CHAPTER 11

Nuclear Chemistry

11.0 INTRODUCTION

Most of chemistry focuses on the changes in the electronic structure of the atoms and molecules because it is those changes that result in bond breaking and bond formation (*i.e.*, in chemical reactivity). In this chapter, we examine reactions that involve changes in the nucleus. This branch of chemistry is called **nuclear chemistry** or **radiochemistry**.

Most of us have a love-hate relationship with nuclear chemistry. Its promise of vast amounts of energy to replace fossil fuels and possible cures for diseases is exciting, but its threat of nuclear war and nuclear waste contamination is threatening. Is its promise worth the risk? The goal of this chapter is to give you a background that may help you answer that question.

THE OBJECTIVES OF THIS CHAPTER ARE TO DISCUSS

- balanced nuclear reactions;
- stable versus unstable nuclei;
- half-lives of radioactive substances;
- effects of nuclear radiation on the human body;
- conversion of mass and energy into one another;
- fission and fusion;
- nuclear power plants; and
- nuclear reactions in stars.

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11.1 THE NUCLEUS

There are three major subatomic particles: electrons, protons and neutrons. Their masses and charges are summarized in Table 11.1. Notice that electrons and protons carry a net charge, but neutrons are neutral. Also, the mass of the neutron and the proton are each very close to 1 amu ($M_m \sim 1$ g·mol^{-1}) while the mass of an electron is much smaller. Because neutrons and protons reside within the nucleus, they are referred to as **nucleons.** The number of protons in the nucleus is given by the **atomic number, Z**, while the number of **nucleons** (protons plus neutrons) in the nucleus is given by the **mass number, A**. The symbol N_n will be used to denote the number of neutrons in the nucleus. Thus,

$$A = Z + N_n \qquad \text{Eq. 11.1}$$

Equation 11.1 simply indicates that the total number of nucleons (A) is the sum of the number of protons (Z) and the number of neutrons (N_n). Because the mass of a neutron and a proton are each nearly equal to 1 amu, the mass number is the integer that is closest to the mass of the nucleus.

The atomic number is the number that characterizes the atom. Two atoms with different atomic numbers are atoms of different elements. Atoms of the same atomic number but with different mass numbers are called **isotopes**. Thus, isotopes differ in the number of neutrons in the nucleus. Isotopes are distinguished by indicating the mass number as a superscript in front of the symbol of the element. For example, ^{13}C (carbon-13), is an isotope of carbon that has seven neutrons ($N_n = A - Z = 13 - 6 = 7$). The atomic mass of ^{13}C is 13.003 amu. The atomic mass scale is based on the assignment of the mass of a carbon-12 atom (^{12}C), which is defined as *exactly* 12.00… amu. The reason that the atomic mass of carbon is 12.011 and not 12.000 is that the **atomic mass** of an element is the mass-weighted average of the masses of all of its naturally occurring isotopes. Naturally occurring carbon is a mixture containing 98.9 % ^{12}C and 1.1% ^{13}C, so a mole of carbon contains 0.989 mol ^{12}C and 0.011 mol ^{13}C and has a mass of

$$(0.989 \text{ mol } ^{12}C)(12.000 \text{ g·mol}^{-1}) + (0.011 \text{ mol } ^{13}C)(13.003 \text{ g·mol}^{-1}) = 12.011 \text{ g}$$

Example 11.1 shows why the molar mass of magnesium is 24.31 g·mol^{-1}.

Table 11.1 The major subatomic particles

Particle	Mass (amu)[a]	Charge[b]
electron	5.49×10^{-4}	-1
proton	1.00728	+1
neutron	1.00867	0

a) An amu is an atomic mass unit. The mass of a nucleus in amu is numerically equal to the mass of a mole of nuclei in grams.

b) The charge is given in terms of the fundamental unit of charge, 1.60×10^{-16} C.

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Example 11.1

Naturally occurring magnesium exists as a mixture of three isotopes. Determine the atomic mass of magnesium given the masses and abundances of the three isotopes.

Isotope	Mass	Abundance
^{24}Mg	23.9850	78.70 %
^{25}Mg	24.9858	10.13 %
^{26}Mg	25.9826	11.17 %

One mole of magnesium contains 0.7870 mol of ^{24}Mg, 0.1013 mol of ^{25}Mg and 0.1117 mol of ^{26}Mg. One mole therefore contains

$$(0.7870 \text{ mol } ^{24}\text{Mg})(23.9850 \text{ g·mol}^{-1}) = 18.88 \text{ g } ^{24}\text{Mg}$$

$$(0.1013 \text{ mol } ^{25}\text{Mg})(24.9858 \text{ g·mol}^{-1}) = 2.53 \text{ g } ^{25}\text{Mg}$$

$$(0.1117 \text{ mol } ^{26}\text{Mg})(25.9826 \text{ g·mol}^{-1}) = 2.90 \text{ g } ^{26}\text{Mg}$$

$$\text{total mass of one mole of Mg} = 24.31 \text{ g Mg}$$

The molar mass of magnesium is 24.31 g·mol^{-1}.

NUCLEAR STABILITY

How stable one nucleus is compared to another is an important consideration in nuclear reactions, and we now address the manner in which relative nuclear stabilities are measured. We do so by analogy with molecular stability. An atomization enthalpy (Section 3.8) is the energy required to breakdown a molecule into its atoms; that is, to break all of the bonds in a molecule. Consequently, the atomization enthalpy depends upon both the number and the strength of the bonds. For example, the enthalpy of atomization (ΔH_{atom}) of methane is the enthalpy change for the following process:

$$CH_4(g) \rightarrow C(g) + 4H(g) \qquad \Delta H_{atom} = 1.66\text{x}10^3 \text{ kJ} \qquad \text{Rxn 11.1}$$

The process involves breaking four C-H bonds. However, a molecule needs only one weak bond to become unstable, so molecular stability is related to the strength of individual bonds not the total energy required to break all of the bonds. In the case of CH_4, the average C-H bond energy ($D_{C\text{-}H}$) is the atomization energy divided by the number of C-H bonds.

$$D_{C\text{-}H} = \frac{\Delta H_{atom}}{\text{number of C-H bonds}} = \frac{1.66\times10^3 \text{ kJ}}{4 \text{ mol CH bonds}} = 415 \text{ kJ}\cdot\text{mol}^{-1}$$

The bonds in CH_4 have high bond energies, which makes CH_4 a stable molecule.

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Nuclear stability is measured in a similar manner. First, the **binding energy** of the nucleus (the energy required to separate the nucleus into its nucleons) is determined. The binding energy of a ^{12}C nucleus is the energy change for the following process:

$$^{12}C \rightarrow 6p + 6n \qquad \Delta E = 8.90\times10^9 \text{ kJ} \qquad \text{Rxn 11.2}$$

Just as it is the energy per bond not the atomization energy of a molecule that dictates its stability, it is the energy per nucleon not the binding energy that dictates the stability of a nucleus. The binding energy per nucleon of a ^{12}C nucleus is

$$\frac{\text{nuclear binding energy}}{\text{number of nucleons}} = \frac{8.90\times10^9 \text{ kJ}\cdot\text{mol}^{-1}}{12 \text{ nucleons}} = 7.42\times10^8 \text{ kJ}\cdot\text{mol}^{-1}\cdot\text{nucleon}^{-1}$$

Figure 11.1 shows that the binding energy per nucleon* (nuclear stability) reaches a maximum for nuclei with mass numbers in the range of 50 - 60. Thus, nuclei with mass numbers close to that of iron (A = 56) are thermodynamically the most stable nuclei.

* There is no general agreement on the sign convention for binding energy. Indeed, most general chemistry books define the binding energy as the energy change for the reverse process and report it as a negative value. We choose the convention above to be consistent with atomization and bond energies, which are always positive and define the same type of processes.

To understand the origin of the binding energy, we examine the nuclear mass in detail. The mass of a ^{12}C nucleus is the mass of the atom (exactly 12 amu) less the mass of the six electrons.

$$\text{mass of } ^{12}C \text{ nucleus} = 12.00000 - 6(0.000549) = 11.9967 \text{ amu}$$

The nucleons of a ^{12}C nucleus, six protons (p) and six neutrons (n), have a mass of

$$\text{mass of nucleons} = (6p)(1.00728 \text{ amu}\cdot\text{p}^{-1}) + (6n)(1.00867 \text{ amu}\cdot\text{n}^{-1}) = 12.0957 \text{ amu}$$

Consequently, the mass of the nucleus is less than the mass of its nucleons; that is, mass is not conserved in Reaction 11.2. The mass difference is called the **mass defect**, **Δm**.

$$\Delta m = \text{final mass} - \text{initial mass} = \text{mass of product} - \text{mass of reactant} \qquad \text{Eq. 11.2}$$

The mass defect for Reaction 11.2, which is the mass defect of a ^{12}C nucleus, is

$$\Delta m = \text{mass of nucleons} - \text{mass of nucleus} = 12.0957 - 11.9967 = 0.0990 \text{ amu}$$

The mass defect for a ^{12}C nucleus is 0.0990 $g\cdot mol^{-1}$ or 9.90×10^{-5} $kg\cdot mol^{-1}$.

