UNIFORM CIRCULAR MOTION

INTRODUCTION

If you have ever been on an amusement park ride that travels in a curved or circular path, then you have experienced a force, called a centripetal force, pushing you into the ride. Whether it's the back wall of the "Roundup"[1] or "Rotor"[2], the ride where the floor drops from beneath your feet, or the seat belt of the "roller coaster"[3] that supplies the force, you are constantly being accelerated toward the ride's center of curvature. If—what you fear most on such a ride—this force were suddenly removed, you would move off in a direction tangent to the circular path. That's what happens when you go over a hill on the roller coaster just before the seat belt takes effect.

Figure 1 compares the motion in the presence of a centripetal force to the resulting motion of a body if the centripetal force were to cease suddenly.

With centripetal force the object moves in a circular path with speed v.

When centripetal force is removed, the object moves along a tangent to the circular path with speed v.
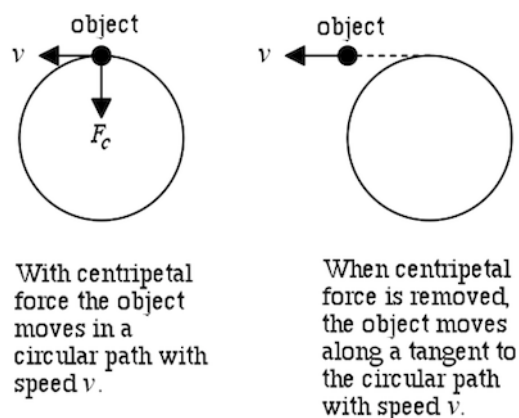
Figure 1: Object in circular motion

As another example consider a ball, attached to a string and whirled in a circle as shown in Fig. 2a. The tension in the string applies the *centripetal force*[4] to the ball, causing it to move in a circular path. The string pulls the ball toward the center of the circle while the ball pulls outward on the string and hence on your hand in accordance with Newton's third law of action and reaction[5]. So this outward force does not act on the ball, though it is commonly and incorrectly referred to as the centrifugal force acting on the ball. When the centripetal force is discontinued, such as in Fig. 2b when the string breaks, the object moves in the direction of the velocity at that instant. This direction is tangential to the circle at that point.

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[1]http://en.wikipedia.org/wiki/Round_up
[2]http://en.wikipedia.org/wiki/Rotor_(ride)
[3]http://en.wikipedia.org/wiki/Roller_coaster
[4]http://en.wikipedia.org/wiki/Centripetal_force
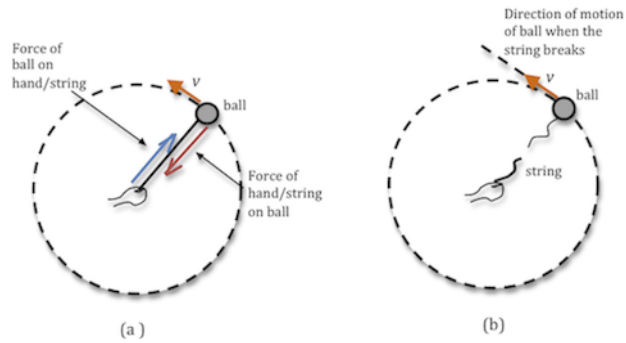[5]http://en.wikipedia.org/wiki/Newton%27s_laws_of_motion

Figure 2: Top view of a ball on a string before and after the string breaks

The centripetal force that holds you in the ride can be determined with a few measurements and calculations. In this experiment you will determine what variables must be known to determine the centripetal force required to keep a mass moving in a circular path with a constant speed.

DISCUSSION OF PRINCIPLES

Newton's second law states that the net force F_{net} exerted on a moving body is equal to the product of a body's mass m and its acceleration a.

$$\vec{F}_{net} = m\vec{a} \tag{1}$$

When applying Newton's second law to circular motion, it is convenient to use coordinates that are parallel and perpendicular to the object's motion.

$$F_{\parallel} = ma_{\parallel} \tag{2}$$

$$F_{\perp} = ma_{\perp} \tag{3}$$

For a body moving in a straight line, the acceleration is due to a change in the magnitude of the velocity. For a body moving in a circular path with constant speed the magnitude $|\vec{v}|$ of the velocity does not change, but the direction of the velocity vector $\vec{v}$ constantly changes. In this case the object has no acceleration in the direction parallel to its motion and $F_{\parallel}$ in Eq. (2) is zero. This motion is called uniform circular motion—motion in a circular path at constant speed. Since the velocity vector is changing in time, the object in uniform circular motion is accelerating. Conceptually, using parallel and perpendicular coordinates is convenient because the parallel force is responsible for changes in speed and the perpendicular force (or centripetal force) is responsible for changes in direction.

