Connecting the Threads of Area and Perimeter

See how sixth-grade students design and create quilt squares for this geometry project.

By Jordan T. Hede and Jonathan D. Bostic

"What is the point of learning this?"
“What will I ever use this again?”

I (Hede) frequently heard these two questions from my students during a math unit focused on area and perimeter. They consistently asked why it was important to know the formula for the area of polygons, connections between formulas, and when to use formulas. Students asked these questions because they did not value learning to apply their geometry knowledge in a variety of situations. I realized they needed (1) more meaningful experiences learning about area and perimeter and (2) understanding the importance of these concepts. They deserved the opportunity to experience a worthwhile task that might help them later use their content knowledge while problem solving.

Their questions also concerned me because my students perceived problem solving as separate from doing mathematics, a perception that had been influenced by a greater focus on exercises than on problem solving in their previous classes (Schoenfeld 2011). Instruction should allow students to regularly engage with problems that require synthesis and the creation of novel products (Kilpatrick, Swafford, and Findell 2001). These opportunities to learn through rich tasks ought to encourage
children to (1) adapt to novel situations, (2) apply efficient and effective strategies, and (3) collaborate to solve a problem (Bostic and Jacobbe 2010). To meet these goals, I designed a project for my students that supported them in synthesizing their knowledge about perimeter and area formulas. The Quilting task described here encourages students to use their imagination to create a quilt square while engaging with the mathematics found in the Common Core State Standards for Mathematics (CCSSI 2010).

Classroom expectations
Approximately twenty sixth-grade students regularly meet in each section of my mathematics classes. Since the beginning of the school year, they have practiced doing mathematics independently, in pairs, and in groups of three or four students. They are accustomed to assisting peers who need help, without expressing judgment toward one another. Two expectations in my classroom are that students should ask questions of their peers if they need assistance and that students should critique the reasoning of their peers to promote growth. I also expect students to share an idea that is still under investigation, even if it is incorrect. Finally, they are expected to use mathematical reasoning when giving explanations. These expectations align with two of the Standards for Mathematical Practice (CCSSI 2010). A student's typical time in my classroom involves independent work as well as student-to-student small-group collaboration on a worthwhile task to learn mathematics concepts and procedures. I continuously voice that our classroom is a safe environment for students to express their ideas during small-group work and whole-class discussions. This encouragement to be creative and use their imagination while working leads to interesting comments about connections between mathematics concepts as well as to remarkable student work products.

Purpose of the task
The aim of this lesson was to give students an experience using area and perimeter to create a unique quilt square. Students did not necessarily become master quilters, but the task's context drew on a situation that was familiar to them. By the end of the activity, they could do the following:

• Calculate the area and perimeter of different regular and irregular polygons
• Justify that summing every shape's area is equal to the area of the quilt square
• Design a quilt square using only quadrilaterals and triangles
• Create a quilt square that matched their design

This project aligns with sixth-grade Common Core geometry topics and was completed during one academic week. Specifically, it addresses aspects of Standards 6.G.A.1 and 6.G.A.3 (CCSSI 2010). The task requires only a few materials and two handouts (see the online appendix), including grid paper and one piece of 8 × 8 inch cardstock for each student, a classroom set of glue and scissors, and books of wallpaper samples. A local hardware store provided discontinued wallpaper sample books for us.
Implementation of the task
Students had learned how to find the area and perimeter of different polygons during a previous week; now they were asked to complete all four instructional phases of this project.

Phase 1
(about 45 minutes)
I prepared my students by showing them examples of real quilts as well as pictures of various quilt squares like those in figure 1. I asked them to share what they noticed in the quilt squares. They talked about common patterns and mathematical features within the quilts and quilt squares. These comments helped initiate a whole-class discussion that touched on various topics students had learned previously in math class and art class. For example, students mentioned that the quilt squares tended to be symmetrical. That is, students were able to talk about the characteristics of a square and that the pattern within the quilt square showed lateral,

Students discussed the visually pleasing contrasting colors of the quilt squares and the pattern characteristics of lateral, rotational, translational, or glide reflective symmetry.

FIGURE 1
Figure 1. Sample quilts and quilt squares shown to students.
rotational, translational, or glide-reflective symmetry. They also noticed that the more visually appealing examples of quilt squares used contrasting colors. As I transitioned to the project, I praised students for their focus in attending to the qualities of quilt squares. I distributed the handouts (see the online appendix), described the requirements for this project, and reviewed the task with them.

Phase 2
(about 60 minutes)
The next stage of the project focused students’ attention on designing their quilt square on grid paper that I provided for them. During this phase, I walked around the classroom to formatively assess students’ progress and understanding of the task. Specifically, I was looking at their ability to talk about their designs, trying to determine whether they were following the directions of the project and applying formulas correctly. Students often asked me during this phase if they must cover the entire piece of grid paper with shapes. This led me to think that they did not understand what to do if their designs and shapes did not fit on the grid paper. We corrected this misconception with the explanation that not having a shape on the grid paper is similar to not having fabric in a section of a quilt. If a student had an unusual shape in his or her design (e.g., an oval), then I asked for an explanation of how the area and perimeter might be calculated. When students realized they did not know how to calculate the area of rounded and unusual shapes, they reworked their designs to include shapes with area formulas they knew.