The origin of mass defect can be understood in terms of Einstein's famous equation that relates mass and energy.

$$E = mc^2 \qquad \text{Eq. 11.3}$$

Or, in terms of changes in energy due to changes in mass:

$$\Delta E = \Delta mc^2 \qquad \text{Eq. 11.4}$$

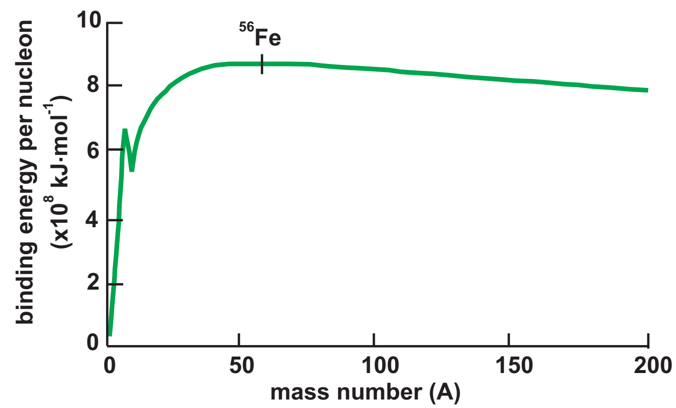

Figure 11.1 Binding Energy per nucleon versus mass number
A nucleus of ^{56}Fe has the highest energy per nucleon, so it is the most stable nucleus.

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Equations 11.3 and 11.4 show the equivalence of mass and energy, and the term **mass-energy** is sometimes used to express the equivalence. Indeed the law of conservation of mass and the law of conservation of energy are combined into the law of conservation of mass-energy: the total mass-energy of the universe is constant.

The binding energy of a nucleus is determined from its mass defect and the application of Equation 11.4. A joule is a $kg \cdot m^2 \cdot s^{-2}$, so, in order to obtain ΔE in joules, Δm *must be expressed in kg*. The speed of light is $c = 2.998 \times 10^8\ m \cdot s^{-1}$.

Example 11.2

What is the binding energy of a ^{12}C nucleus?

The mass defect for ^{12}C was determined in the preceding discussion to be $\Delta m = 0.0990$ amu. An amu is numerically equal to the molar mass expressed in grams, so we can also write that $\Delta m = 0.0990\ g \cdot mol^{-1} = 9.90 \times 10^{-5}\ kg \cdot mol^{-1}$. Applying Equation 11.4, we obtain the binding energy.

$$\Delta E = \Delta mc^2 = (9.90 \times 10^{-5}\ kg \cdot mol^{-1})(2.998 \times 10^8\ m \cdot s^{-1})^2 = 8.90 \times 10^{12}\ J \cdot mol^{-1}$$

Example 11.3

What is the mass defect for the atomization of CH_4?

$$CH_4(g) \rightarrow C(g) + 4H(g) \qquad \Delta E = 1.7 \times 10^3\ kJ \cdot mol^{-1}$$

The energy change of the reaction must be converted to a mass defect, so we apply Equation 11.4.

$$\Delta m = \frac{\Delta E}{c^2} = \frac{1.7 \times 10^6\ J \cdot mol^{-1}}{(2.998 \times 10^8\ m \cdot s^{-1})^2} = 1.9 \times 10^{-11}\ kg \cdot mol^{-1} = 1.9 \times 10^{-8}\ g \cdot mol^{-1}$$

The mass defect for this reaction is negligible because it is far less than can be measured on a laboratory balance. Indeed, the mass defects for all chemical reactions are negligible, which is the reason that the law of conservation of mass states that mass is conserved in *chemical reactions*. It is really an approximation, but it is a very good one for chemical reactions.

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Example 11.4

Show that a ^{56}Fe nucleus is thermodynamically favored over a ^{209}Bi nucleus. The atomic masses are: ^{56}Fe = 55.9349 and ^{209}Bi = 208.9804

The more stable nucleus is the one with the greater binding energy per nucleon, so we must first determine the number of protons and neutrons present in each nucleus from the periodic table.

$Z_{Fe} = 26$ and $Z_{Bi} = 83$ **protons**

The number of neutrons in each nucleus is then obtained by subtraction.

$N_{Fe} = A_{Fe} - Z_{Fe} = 56 - 26 = 30$ **neutrons**

$N_{Bi} = A_{Bi} - Z_{Bi} = 209 - 83 = 126$ **neutrons**

Next, determine the mass defect and then use Equation 11.4 to obtain the binding energy. Finally, divide the binding energy by the number of nucleons (A) to get the binding energy per nucleon.

^{56}Fe

$m_{nucleus} = m_{atom} - m_{electrons} = 55.9349 - 26(0.000549) = 55.9206\ g{\cdot}mol^{-1}$

$m_{nucleons} = m_{protons} + m_{neutrons} = 26(1.00728) + 30(1.00867) = 56.4494\ g{\cdot}mol^{-1}$

$\Delta m = m_{nucleons} - m_{nucleus} = 56.4494 - 55.9206 = 0.5288\ g{\cdot}mol^{-1} = 5.285\times10^{-4}\ kg{\cdot}mol^{-1}$

$\Delta E = mc^2 = (5.285\times10^{-4}\ kg{\cdot}mol^{-1})(2.998\times10^8\ m{\cdot}s^{-1})^2 = 4.750\times10^{13}\ J{\cdot}mol^{-1}$

$$\frac{\Delta E}{A} = \frac{4.750\times10^{13}\ J\cdot mol^{-1}}{56\ nucleons} = 8.482\times10^{11}\ J\cdot mol^{-1}\cdot nucleon^{-1}$$

^{209}Bi

$m_{nucleus} = m_{atom} - m_{electrons} = 208.9804 - 83(0.000549) = 208.9348\ g{\cdot}mol^{-1}$

$m_{nucleons} = m_{protons} + m_{neutrons} = 83(1.00728) + 126(1.00867) = 210.6967\ g{\cdot}mol^{-1}$

$\Delta m = 210.6967 - 208.9348 = 1.7619\ g{\cdot}mol^{-1} = 1.7619\times10^{-3}\ kg{\cdot}mol^{-1}$

$\Delta E = mc^2 = (1.7619\times10^{-3}\ kg{\cdot}mol^{-1})(2.998\times10^8\ m{\cdot}s^{-1})^2 = 1.5836\times10^{14}\ J{\cdot}mol^{-1}$

$$\frac{\Delta E}{A} = \frac{1.5836\times10^{14}\ J\cdot mol^{-1}}{209\ nucleons} = 7.5770\times10^{11}\ J\cdot mol^{-1}\cdot nucleon^{-1}$$

Although the total binding energy of ^{209}Bi is greater than that of ^{56}Fe, the binding energy per nucleon is greater for the ^{56}Fe nucleus. Consequently, the ^{56}Fe nucleus is thermodynamically favored.

PRACTICE EXAMPLE 11.1

Determine the binding energy per nucleon for a ^{64}Zn nucleus (atomic mass = 63.9291).

mass of protons:

mass of neutrons:

mass of nucleus:

mass defect:

binding energy:

binding energy per nucleon:

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11.2 NUCLEAR REACTIONS AND RADIOACTIVITY

In Example 11.4, we showed that ^{56}Fe is thermodynamically favored over ^{209}Bi, but both nuclei are stable. Indeed, most nuclei found in nature are stable. Nuclei that are not stable are said to be **radioactive**. Radioactive nuclei spontaneously emit particles and electromagnetic radiation to change into other more stable nuclei. Radioactive nuclei are also called **radioisotopes**. All of the first 83 elements except technetium (Z = 43) have at least one stable nucleus. However, the ^{209}Bi nucleus is the heaviest stable nucleus. Furthermore, many of the elements that have stable nuclei also have radioisotopes. In this section, we examine the different types of radioactive decay and present some observations that help us predict the mode of decay that a particular radioisotope is likely to undergo. We begin with a discussion about how nuclear reactions are written.

WRITING NUCLEAR REACTIONS

The atomic number (Z) is the number that characterizes an element. If a nucleus contains 17 protons, then it is the nucleus of a chlorine atom. The symbol Cl means Z =17 and *vice versa*. Thus, there is little reason to include both the atomic number and the symbol of an element. We include the mass number when referring to a specific isotope (*e.g.*, chlorine-35 is ^{35}Cl), but we do not usually include the atomic number. However, as in chemical reactions, nuclear reactions involve balancing both mass and charge. In a chemical reaction, the charge is given explicitly on each ion, but in a nuclear reaction, the charge is the charge on the nucleus, and that is given by the atomic number. Thus, the atomic number is included with the symbol in nuclear reactions to aid in charge balance. The element with the symbol X, a mass number A, and an atomic number Z is represented as ${}^{A}_{Z}X$. For example, the two isotopes of chorine are ${}^{35}_{17}Cl$ and ${}^{37}_{17}Cl$.