Centripetal Acceleration

What acceleration is needed to keep the body moving in a circle with constant speed? Refer

to Fig. 3a below. At a certain instant the particle is located at the tip of the radius vector $\vec{r}$. At another instant Δt later the particle is located at the tip of the radius vector $\vec{r}'$. In the small time interval Δt the body moves along an arc of length $v\Delta t$. If Δt is small enough, the arc length is approximately equal to the distance between the position of the body at the beginning of the interval Δt and the position at the end of the interval. This straight-line segment, together with the two radii, forms the isosceles triangle shown in Fig. 3a.
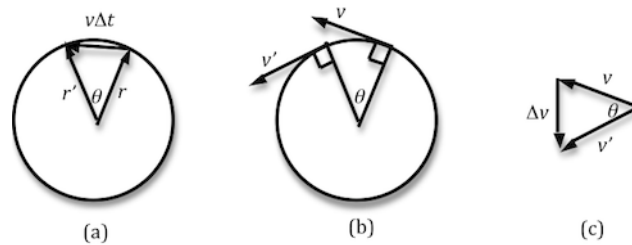

Figure 3: Geometrical considerations

Figure 3b shows the velocity vectors at the two positions. Notice that the magnitude of the velocity vector is not changing. In Fig. 3c the two velocity vectors are redrawn, without changing their lengths or orientations. The arrow joining the tips of the velocity vectors represents the change Δv in the velocity of the particle due to the change in direction during the time interval Δt. Note that as Δt gets smaller, the angle between the direction of Δv and both v and v' comes close to a right angle $90°$.

Since the tangent to a circle is perpendicular to its radius, it follows that the angle θ between v and v' as shown in Fig. 3c is the same as the angle between the two radii in Fig. 3a. From trigonometry we know that these two triangles are congruent, and therefore the ratios of their sides must be equal. So we can write the following expression

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r} \tag{4}$$

Dividing both sides by $\Delta t/v$ gives

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r} \tag{5}$$

Of course $\Delta v/\Delta t$ is the acceleration of the body, and it is in the direction of Δv. From Fig. 3c we can see that the acceleration is directed toward the center of the circle. The magnitude of the centripetal acceleration a_c is given by

$$a_c = \frac{v^2}{r} \tag{6}$$

and the centripetal force is

$$F_c = ma_c = m\frac{v^2}{r} \tag{7}$$

Since it is difficult to measure the velocity of the body directly, you will instead compute the velocity from quantities that are easier to measure. The magnitude of the velocity vector can be determined by measuring the distance that the object travels per unit time. If T is the period (length of time needed for the object to make one complete revolution), then the speed is equal to the distance traveled in that one revolution divided by the period.

$$v = \frac{2\pi r}{T} \tag{8}$$

Substituting this into Eq. (6) for the centripetal acceleration gives

$$a_c = \frac{(2\pi r f)^2}{r} = 4\pi^2 f^2 r = 4\pi^2 T^{-2} r \tag{9}$$

where $f = 1/T$ is the number of revolutions per second measured in Hertz. Now Eq. (7) can be written in terms of the measurable quantities m, T, and r as

$$F_c = 4\pi^2 m f^2 r \tag{10}$$

OBJECTIVE

In this experiment you will measure the period of rotation for an object and calculate the centripetal force acting on it. You will compare this centripetal force with an equivalent force needed to maintain the object at the same radius.

EQUIPMENT

Circular motion apparatus
Assorted weights
Bubble level
Balance
Stop watch
Meter stick
Spring

PROCEDURE

While it might be more fun to carry out this study on a midway ride, a simple laboratory apparatus shown in Fig. 4 below will be used to examine the nature of centripetal force. By using the centripetal force apparatus you can measure the frequency of rotation of an object moving in a circular path of a known radius. Eq. (10) can then be used to compute the centripetal force acting on the object.

