banks of fog or cloud just above them that do not reach as high as the mountain top. The low-lying sun then casts a huge shadow of the mountain peak onto the upper surface of the mist. This silhouette of the peak appears surrounded by rings of colored light, as rays passing around the edge of the peak are bent and separated out according to their wavelength (Fig. 26–H).

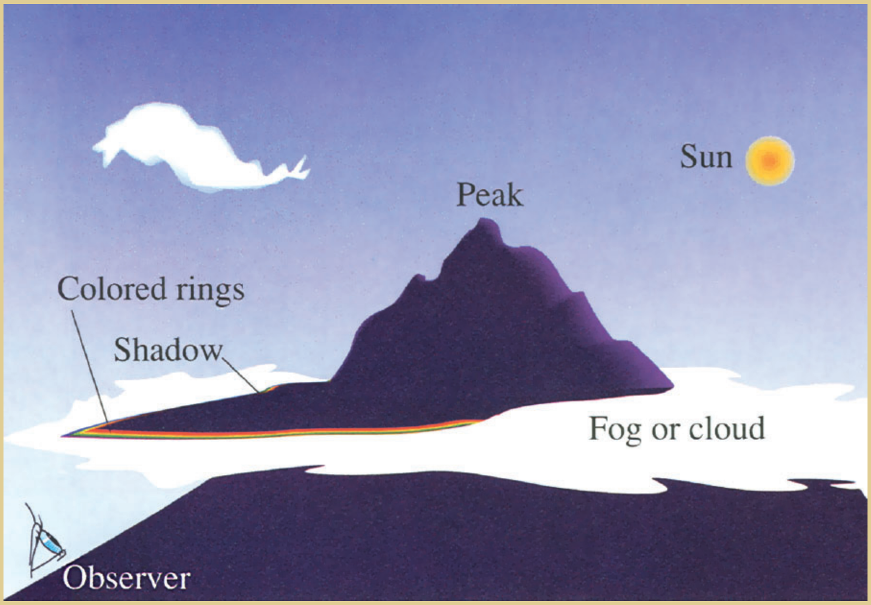


Fig. 26–H Viewing the Brocken Specter.