Table 11.2 lists the names and symbols of several small particles that are encountered in nuclear reactions. A neutron is represented as ${}^{1}_{0}n$ because it has a mass number of 1 but carries no charge. A proton is ${}^{1}_{1}p$, which indicates a mass number of 1 and a +1 charge. An electron has a mass number of zero and a charge of -1, so its symbol is ${}^{0}_{-1}e$. An electron is also called a β (beta) particle and is often represented as ${}^{0}_{-1}\beta$ or simply β^-. A **positron** has the same mass as the electron, but it is positively charged. It is represented as ${}^{0}_{+1}\beta$ or ${}^{0}_{+1}e$ or simply β^+. An **alpha particle** ($\alpha = {}^{4}_{2}\alpha = {}^{4}_{2}He$) is a helium-4 nucleus.

Table 11.2 Common particles in nuclear reactions

Particle	Second name	Symbol
proton		${}^{1}_{1}p$
neutron		${}^{1}_{0}n$
electron	beta particle	${}^{0}_{-1}e = {}^{0}_{-1}\beta = \beta^-$
positron		${}^{0}_{+1}e = {}^{0}_{+1}\beta = \beta^+$
helium nucleus	alpha particle	${}^{4}_{2}He = {}^{4}_{2}\alpha = \alpha$

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Like chemical equations, nuclear equations are balanced so as to conserve both mass and charge. However, in nuclear equations, mass is given by the mass numbers and charge by the atomic numbers. Thus, a balanced nuclear equation must have both

- *charge balance:* the sum of the atomic numbers (Z) of the products must equal the sum of the atomic numbers of the reactants; i.e., $\sum Z(\text{products}) = \sum Z(\text{reactants})$, and
- *mass balance:* the sum of the atomic masses of the products must equal the sum of the atomic masses of the reactants; i.e., $\sum A(\text{products}) = \sum A(\text{reactants})$.

Example 11.5

Identify the unknown particle, X, in each of the following reactions.

a) $^{69}Zn \rightarrow {}^{69}Ga + X$

First, rewrite the reactions to include the atomic numbers: $^{69}_{30}Zn \rightarrow {}^{69}_{31}Ga + {}^{A}_{Z}X$

Then apply charge balance to obtain Z: $30 = 31 + Z \Rightarrow Z = 30 - 31 = -1$

and mass balance to obtain A: $69 = 69 + A \Rightarrow A = 0$.

The particle with a mass number of zero and -1 charge is the electron, $^{0}_{-1}e$ or β^-

b) $^{27}Al + {}^{4}He \rightarrow n + X$

Rewriting with atomic numbers, we obtain $^{27}_{13}Al + {}^{4}_{2}He \rightarrow {}^{1}_{0}n + {}^{A}_{Z}X$

Applying charge balance, we obtain Z. $13 + 2 = 0 + Z \Rightarrow Z = 15$.

An atomic number of 15 indicates that the particle is a phosphorus nucleus.

Apply mass balance to obtain A. $27 + 4 = 1 + A \Rightarrow A = 27 + 4 - 1 = 30$

The particle is ^{30}P.

PRACTICE EXAMPLE 11.2

Identify X in each of the following:

$^{65}Zn \rightarrow {}^{65}Cu + X$

$^{59}Fe \rightarrow X + \beta^-$

$^{213}Bi \rightarrow X + \alpha$

TRENDS IN NUCLEAR STABILITY

Nuclear forces are not understood well enough to allow us to predict whether a nucleus is stable or not. However, we present two empirical observations about nuclear stability that indicate the importance of the neutron to proton ratio and the size of the nucleus.

Neutron/proton ratios: Neutrons play an important role in holding the nucleus together, and every stable nucleus (except ^{1}H and ^{3}He) contains at least one neutron per proton. Figure 11.2 shows the number of protons and neutrons in the stable nuclei. The relative number of protons and neutrons in the stable nuclei lie in a narrow band, referred to as the **band** or **belt of stability**. Only one neutron per proton is sufficient for the lighter elements. However, the number of neutrons exceeds the number of protons in the stable

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nuclei of the larger elements. Thus, the neutron/proton ratio remains near one through the third period (Z = 18) then it begins to rise reaching a maximum of 1.52 for $^{209}_{83}\text{Bi}\left(\frac{N}{Z}=\frac{209-83}{83}\right)$, the heaviest stable nucleus.

Total number of protons: There are no stable nuclei with atomic numbers greater than 83. All elements with Z > 83 are radioactive.

TYPES OF DECAY

We now consider how an unstable nucleus decays to a stable one. The decay pathway may involve many steps, but each step involves either the emission of one of three particles (α, β^-, or β^+) or the capture of an electron. The decay of an unstable nucleus to a more stable nucleus is an exothermic process, and the energy released in the decay is often carried away by the emitted particle; but, if the energy release is large, much of the energy is released in the form of gamma rays. γ-ray photons, which are represented as $^0_0\gamma$ or simply γ, do not enter into charge or mass balance considerations; they simply represent energy released in the process. In nuclear decay reactions, the decaying nucleus is referred to as the **parent** and the produced nucleus as the **daughter**. Refer to Figure 11.2 for the regions expected for each type of decay.

Alpha decay is the loss of an alpha particle. The loss reduces the mass number by four and the atomic number by two. *Alpha decay is the most common mode of decay for the heavy nuclei* because the alpha particle is the most massive particle of the common decay particles. ^{238}U undergoes α-decay to ^{234}Th:

$$^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^4_2\alpha$$

α-decay is not limited to heavier nuclei, but it is found in only a few of the lighter elements. ^{8}Be is the lightest element to undergo alpha decay.

$$^8_4\text{Be} \rightarrow 2\,{}^4_2\alpha$$

Beta decay is the ejection of an electron *by the nucleus*. It results in an increase of one in the atomic number. The electron that is emitted is produced by the disintegration of a neutron, $^1_0\text{n} \rightarrow {}^1_1\text{p} + {}^0_{-1}\text{e}$. Because β-decay results from the conversion of a neutron into a proton, it decreases the neutron/proton ratio. As such, *β-decay is the common mode of decay for those nuclei lying above the belt of stability*. For example, the neutron/proton ratio of ^{14}C is 8/6 = 1.3, which is well above the value of 1.0 found for stable nuclei of the

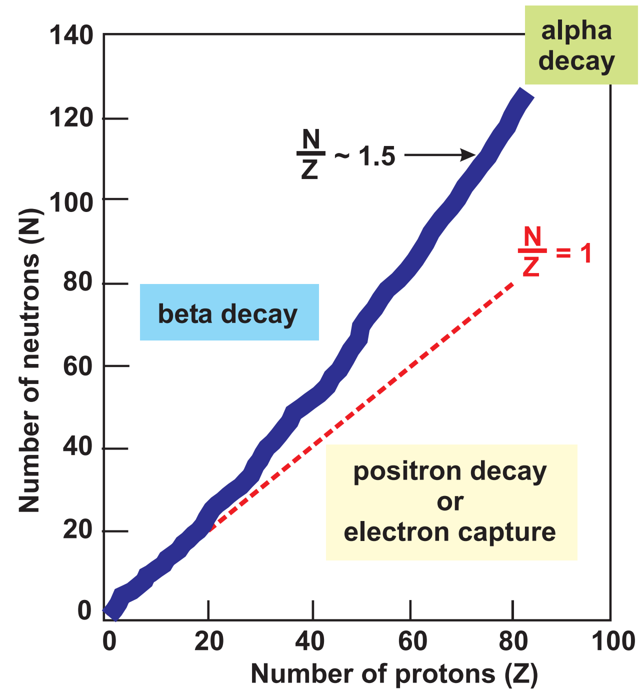

Figure 11.2 $^N/_Z$ ratios for the stable nuclei

The $^N/_Z$ = 1 line is shown for comparison. The expected modes of decay is given in each region.

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first three periods. Consequently ^{14}C undergoes β-decay to a stable ^{14}N nucleus with $N_n/Z = 7/7 = 1.0$.

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e}$$

α-decay, the most common decay among the heavy elements, is the loss of two protons and two neutrons, which increases N_n/Z slightly. Thus, successive α-decays produce isotopes with unfavorable N_n/Z ratios. Consequently, some heavy nuclei formed by α-decay undergo β-decay in order to maintain $N_n/Z \sim 1.5$. ^{234}Th, formed from the α-decay of ^{238}U, is a heavy nucleus and might be expected to undergo α-decay, but it also has a very high neutron/proton ratio of (234-90)/90 = 1.60. *Reducing a high neutron/proton ratio is usually favored over reducing the mass in heavy nuclei*. Consequently, ^{234}Th undergoes β-decay to ^{234}Pa, which has an N_n/Z ratio 1.57.

$$^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}\mathrm{e}$$

Positron decay is the emission of a positron. It has the opposite effect of β-decay. That is, positron decay converts a proton into a neutron ($^{1}_{1}\mathrm{p} \rightarrow {}^{1}_{0}\mathrm{n} + {}^{0}_{+1}\mathrm{e}$), which reduces the atomic number by one. As a result, *positron emission is a common mode of decay for nuclei below the belt of stability*. Positron emission of ^{13}N produces ^{13}C, which results in an increase of N_n/Z from 0.86 to 1.2.

$$^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + {}^{0}_{1}\mathrm{e}$$

A positron is the **antimatter** analog of the electron because it is identical to the electron in every respect except charge. Occasionally, the emitted positron collides with an orbital electron. The collision results in the annihilation of the two antiparticles (*i.e.*, the particles disappear as their mass is converted to energy: $\beta^- + \beta^+ \rightarrow \gamma$).