Description of apparatus

Figure 4 shows the apparatus and its various components. The bottom of the base is provided with adjustable screws that can be used to level the entire apparatus. The crossbar with the counter-weight at its end can be moved to change the position of the rotating mass. The pointer can be moved along the slot and positioned just under the tip of the rotating mass. This helps to measure the radius of the circular path of the rotating mass.
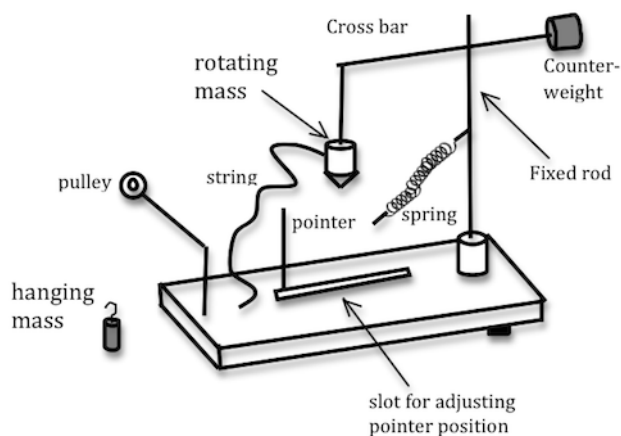

Figure 4: Sketch showing the components of the apparatus

In this apparatus the centripetal force is provided by the spring. When the spring is attached to the rotating mass M_R, it gets pulled in as shown in Fig. 5a and Fig. 6b. To bring the rotating mass back above the pointer or radius indicator, the string is passed over the pulley and enough mass added at the end of the string to bring the rotating mass back above the radius indicator.
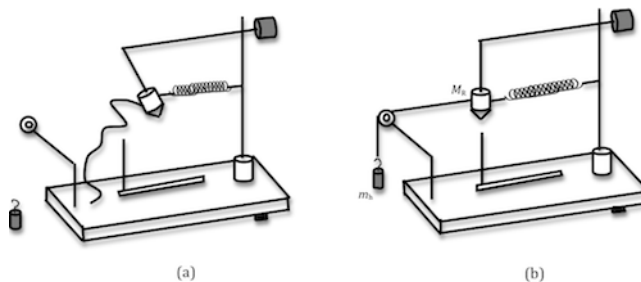

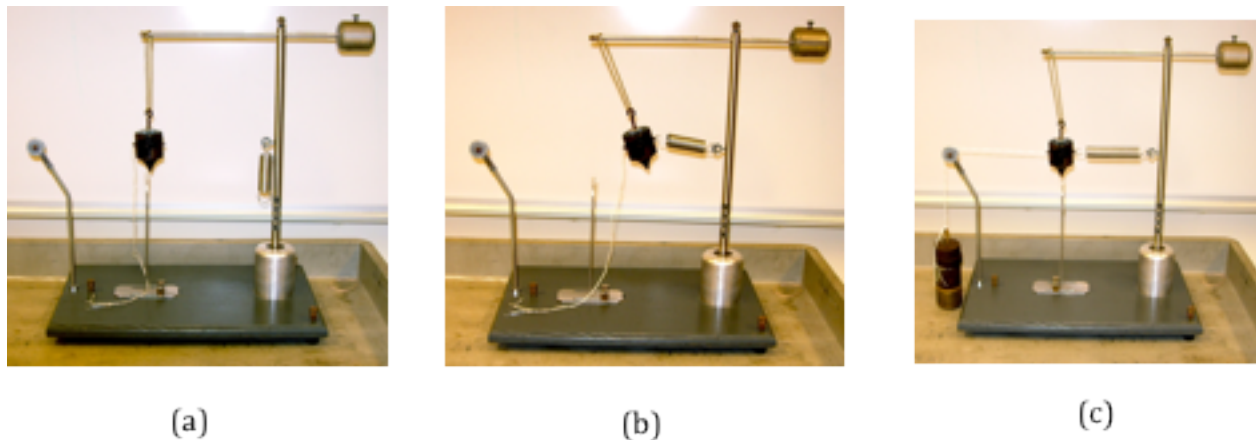Figure 5: Sketch showing initial set-up

(a)   (b)   (c)

Figure 6: Photo of initial set-up

In Fig. 5b and Fig. 6c the weight $m_{\mathrm{h}}g$ of the hanging mass m_{h} just balances the elastic force of the spring when the rotating object M_{R} rests above the pointer or radius indicator post.