Phase 3
(about 25 minutes)
Next was the synthesis portion of the project. After completing a drawing of their quilt square, students examined the second handout. The chart directed them to find the area and perimeter of each shape and to show their work (see fig. 2). I expected them to show that the sum of their polygons was the same as the area of a square measuring eight inches on each side. This was an important step because it presented an opportunity to formatively assess whether students accurately calculated the sum of 64 units². After students completed their design and chart, they submitted them for approval. During the approval process, I checked students’ strategies to determine whether they used appropriate methods for calculating area and perimeter. This process allowed more time to probe students’ thinking about the polygons’ area and perimeter. For instance, I asked one student during the one-on-one review process, “Why might your total area not be 64 inches²?”

The student replied, “Because some of my shapes did not cover the entire square.”

Students’ areas usually did not sum to 64 inches² because they had approximated their measurements to the nearest fraction of an inch. When I asked them why their sums did not equal 64 inches², one student responded as follows:

![Figure 2](image-url)
I did not get 64 inches\(^2\). I got very close. The area of one of my shapes was not a whole number, so I got a decimal for my total area, which was close to 64 inches\(^2\).

Other students shared that their designs required rounding and that they expected their total area would not equal 64 inches\(^2\). I expected students to be precise with their measurements and calculations and to spend ample time persevering in making the sum as close to 64 inches\(^2\) as possible. Because of this activity, they showed evidence of attending to precision, which is the sixth Standard for Mathematical Practice (CCSSI 2010). During these conversations, I accepted students’ work if the total sum was not exactly 64 inches\(^2\) if and only if they provided ample evidence indicating why the sum was slightly more or less than 64 inches\(^2\). The conversations were evidence that students were able to make sense of the problem, which is an aspect of the first Standard for Mathematical Practice (CCSSI 2010).

While phase 3 was happening in the classroom, students saw that I was the only person who checked their work before they proceeded to the next phase. Because I could verify only one student’s work at a time, students asked one another for help to ensure an accurate table before meeting with me. They reviewed their peers’ work and showed respect while critiquing others’ designs, reasoning, and arithmetic. They posed questions to one another and displayed kindness and thoughtfulness with their remarks. Drawing on the language of the third Standard for Mathematical Practice, they “justified their conclusions, communicated them to others, and responded to others’ questions and concerns” (CCSSI 2010, pp. 6–7). Two frequent occurrences were incorrect arithmetic and measurement errors. When a student noticed someone’s error, he or she addressed it with the peer and asked for clarification. The positive learning environment and established norms in my classroom facilitated peer-to-peer mathematical discourse to complete the project successfully and on time.

Phase 4 (about 60 minutes)

After designs and charts had been approved, students began phase 4 of the project. Using wallpaper, they created a quilt square. Students cut out each individual shape from their grid paper, traced the shapes onto the wallpaper, and cut the shapes from the wallpaper. This ensured that students’ polygons fit together.

Outcomes from the Quilting task

Students created amazing quilt squares (see fig. 3). They applied their knowledge of area
Students’ conceptual and procedural understandings are enhanced by mathematics projects and collaboration.

and perimeter while problem solving, creatively applied mathematical procedures, and produced artwork that we displayed in the school. I encouraged them to use valid approaches for calculating area and perimeter rather than algorithms. Although some used algorithms, others employed alternate strategies, such as counting the squares on grid paper, evidence of learning to adapt their reasoning and use efficient, effective strategies when appropriate—two features of mathematical proficiency. When a student could not remember a formula for a specific polygon, instead of giving it to them, I provided scaffolding by asking questions to push his or her thinking forward. For example, when I asked, “How else could you find the area without using the formula?” a student responded, “By counting up the number of whole spaces a shape takes up and then adding any half spaces the shape takes up.” Their comments and actions suggested that they were aware that multiple ways exist for finding area, albeit some are less precise approaches than others.

One student struggled with accurately calculating the area because he did not count the half of each square in his drawing. When questioned about how he could do so, he told me, “I could count the number of half-squares I have and add them up and then add that to the number of whole squares I have.” I praised him for his approach and asked if he might be able to use his answer to remember the area formula for a parallelogram. He paused to think for a moment and replied, “Oh, yeah! I need to multiply the length by the height and not the side. I was trying to multiply the length by the side and kept getting a decimal.”

Scaffolding and encouragement supported students in using various methods for calculating area and making connections to formulas.

Creating a new product

Many students complete exercises flawlessly and yet often struggle when asked to apply this knowledge (Schoenfeld 2011). They may be able to complete exercises with accuracy but cannot use this knowledge to create something. Students should be able to apply their math knowledge. Math tasks that involve creativity while problem solving and that draw on multiple mathematics concepts and procedures can support the idea that doing math involves more than completing exercises (Bostic and Jacobbe 2010). The benefits of such tasks are greatest when students know how to work together to solve a problem (Bostic and Jacobbe 2010). Thus, students’ conceptual and procedural understandings are enhanced by mathematics projects and collaboration.

The Quilting task gives students an opportunity to synthesize their knowledge of area and perimeter with an application problem. Creating a new product is the highest level of Bloom’s taxonomy (Krathwohl 2002). Hence, students need to have novel opportunities to
demonstrate their mathematical knowledge. Just imagine what you might learn about your students when they make their quilt squares.

REFERENCES


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