The last mode of decay we consider is **electron capture (EC),** the capture by the nucleus of an electron from an inner-shell orbital. EC, like positron emission, increases the neutron/proton ratio by converting a proton into a neutron, $^{1}_{1}\mathrm{p} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{1}_{0}\mathrm{n}$. It is a common decay used by nuclei below the belt of stability. For example, ^{7}Be ($N_n/Z = 0.75$) undergoes electron capture to become ^{7}Li ($N_n/Z = 1.3$).

$$^{7}_{4}\mathrm{Be} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^{7}_{3}\mathrm{Li}$$

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Example 11.6

a) **The most abundant isotope of krypton is ^{84}Kr. Predict the mode of decay of the radioactive nucleus ^{76}Kr and write the nuclear reaction for the decay.**

The atomic number of krypton is 36. N_n/Z = 1.33 for the stable isotope and 1.11 for the unstable isotope. Thus, N_n/Z is low for ^{76}Kr, so a proton must be converted into a neutron. This can be accomplished by either electron capture or positron emission. We cannot predict more than that with only the guidelines given above. However, ^{76}Kr undergoes electron capture, and the reaction is

$$^{76}_{36}\text{Kr} + ^{0}_{-1}\text{e} \rightarrow ^{76}_{35}\text{Br}$$

b) **Cobalt occurs naturally as ^{59}Co. Predict the mode of decay of the radioisotope ^{62}Co.**

The atomic number of cobalt is 27. The naturally occurring isotope contains 32 neutrons (59 - 27), so N_n/Z = 32/27 =1.19. ^{62}Co contains 35 protons, so N_n/Z = 1.30 for the unstable isotope. Thus, the N_n/Z ratio is high for ^{62}Co, and β^- decay is predicted. The decay process is

$$^{59}_{27}\text{Co} \rightarrow ^{0}_{-1}\text{e} + ^{59}_{28}\text{Ni}$$

The product, ^{59}Ni, has a neutron/proton ratio of $^{31}/_{28}$ = 1.11 and is stable.

c) **Identify the particle emitted by a chlorine-33 nucleus.**

The neutron/proton ratio for this second-period nucleus is 16/17 = 0.94, which is less than the stable value of 1.0, so we predict either electron capture or positron emission. The particle is emitted, so it must be a positron.

$$^{33}_{17}\text{Cl} \rightarrow ^{0}_{+1}\text{e} + ^{33}_{16}\text{S}$$

The product of the positron emission is ^{33}S, a stable isotope.

d) **Predict the mode of decay of ^{220}Fr and write the nuclear reaction.**

The atomic number exceeds 83, so α-decay is a possibility. However, we first check the neutron/proton ratio. N_n/Z = (220 – 87)/87 = 1.53, which is fairly close to the stable ratio of ~1.55 for large nuclei. Consequently, α-decay is the predicted mode of decay.

$$^{220}_{87}\text{Fr} \rightarrow ^{216}_{85}\text{At} + ^{4}_{2}\text{He}$$

PRACTICE EXAMPLE 11.3

^{56}Fe is the most abundant isotope of iron. Predict the mode of decay of ^{59}Fe and write the decay equation.

Z = ______ N_n(^{56}Fe) = ______ N_n(^{59}Fe) = ______

N_n/Z for ^{56}Fe =

N_n/Z for ^{59}Fe =

mode of decay expected is __________ decay

Decay Equation: ________________________________

What particle is emitted by ^{58}Cu? What stable nucleus results?

molar mass of copper = _____________

Based on the molar mass of copper, does a ^{58}Cu nucleus contain too many or too few neutrons?

The predicted mode of decay is __________ decay.

Decay Equation: ________________________________

What particle is emitted by ^{221}Ra?

Z = ______ N_n = ______ N_n/Z = ______

The likely mode of decay is ________ decay.

Decay Equation: ________________________________

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11.3 KINETICS OF RADIOACTIVITY

Radioactive decay is unimolecular, so it follows first-order kinetics as discussed in Section 10.4. One form of the integrated rate law for first-order kinetics was given in Equation 10.4b as the following:

$$ln\left(\frac{[A]}{[A]_o}\right) = -kt$$

[A] is the concentration of the radioisotope at time t, and $[A]_o$ is the radioisotope concentration at the beginning of the experiment. The concentrations can be expressed as the number of moles (n) divided by the volume (V).

$$ln\left(\frac{n/V}{n_o/V}\right) = -kt$$

The volumes in the above equation are identical and cancel. Furthermore, the ratio of moles of nuclei equals the ratio of their masses because the molar masses also cancel in the ratio. Making these two substitutions yields Equation 11.5,

$$ln\left(\frac{n}{n_o}\right) = ln\left(\frac{m}{m_o}\right) = -kt \qquad \textbf{Eq. 11.5}$$

where n is the number of moles of radioactive nuclei and m is its mass at time t. Equation 11.5 is the rate law for radioactive decay. It can be expressed without logarithms as shown in Equation 11.6.

$$n = n_o e^{-kt} \qquad \textbf{Eq. 11.6}$$

The rate constants for radioactive decays are most frequently given in terms of the half-life as given in Equation 10.6 and reproduced below.

$$t_{½} = \frac{ln\ 2}{k} = \frac{0.693}{k}$$

Example 11.7

a) Magnesium-23 undergoes positron decay. What is the product of the decay, and what is its half-life if 17.9% of the ^{23}Mg remained in a sample after 30.0 seconds?

We represent the decay as $^{23}_{12}Mg \rightarrow {}^{A}_{Z}X + {}^{0}_{1}e$. Balancing charge, we determine that for Na, Z = 11. Mass balance indicates that A = 23. Thus, the decay can be written as $^{23}_{12}Mg \rightarrow {}^{23}_{11}Na + {}^{0}_{1}e$.

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We now use Equation 11.5 to obtain the rate constant. We are told that $N/N_o = 0.179$ when t = 30.0, so we may write the following:

$$\mathit{ln}\ 0.179 = -k(30.0\ s) \Rightarrow k = -\frac{\mathit{ln}\ 0.179}{30.0\ s} = 0.0573\ s^{-1}$$

We can now use this rate constant and Equation 10.6 to determine the half-life.

$$t_{1/2} = \frac{0.693}{0.0573\ s^{-1}} = 12.1\ s$$

Every 12.1 seconds half of a ^{23}Mg sample decays no matter how large the sample is.

b) How long would it take for 99.9% of the ^{23}Mg to disintegrate?

We now know the rate constant and are asked for the time at which n/n_o reaches a particular value. We again apply Equation 11.5. A common mistake in a problem like this is to substitute the given fraction for n/n_o. However, n/n_o is the fraction remaining, not the fraction disintegrating. We use the fact that the fraction remaining is equal to one minus the fraction disintegrating.

$$\frac{n}{n_o} = 1.000 - 0.999 = 0.001$$

We use the preceding value in Equation 11.5 along with the rate constant determined in Part A to obtain the following.

$$\mathit{ln}\ 0.001 = -(0.0573\ s^{-1})t \Rightarrow t = -\frac{\mathit{ln}\ 0.001}{0.0573\ s^{-1}} = 120\ s$$

Thus, it would take only 2 minutes for 99.9% of this isotope to disappear.

RADIOACTIVE DATING

Radioactive dating is the process of determining the age of an object from its radioactive components. It is based on Equation 11.5, which indicates that the time required for some known initial amount of radioisotope to decay to another known amount can be determined if the rate constant (half-life) for the decay is known. Determining the amount of radioisotope present in the object today is straightforward, and we outline the approximations used in two techniques to obtain the initial amounts. One technique is used for historical time scales and the other for geological time scales.

Historical ages are frequently determined with carbon-14 dating, which is based on the fact that there is a constant exchange of carbon containing compounds between living organisms and the atmosphere. Atmospheric CO_2 is used in photosynthesis to produce

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organic compounds that are ingested by animals, and the carbon that was in the CO_2 becomes incorporated into the compounds in the organism. The organism returns some of the carbon back to the atmosphere in the form of CO_2 to continue the cycle. A small fraction of the carbon is in the form of radioactive ^{14}C, which is formed in the upper atmosphere by the following reaction:

$$^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}p$$

^{14}C then undergoes β-decay with a half-life of 5730 years ($k = \frac{ln\ 2}{5730\ yr} = 1.21\times10^{-4}\ yr^{-1}$).

$$^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e$$

The two competing processes have resulted in an equilibrium ^{14}C:^{12}C ratio of 1:10^{12} in the atmosphere. This ratio is also maintained in all living organisms and results in a ^{14}C radioactivity of 15.3 disintegrations per minute per gram of carbon ($d\cdot min^{-1}\cdot g^{-1}$). However, when the organism dies, it no longer replaces the decaying ^{14}C, and the disintegration rate drops. As shown in Example 11.8, the age of material can be estimated by measuring the rate of ^{14}C disintegration and assuming that the rate has remained constant.