In Fig. 7 the spring is stretched by the same amount as the mass M_{R} rotates in a path with the same radius as that set in Fig. 5b. The magnitude of the two forces F_{c} and $m_{\mathrm{h}}g$ should be in close agreement since both forces produce the same elongation of the spring.
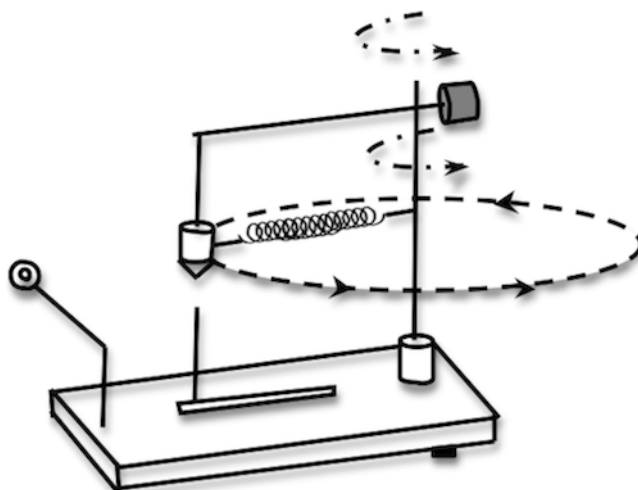

Figure 7: Sketch showing the circular path of the object

Procedure A: Measuring the period of rotation

1. Level the apparatus with the three adjusting screws in the base. The apparatus is level when the crossbar remains fixed in any position. If necessary, use a bubble level to assist you in this process.

2. Weigh the rotating mass M_{R} and enter this value on the worksheet.

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3. Set the radius indicator for the smallest radius and measure r. The radius is measured from the center of the post (axis of rotation) to the center of the radius indicator.

Record r in the Data Table on the worksheet.

4. Move the crossbar until the tip of the rotating mass is above the radius indicator.

5. Attach the spring to the rotating mass.

Attach the string to the rotating mass, pass the string over the pulley, and attach it to the mass hanger at the other end.

Add masses to the hanger until the tip of the rotating mass is above the radius indicator.

Record the value of the total hanging mass m_h in Data Table 1 on the worksheet.

6. Remove masses and hanger and rotate the crossbar.

Make sure the tip of the rotating mass hits the radius indicator every revolution. You may have to practice a little before you start taking data. It is important to spin the shaft uniformly.

CAUTION: Make sure to keep hair and clothing away from the apparatus when it is in motion!

Using a timer record the time required for 50 revolutions. Record this time in Data Table 1 on the worksheet.

7. Unhook the spring and move the radius indicator out by about 1 cm. Measure and record this new radius value in Data Table 1.

8. Repeat steps 4 through 6 and enter the values in Data Table 1.

9. Repeat step 7 for three more positions of the radius indicator, for a total of 5 radii.

CHECKPOINT 1: Ask your TA to check your Data Table 1 values before proceeding.

10. Calculate and record the period T of rotation in Data Table 2.

11. Calculate and record the frequency f of rotation in Data Table 2.

12. Calculate and record the centripetal acceleration in Data Table 2.

13. Calculate the centripetal force.

14. Calculate the percent difference between the experimental value of the centripetal force and the force of the hanging mass, $m_h g$ and record the values in Data Table 2. See Appendix B.

Note: If your percent difference is greater than 15% for one trial, you *must* redo that trial.

CHECKPOINT 2: Ask your TA to check your Data Table 2 values and calculations.

Procedure B: Plot of $m_{\mathrm{h}}g$ versus a_{c}

15. Using Excel, graph $m_{\mathrm{h}}g$ versus a_{c}. See Appendix G.

16. Use the trendline option in Excel to find the slope. See Appendix H.

17. From the slope determine the value of the rotating mass.

18. Compare the value of the rotating mass obtained from the slope with the measured value by computing the percent difference.

CHECKPOINT 3: Ask your TA to check your graph and calculations.