Example 11.8

A piece of charred bone found in the ruins of an American Indian village has a ^{14}C disintegration rate of 9.2 $d\cdot min^{-1}\cdot g^{-1}$. What is the approximate age of the bone?

We assume that, when the organism died, the disintegration rate was 15.3 $d\cdot min^{-1}\cdot g^{-1}$, so n/n_o is determined as follows:

$$\frac{n}{n_o} = \frac{9.2}{15.3} = 0.60$$

Use this ratio and the known rate constant for ^{14}C decay ($k = 1.21\times10^{-4}\ yr^{-1}$) in Equation 11.5 to determine how many years have passed since the animal died.

$$ln\ 0.60 = -(1.21\times10^{-4}\ yr^{-1})t \Rightarrow t = -\frac{ln\ 0.60}{1.21\times10^{-4}\ yr^{-1}} = 4.2\times10^{3}\ yr$$

The bone belonged to an animal that died over four thousand years ago.

PRACTICE EXAMPLE 11.4

^{132}I undergoes beta decay to ^{132}Xe with a half-life of 2.3 hours. How old is a sample that is 85% ^{132}Xe if it was pure ^{132}I initially?

k =

Carbon dating can be used to estimate the age of materials that are up to 50,000 years old. The rate of decay for older objects is too slow to be measured. Thus, when a geological age is required, a radioisotope with a much longer half-life must be used. One method used to determine the age of rocks is based on the decay of ^{238}U to ^{206}Pb, a process with a half-life of 4.5×10^{9} (4.5 billion) years. In this method, it is assumed that all

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of the ^{206}Pb found in the rock originated from ^{238}U, so

$$\frac{n}{n_o} = \frac{\text{moles of } ^{206}\text{Pb in the sample}}{(\text{moles of } ^{206}\text{Pb + moles of } ^{238}\text{U) in the sample}}$$

This presumes that none of the lead was in the rock when it was formed, which is an acceptable assumption if there is not much of the more abundant ^{208}Pb present.

11.4 NUCLEAR RADIATION AND LIVING TISSUE

When a substance absorbs visible or ultraviolet light, one of its electrons is excited into an excited state, but the energy is soon given off as heat or light when the electron returns to the ground state. The electron remains in the atom during the process, so the radiation is said to be **non-ionizing radiation**. Radio and TV waves, microwaves, and infrared radiation are also non-ionizing. However, the energy of x-rays and γ-rays is so great that their absorption results in the loss of the electron and the production of an ion. Thus, x-rays and γ-rays are said to be **ionizing radiation**. α-particles and β-particles are also ionizing. In this section, we discuss some of the harmful effects of ionizing radiation.

In order for ionizing radiation to be harmful, it must encounter tissue. Thus, ionizing radiation produced in an experiment conducted in a laboratory next door would have to pass through at least one wall and your clothing before it could harm you. The ability of radiation to pass through material is called its **penetrating power**. The penetrating power decreases as the mass and charge of the particle increases. Alpha particles are both highly charged and massive, which results in very poor penetrating power. α-particles are stopped by a piece of paper or by the layer of dead skin cells covering the body. They can be very damaging to internal organs, but they must be ingested or inhaled to do so.

Approximately 40% of the background radiation to which humans are exposed is produced by radon that is formed by the decay of ^{238}U to ^{206}Pb. The other members of the decay pathway are also radioactive, but they are solids and remain in the rock. However, radon is a gas and can pass from the rocks into our homes. It is a source of α-particles $\left({}^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\text{He},\ t_{1/2} = 3.8 \text{ days}\right)$ that has been attributed to up to 10% of lung cancer deaths. As a gas, radon is readily inhaled and, after inhalation, the resulting α-particles can bombard the lung tissue. In addition, ^{218}Po is also an alpha emitter ($t_{1/2}$ = 3 minutes), but it is a solid and is not exhaled. ^{218}Po in the lungs bombards the lung tissue constantly, which damages the tissue and the growth-regulation mechanism of the cells; causing the uncontrolled cell reproduction called cancer.

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Beta particles are not as highly charged and not nearly as massive as α-particles. Consequently, they have greater penetrating power. However, even β-particles are stopped by a sheet of metal or wood. β-particles can cause damage to the skin and the surface of organs, but they also do their worst damage if ingested or inhaled. Gamma rays are photons and have excellent penetrating power because they have neither charge nor mass. Dense materials like lead or concrete are required to stop γ-rays. Recall that γ-rays are used to carry excess energy away from a nuclear reaction. Consequently, many radioisotopes emit γ-rays. ^{60}Co is a γ-emitter that is used in cancer treatment by bombarding the tumor with γ-rays to destroy the cancerous cells.

11.5 NUCLEAR FISSION

Nuclear fission is the process of splitting a large nucleus into smaller nuclei. These processes are extremely exothermic and are the basis for nuclear power plants and weaponry. We consider the most common example, the fission of a ^{235}U nucleus after neutron capture.

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{92}_{36}\text{Kr} + ^{141}_{56}\text{Ba} + 3^{1}_{0}\text{n}$$

As with all nuclear reactions, the energy change for this reaction can be determined from the mass defect as discussed in Section 11.1 and demonstrated in Example 11.9.

Example 11.9

Determine how much energy is released when 1.00 g of ^{235}U undergoes the above fission reaction.

The atomic masses are: ^{235}U = 235.0439; ^{92}Kr = 91.9263; and ^{141}Ba = 140.9144

The mass defect for a reaction is Δm = mass of products – mass of reactants.

mass of products: **3(1.0087) + 91.9263 + 140.9144 = 235.8668 g·mol^{-1}**

mass of reactants: **235.0439 + 1.0087 = 236.0526 g·mol^{-1}**

Δm = 235.8668 – 236.0526 = -0.1858 g·mol^{-1} = -1.858x10^{-4} kg·mol^{-1}

The mass defect is converted to energy by using Equation 11.4: $\Delta E = \Delta mc^2$

Δm must be expressed in kg and c is the speed of light in m·s^{-1}.

$\Delta E = (-1.858\text{x}10^{-4}\ \text{kg·mol}^{-1})(2.998\text{x}10^{8}\ \text{m·s}^{-1})^2 = -1.670\text{x}10^{13}\ \text{J·mol}^{-1}$

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Finally, use the molar ΔE above to determine ΔE for 1.00 g of ^{235}U

$$1.00\text{ g U}\times\frac{1\text{ mol U}}{235.0439\text{ g U}}\times\frac{-1.670\times10^{13}\text{ J}}{\text{mol U}}=-7.11\times10^{10}\text{ J}$$

Thus, the fission of 1 g of uranium-235 *releases* $7.11\text{x}10^7$ kJ of energy. Compare that with burning 1 gal of octane (a component of gasoline), which liberates 10^5 kJ of heat. In other words, the fission of one gram of uranium-235 produces the same amount of energy as the combustion of about 600 gallons of gasoline.

The fission reaction of ^{235}U considered above is represented in Figure 11.3. A single neutron (a) starts the reaction, which produces three more neutrons (b). Reaction of each of these three neutrons produces the nine neutrons shown in (c). If each of these nine neutrons goes on to react with nine ^{235}U nuclei, 27 neutrons would be produced in the next step. In general, 3^n neutrons are produced in the n^{th} step. Thus, in the 10^{th} step, 3^{10} or 59,049 neutrons are produced. Reactions like the fission of ^{235}U in which one of the products of the reaction initiates further reaction are called **chain reactions**.

The ^{235}U fission reaction involves a bimolecular collision between a neutron and a ^{235}U nucleus. Consequently, the rate of this elementary reaction is proportional to the product of the two concentrations:

rate = k[n][^{235}U] **Eq. 11.7**

where [n] is the concentration of neutrons. As the reaction proceeds, the concentration of neutrons increases faster than the concentration of ^{235}U decreases, which causes the rate of the reaction to increase. Furthermore, each step of the reaction produces three times the energy of the previous step. If it is not controlled, the chain reaction results in an explosion as a vast amount of energy is released in a very short period of time.

Equation 11.7 indicates that the rate of fission can be reduced by reducing either the neutron concentration or the uranium-235 concentration. ^{235}U does not undergo a chain reaction in nature because both concentrations are low. The natural abundance of ^{235}U in uranium ore is only 0.7%, which means that [^{235}U] is low. Indeed, the uranium must be enriched to levels of around 4% if it is to serve as a nuclear fuel. Even enriched uranium does not get out of control if the sample size is kept small. This is because many of the neutrons produced in the fission process are near the surface and escape the sample without colliding with other ^{235}U nuclei. However, as the sample size increases, the fraction of neutrons initiating fission increases. The minimum mass of uranium required

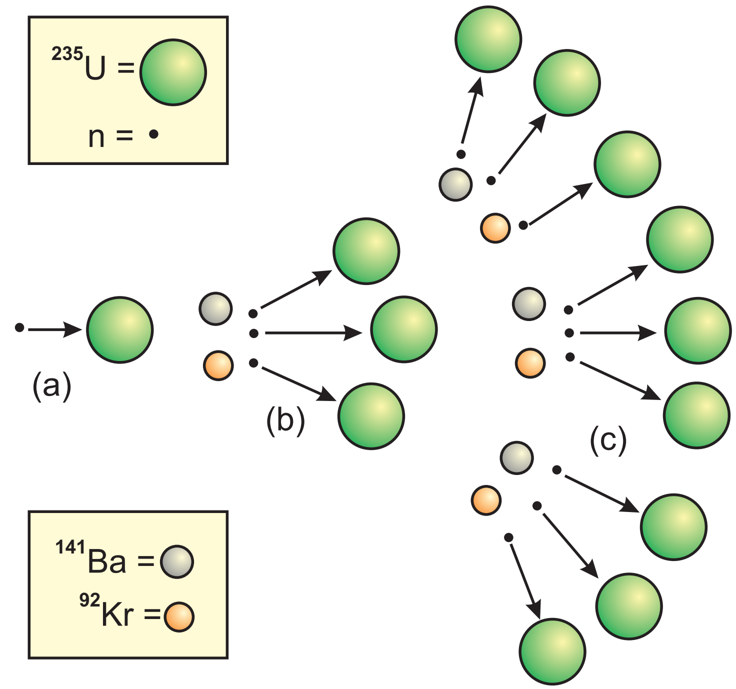

Figure 11. 3 Fission of ^{235}U results in a chain reaction

a) A single neutron collides with a single ^{235}U nucleus to produce $^{141}Ba + {}^{92}Kr + 3n$.
b) The three neutrons produced in (a) collide with three other ^{235}U nuclei to produce $3{}^{141}Ba + 3{}^{92}Kr + 9n$.
c) The nine neutrons produced in (b) collide with nine other ^{235}U nuclei. The result would be $9{}^{141}Ba + 9{}^{92}Kr + 27n$.

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to maintain a chain reaction is called the **critical mass**. At the critical mass, one neutron from each fission encounters a uranium nucleus. Masses that are less than the critical mass are said to be **subcritical**. Subcritical masses cannot sustain a chain reaction because less than one neutron per fission initiates a subsequent fission. Masses in excess of the critical mass are called **supercritical**. In a supercritical mass, most of the neutrons initiate further reaction. The critical mass of ^{235}U, which depends upon its purity, the shape of the sample, and the energy of the neutrons, ranges from about 15 kg to over 50 kg.

The atomic bomb is an example of uncontrolled fission. The design of the first bomb, shown schematically in Figure 11.4, is quite simple. It is transported with the fissionable uranium divided into two sections, each with a subcritical mass and located at the opposite ends of a large gun barrel. A chemical explosive, TNT, is used to send one subcritical mass into the other. The combined mass of the two samples exceeds the critical mass, and an uncontrolled chain reaction is initiated. The first bomb dropped on Japan at the end of World War II produced an explosion equivalent to 19, 000 tons of TNT.

A **nuclear reactor**, which is shown schematically in Figure 11.5, is a controlled chain reaction. Enriched ^{235}U in the form UO_2 is contained in fuel rods, which are tubes made of a zirconium alloy. The reaction is controlled by maintaining a constant rate of reaction (Equation 11.7). At the beginning of the reaction, the concentration of ^{235}U is relatively high, but it drops throughout the reaction. In order to maintain a constant rate of reaction, the concentration of neutrons used in the reaction must increase at the same rate that the concentration of ^{235}U decreases. The number of neutrons is controlled with rods made from cadmium or boron, both of which absorb neutrons. By adjusting the height of these **control rods,** the rate at which neutrons strike ^{235}U nuclei can be maintained at a constant level. When there is new fuel present, the rods are lowered to capture a greater number of neutrons, but as the fuel is consumed, the rods are raised to increase the number of neutrons available to initiate fission. The control rods can also be lowered completely to shut off the reactor.

Heat generated by the nuclear reaction is carried out of the reactor core by high-pressure water (300 °C, 2250 psi) in the primary water loop. Over 300,000 gal·min^{-1} can flow through this loop in a large reactor. The heat is used to boil water in a steam generator. The escaping steam in a secondary water loop drives a turbine connected to a generator. The steam leaving the turbine is condensed and cooled in the condenser with cooling water from a lake or river. The cooled water is then returned to the steam generator. The cooling water leaves the condenser about 20 °C warmer than it enters, an

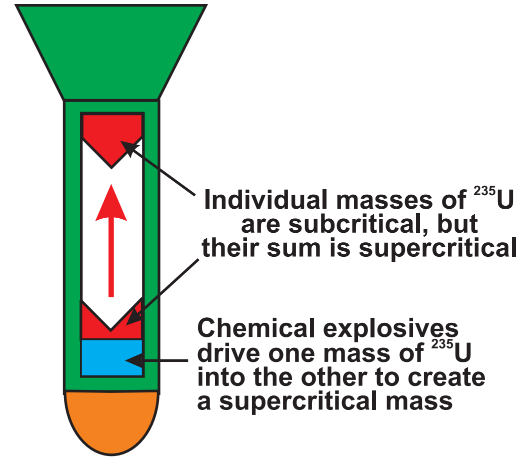

Figure 11.4 Schematic of an atomic bomb
Chemical explosive (TNT) is used to drive one subcritical mass into another. If the sum of the two subcritical masses exceeds the critical mass, an uncontrolled chain reaction is initiated.

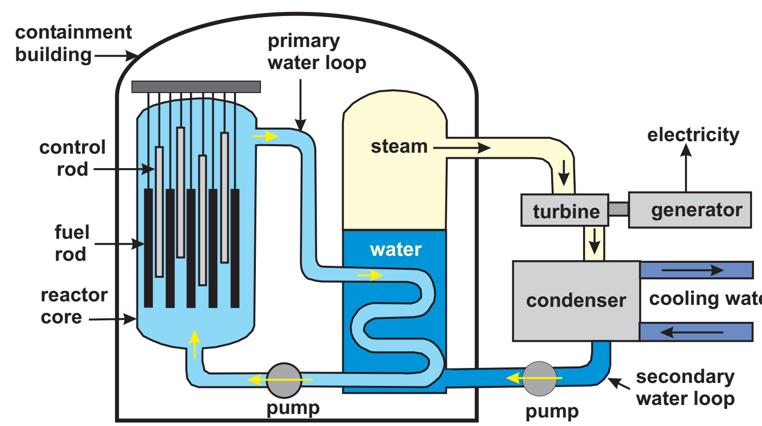

Figure 11.5 Schematic of a nuclear power plant

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increase that would heat the lake or river beyond safe levels. Consequently, cooling towers or canals are constructed to allow the heat in the water to dissipate prior to returning the water to the river or lake. It should be noted that no mixing occurs between the primary loop, secondary loop, or cooling water. Both the primary and secondary loops are self-contained.

The fuel in a nuclear plant cannot explode like an atomic bomb, but if the reaction gets out of control, the reactor can experience a ‘melt down’. The worst nuclear disaster occurred at Chernobyl in the Ukraine in 1986. Operators disabled the safety system to carry out some tests. During the tests, the reactor cooled and nearly shut down, so, to avoid a costly shut down, they removed most of the control rods. In the absence of the control rods and with the safety system disabled, the reactor heated beyond safe limits. The excess heat boiled the superheated water and melted the fuel rods, which then mixed with the superheated water. High-pressure steam generated by boiling the superheated water blew off the top of the reactor, and spread the radioactive fuel into the atmosphere. A malfunction of the cooling system was also responsible for the Three Mile Island accident in 1979, but no explosion accompanied that partial melt down and only a very small amount of radiation was released.

Nuclear reactors are built with many levels of safeguards that have proved effective in preventing accidents except in the case of gross operator error. However, there is one other problem presented by the use of nuclear power. The major concern surrounding nuclear power today is nuclear waste disposal. Not all of the radioactive fuel in the fuel rod can be consumed, and many of the products of the fission reactions are radioactive with long half-lives. Three problems arise: where do you store this radioactive waste, how do you get it there, and how do you keep it secure? Nobody wants to live near a nuclear waste site, and there is major opposition to the transport of radioactive material along our highways and railways. Consequently, the spent fuel rods are usually kept at sites close to the reactor. There are bills before congress to build a national repository for radioactive waste. A nuclear waste repository has been approved for Yucca Mountain, Nevada. The plan for nuclear waste storage calls for the transportation of the waste by railroad. However, as with any plan calling for cross-country transportation of nuclear waste, there is still a great amount of opposition due to safety and security concerns.

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11.6 NUCLEAR FUSION

In **nuclear fusion**, two lighter atoms combine, or *fuse,* to form a heavier atom. It is the process that powers the sun and other stars. As in fission, some of the mass of the fusing nuclei is converted into energy. The most studied fusion reaction is the fusion of *deuterium* (^{2}H) with *tritium* (^{3}H) to form helium and a neutron:

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n \qquad \Delta E = -1.7\times10^{9}\ kJ$$

Even with a natural abundance of only 0.015 %, deuterium is a readily available isotope because it is present in water. Tritium atoms can be prepared by bombarding lithium atoms with the neutrons released in the above reaction:

$$^{6}_{3}Li + ^{1}_{0}n \rightarrow ^{3}_{1}H + ^{4}_{2}He \qquad \Delta E = -4.6\times10^{8}\ kJ$$

The fusion of deuterium and tritium offers almost limitless energy.

The reason we do not have fusion power plants is that the activation energy for a fusion reaction is enormous. The potential energy of two nuclei as a function of the distance between them rises very sharply at distances less than the bond length. The rise in energy is due to the repulsion between the two positively charged nuclei. In order for fusion to occur, this high repulsion energy must be overcome. Consequently, extremely high temperatures are required to bring about fusion. For this reason, fusion reactions are also called **thermonuclear**. Instead of a critical mass that must be exceeded, fusion reactions have temperatures that must be exceeded. The fusion of deuterium and tritium has the lowest threshold temperature for any fusion reaction, a mere 40,000,000 K! The uncontrolled fusion of deuterium and tritium is called a hydrogen bomb. The threshold temperatures required for the fusion in a hydrogen bomb are achieved by first detonating a fission bomb!

In order to achieve controlled fusion, the nuclei not only have to have sufficient energy to fuse, they must also be held together long enough for fusion to occur. As we shall see in Section 11.7, stars use enormous gravitational fields to both heat the nuclei and to confine them. Scientists are trying two techniques to produce fusion in the laboratory. In **magnetic confinement**, the nuclei are confined by a strong magnetic field and heated by powerful microwaves. In **inertial confinement**, a pellet of frozen hydrogen is compressed and heated by an intense energy beam so quickly that fusion occurs before the atoms can fly apart. Fusion has been achieved in the laboratory, but the nuclei fly apart before a self-sustained reaction can be initiated. Consequently, more energy is pumped

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into the system than is extracted from it. However, it is expected that fusion reactions that produce more energy than they consume will be achieved relatively soon, but the predictions are that commercial fusion will not be available for decades.

11.7 ORIGIN OF THE HEAVY ELEMENTS

Nature has mastered fusion in nuclear reactors called stars, and the by-products of these thermonuclear reactions are the elements that populate the periodic table. The universe is comprised mostly of hydrogen, and the story of how the heavier elements came into being is illuminating.

Hydrogen atoms in space are attracted to one another by gravitational forces. As the number of atoms that are attracted to one another increases, the gravitational forces between the atoms also increases, causing the system to begin to collapse. As the body of hydrogen atoms collapses, the pressure at the center begins to build, and the increase in pressure results in an increase in temperature. If there is sufficient mass, the system continues to collapse until the temperature reaches about $4\text{x}10^7$ K, at which point the density is about 100 $\text{g}\cdot\text{cm}^{-3}$. At this temperature, the protons begin to fuse, and a star is born. Further collapse of the star is offset by the enormous energy released by the fusion process, and the star stabilizes as long as the fuel lasts. The overall reaction is

$$4\,{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{0}_{1}\text{e} + 2\gamma$$

After about 10% of the hydrogen has been consumed, the core again begins to collapse. When the temperature reaches about $2\text{x}10^8$ K and the density is around 10,000 $\text{g}\cdot\text{cm}^{-3}$, ^{4}He begins to burn:

$$3\,{}^{4}_{2}\text{He} \rightarrow {}^{12}_{6}\text{C}$$

The energy released by burning helium expands the hydrogen into a sphere over a hundred times larger than the original star. At this point, the star is called a *red giant*. When the concentration of ^{12}C gets sufficiently high, it begins to burn and produce other elements.

$$^{12}\text{C} + {}^{4}\text{He} \rightarrow {}^{14}\text{N} + {}^{2}\text{H}$$
$$^{12}\text{C} + {}^{4}\text{He} \rightarrow {}^{16}\text{O}$$
$$^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{24}\text{Mg}$$
$$^{12}\text{C} + {}^{16}\text{O} \rightarrow {}^{28}\text{Si}$$
$$^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{23}\text{Na} + {}^{1}\text{H}$$

Further collapse and heating produces elements up to ^{56}Fe. Reactions of this type are

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highly exothermic, but reactions to form elements heavier than ^{56}Fe are endothermic (refer to Figure 11.1) and are produced by neutron capture, which is a very slow process. Thus, once a star contains mostly ^{56}Fe there is no further nuclear fuel and the star collapses to a *white dwarf* with a radius similar to earth's and a density of 10^4 to 10^8 g·cm^{-3}. This is the fate that awaits our sun.

However, if the star is large enough, the collapse continues to even greater densities and temperatures of 4x10^9 K, where many neutron-releasing reactions are initiated:

$$^{56}\text{Fe} + \text{energy} \rightarrow 13\ ^{4}\text{He} + 4\ ^{1}\text{n}$$

This final collapse occurs in minutes or less with the release of immense amounts of energy and neutrons. The elements in the outer shell of the star absorb many neutrons almost simultaneously and very large masses (A = 238) are achieved. The shell is then blown off at near the speed of light in a *supernova*, leaving a core of many solar masses, a diameter ~10 km, and a density of 10^{14} g·cm^{-3}. At such pressures, electrons are captured by the protons to form neutrons. Eventually, the core consists of nothing but neutrons and is called a *neutron star*. It is interesting to realize that all of the atoms that are heavier than iron were formed in supernovas, which makes a gold necklace all the more interesting.

11.8 CHAPTER SUMMARY AND OBJECTIVES

Atoms with unstable nuclei emit particles to become different atoms that have more stable nuclei in a process known as nuclear decay. We considered three types of decay:

- α-decay (loss of a ^{4}He nucleus);
- β^--decay (loss of an electron produced by the conversion of a neutron to a proton); and
- β^+-decay (loss of a positron produced when a proton is converted into a neutron.

Nuclei can also capture inner shell electrons in a process known as electron capture. The captured electron converts a proton into a neutron.

The mode of decay of an unstable nucleus depends upon the ratio of neutrons to protons. If N_n/Z is greater than the stable ratio, a neutron is converted into a proton by β^- decay. If N_n/Z is less than the stable ratio, a proton is converted into a neutron by positron emission or electron capture. The most common mode of decay for nuclei with Z > 83 is α-decay.

Nuclei are held together by their binding energy, $\Delta E = \Delta mc^2$, where Δm, the mass defect, is equal to the mass of the nucleons minus the nuclear mass. The binding energy per nucleon is a measure of the thermodynamic stability of the nucleus.

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All nuclear decay is unimolecular and follows first order kinetics. The first-order rate constants are usually given in terms of half-lives. The age of certain materials can be determined by measuring the relative amounts of certain isotopes that they contain in a process called radioactive dating. The ratio of ^{14}C present in the sample to the amount present in a living organism is used to estimate the age of the sample.

Nuclear fission is the breaking apart of a nucleus into smaller nuclei. The energy changes that accompany nuclear fission can be enormous. Fission of one gram of uranium-235 releases as much energy as the combustion of 600 gallons of gasoline. A chain reaction is a reaction that produces more reactant than it consumes. Fission of ^{235}U produces more neutrons than it consumes, so their concentration increases and the rates of reaction and energy production also increase with time. In the controlled fission of a nuclear reactor, the rate of reaction is regulated with control rods to absorb excess neutrons. Atomic bombs are examples of uncontrolled fission.

In fusion (or thermonuclear reactions), two smaller nuclei are combined to form a larger one. These processes also produce vast amounts of energy. However, they also have extremely high activation energies, and the lowest temperature at which fusion can occur is forty million degrees. Fusion offers an almost limitless supply of energy, but the enormous technological barriers, such as temperatures in excess of forty million degrees and a way to confine the nuclei at these temperatures long enough for them to fuse, must be overcome.

After studying the material presented in this chapter, you should be able to:

1. determine the number of protons and neutrons in a nucleus given its symbol or atomic number and its mass number (Section 11.1);
2. determine the atomic weight of an element from the masses and natural abundance of its isotopes (Section 11.1);
3. calculate the mass defect of a nucleus from the number of protons and neutrons it contains and the mass of the nucleus (Section 11.1);
4. convert between mass and energy (Section 11.1);
5. determine the binding energy per nucleon of a nucleus and predict which of several nuclei is most stable (Section 11.1);
6. define the terms radioactive and radioisotopes (Section 11.2);
7. identify a missing particle in a nuclear reaction (Section 11.2);
8. identify the decay particles by name and symbol (Section 11.2);
9. predict the probable mode of decay of an unstable nucleus (Section 11.2);

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10. determine the time required for a given fraction of a radioactive material to disappear given the half-life or rate constant for the decay (Section 11.3);
11. determine the age of an organic material given its ^{14}C rate of decay, the rate of ^{14}C decay in living organisms and the half-life of ^{14}C (Section 11.3);
12. distinguish between ionizing and non-ionizing radiation (Section 11.4);
13. compare the penetrating power of α-particles, β-particles and γ-rays (Section 11.4);
14. describe nuclear fission and chain reactions (Section 11.5);
15. define critical mass and explain its origin (Section 11.5);
16. explain how fission is controlled in a nuclear reactor (Section 11.5);
17. describe what is meant by ‘melt down’ (Section 11.5);
18. describe nuclear fusion and the problems associated with controlling it (Section 11.6); and
19. explain where and how elements are formed (Section 11.7).

ANSWERS TO PRACTICE EXAMPLES

11.1 8.434×10^{-11} J·mol^{-1}·nucleon^{-1}

11.2 β^{+}; ^{59}Co; ^{209}Tl

11.3 $^{59}Fe \rightarrow \beta^{-} + {}^{59}Co$

$^{58}Cu \rightarrow \beta^{+} + {}^{58}Ni$

$^{221}Ra \rightarrow \alpha + {}^{217}Rn$

11.4 6.3 hours

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11.9 EXERCISES

THE NUCLEUS

1. Indicate the number of neutrons in each of the following nuclei.

a) ^{10}Be **b)** ^{100}Mo **c)** ^{75}As **d)** ^{197}Au

2. Indicate the number of neutrons in each of the following nuclei.

a) ^{192}Hg **b)** ^{115}Sn **c)** ^{34}S **d)** ^{85}Rb

3. Write the symbol, including atomic number and mass, for each of the following isotopes.

a) $Z = 26, A = 56$ **b)** $A = 74, N_n = 40$ **c)** $Z = 54, N_n = 78$

4. Write the symbol, including atomic number and mass, for each of the following isotopes.

a) $Z = 46, N_n = 64$ **b)** $A = 110, Z = 48$ **c)** $A = 212, N_n = 129$

5. There are three naturally occurring isotopes of silicon. Use the data below to determine the atomic mass of silicon.

	Mass (amu)	Abundance
^{28}Si	27.97693	92.21%
^{29}Si	28.97649	4.70%
^{30}Si	29.97376	3.09%

6. There are two naturally occurring isotopes of lithium: ^{6}Li and ^{7}Li, with atomic masses of 6.01512 and 7.01600, respectively. If the atomic mass of lithium is 6.939, what is the natural abundance of ^{6}Li?

7. The natural abundance of deuterium is 0.015%. How many deuterium nuclei are present in 100. mL of water?

NUCLEAR STABILITY

8. What is meant by the term 'band of stability'?

9. Determine the mass defects (in $kg \cdot mol^{-1}$) for the following nuclei.

a) ^{79}Br (Mass = 78.9183 amu)

b) ^{99}Ru (Mass = 98.9061 amu)

10. Determine the mass defects (in $kg \cdot mol^{-1}$) for the following nuclei.

a) ^{142}Ce (Mass = 141.9090 amu)

b) ^{40}Ca (Mass = 39.96259 amu)

11. What are the binding energies and binding energies per nucleon for each of the nuclei in Exercise 9?

12. What are the binding energies and binding energies per nucleon for each of the nuclei in Exercise 10?

13. Which nucleus in Exercise 11 is thermodynamically more stable?

14. Which nucleus in Exercise 12 is thermodynamically more stable?

NUCLEAR REACTIONS AND RADIOACTIVITY

15. Predict the mode of decay for each of the following:

a) ^{233}U **b)** ^{197}Pb **c)** ^{231}Ac **d)** ^{225}Th

16. Predict the mode of decay for each of the following:

a) ^{110}Rh **b)** ^{98}Pd **c)** ^{6}He **d)** ^{25}Al

17. Identify X in each of the following nuclear reactions:

a) $^{144}Nd \rightarrow {}^{140}Ce + X$

b) $^{238}U + n \rightarrow 3n + {}^{81}Ge + X$

c) $^{16}O + \alpha \rightarrow X$

18. Identify X in each of the following nuclear reactions.

a) $^{69}Ga + n \rightarrow X$

b) $^{235}U + n \rightarrow 2n + {}^{100}Mo + X$

c) $^{35}Cl + p \rightarrow \alpha + X$

19. Write complete nuclear reactions for the following:

a) Potassium-40 undergoes beta decay.

b) Chlorine-34 emits a positron.

c) Arsenic-73 undergoes electron capture.

d) Bismuth-214 decays to thallium-210.

20. Write complete nuclear reactions for the following.

a) Thorium-229 undergoes alpha decay.

b) Gold-198 emits a beta particle.

c) Antimony-118 emits a positron.

d) Cadmium-115 decays to indium-115.

21. Radon-222 undergoes the following decay sequence to a stable nucleus: α, α, β, β, α, β, β, α. What is the identity of the resulting nucleus?

22. Uranium-238 undergoes the following decay sequence: α, β, β, α, α, α, α, β, β, α, β. What is the identity of the last nucleus?

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KINETICS OF RADIOACTIVITY

23. ^{239}Pu is a very toxic material used in nuclear weapons that has a half-life of 2.44×10^4 years. How long will a sample of Pu have to be stored before only 1% of the original sample remains?

24. ^{131}I is a β-emitter that is used to treat thyroid disorders. If its half-life is 8.070 days, how many days are required to rid the body of 95% of any ingested ^{131}I?

25. A 12.30-mg sample of ^{47}Ca is found to contain 3.24 mg of ^{47}Sc after 2.00 days, what is the half-life of ^{47}Ca in days? What type of decay does ^{47}Ca undergo?

26. A 4.56-mg sample of ^{228}Th, an α emitter, contains 2.58 mg of ^{228}Th after 575 days. What is the half-life of ^{228}Th in years?

27. The Shroud of Turin is a long linen cloth that bears an image of a bearded, longhaired man, with numerous lacerations over his body. Tradition, dating back to the fourteenth century, has it that the fabric is the burial shroud of Jesus Christ. In 1988, its age was determined by carbon dating. If a fiber of the shroud had a ^{14}C disintegration rate of 14.0 $d\cdot min^{-1}\cdot g^{-1}$, how old was the cloth. What conclusion can be drawn about the authenticity of the claim that it is the burial cloth of Jesus Christ? (The rate of decay of living organisms is 15.3 $d\cdot min^{-1}\cdot g^{-1}$, and the half-life of ^{14}C is 5730 years.)

28. The wood on an Egyptian coffin had a ^{14}C disintegration rate of 11.7 $d\cdot min^{-1}\cdot g^{-1}$, how old is the coffin? (The rate of decay of living organisms is 15.3 $d\cdot min^{-1}\cdot g^{-1}$, and the half-life of ^{14}C is 5730 years.)

29. How old is a rock sample from a meteor if it contains 73.2 mg of ^{238}U and 20.2 mg of ^{206}Pb? Assume that all of the ^{206}Pb was formed from ^{238}U. The half-life of the $^{238}U \rightarrow {}^{206}Pb$ process is 4.5×10^9 years.

30. Geological times can also be estimated by Argon dating. ^{40}K undergoes electron capture to ^{40}Ar with a half-life of 1.28×10^9 years. Estimate the age of a moon rock sample if its $^{40}Ar/^{40}K$ mass ratio is 10.4.

NUCLEAR RADIATION

31. List beta particles, gamma rays and alpha particles in order of increasing penetrating power.

32. Why are houses checked for radon? How does radon get into a home?

NUCLEAR FISSION AND NUCLEAR FUSION

Use the following atomic masses and those in Table 11.1 for Exercises 33 and 34.

^{4}He	4.0026	^{13}C	13.0034	^{31}P	30.9737
^{16}O	15.9949	^{24}Mg	23.9850	^{226}Ra	226.0254
^{68}Zn	67.9248	^{72}Ge	71.9221	^{238}U	238.0508
^{230}Th	230.0331	^{234}Th	234.0436		
^{239}Pu	239.0522	^{242}Cm	242.0588		

33. Determine the mass defect in kilograms of each of the following reactions.

a) $2\,^{12}C \rightarrow {}^{24}Mg$ **b)** $^{238}U \rightarrow {}^{234}Th + \alpha$

c) $\beta^+ + \beta^- \rightarrow \gamma$ **d)** $^{239}Pu + {}^{4}He \rightarrow {}^{242}Cm + n$

34. Determine the mass defect in kilograms of each of the following reactions.

a) $^{230}Th \rightarrow {}^{226}Ra + \alpha$ **b)** $2\,^{16}O \rightarrow {}^{31}P + {}^{1}H$

c) $^{13}C + {}^{4}He \rightarrow {}^{16}O + n$ **d)** $^{68}Zn + {}^{4}He \rightarrow {}^{72}Ge$

35. What is the energy change of each reaction listed in Exercise 33?

36. What is the energy change of each reaction listed in Exercise 34?

37. Classify each reaction in Exercise 33 as fission, fusion, decay or annihilation. If it is a decay, indicate what kind.

38. Classify each reaction in Exercise 34 as fission, fusion, decay or annihilation. If it is a decay, indicate what kind.

39. What is a chain reaction? How is the chain reaction in a nuclear power plant controlled?

40. Why is controlled fusion so difficult to achieve? Describe the two methods that are being used to produce controlled fusion.

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