

Rotational Inertia of Different Objects

TOPICS AND FILES

Mechanics Topic

Rotational motion; position, velocity, acceleration

Capstone File

39 Rotational Motion.cap

EQUIPMENT LIST

Qty	Items	Part Numbers
1	PASCO Interface (for one sensor)	
1	Rotary Motion Sensor	CI-6538
1	Mini Rotational Accessory	CI-6691
1	Rod, 45 cm	ME-8736
1	Large Rod Base	ME-8735
1	Mass and Hanger Set	ME-9348
1	Ruler	
1 m	Thread	

INTRODUCTION

This lab has two parts.

The purpose of Experiment 1 is to measure the angular position and velocity of a rotating body. Use the rotary motion sensor to measure the rotation of a disk as the disk undergoes a constant angular acceleration. Use *Capstone* to record and display the data, then plot the angular position and angular velocity and analyze them. Compare the plots of angular position and angular velocity for the accelerating disk to plots of position and velocity for an accelerating fan cart.

The purpose of the Experiment 2 is to measure the rotational inertia of a disk and a ring. Use a rotary motion sensor to measure the rotation of each object as the object undergoes a constant angular acceleration. Use *Capstone* to record and display the data. Plot angular velocity use the slope to determine the angular acceleration. Use the amount of torque applied and the angular acceleration to find the rotational inertia. Calculate the theoretical rotational inertia based on mass and radius of the object and compare to the measured rotational inertia.

BACKGROUND

Kinematic Equations and Angular Analogies

For each kinematic quantity (i.e. displacement, velocity, etc.) there is an analogous quantity in rotational kinematics. The rotational version of position (x) is the “angular position” that is given by the Greek letter theta (θ). The rotational version of velocity (v) is “angular velocity” that is given by the Greek letter omega (ω). All translational (linear) quantities have rotational counterparts.

$$\begin{aligned}
v &= v_0 + at & \omega &= \omega_0 + \alpha t \\
x &= \frac{1}{2}(v_0 + v)t & \theta &= \frac{1}{2}(\omega_0 + \omega)t \\
x &= v_0 t + \frac{1}{2}at^2 & \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\
v^2 &= v_0^2 + 2ax & \omega^2 &= \omega_0^2 + 2\alpha\theta
\end{aligned} \tag{1}$$

The equations of kinematics for constant linear acceleration can be used for solving problems involving linear motion in one and two dimensions. For example, the motion of a fan cart accelerating on a flat track can be described by the equations of translational kinematics. The ideas of angular displacement, angular velocity, and angular acceleration can be brought together to produce a set of equations called the equations of kinematics for constant angular acceleration. The equations of kinematics for constant angular acceleration can be used for solving problems involving rotational motion. For example, the motion of the blades on a fan cart as they start to rotate faster and faster can be described by the equations of rotational kinematics.

Rotational Inertia

A quarterback on an American football team throws the ball so it spirals in flight. A figure skater performs an elegant spin on the ice and increases her rotation rate by moving her outstretched arms closer to her body. Rotational inertia plays an important role in both of these phenomena.

The rotational inertia of an object depends on the mass and the distribution of mass. In general, the more compact an object, the less rotational inertia it has. Theoretically, the rotational inertia, I , of a ring is given by

$$I = \frac{1}{2}M(R_1^2 + R_2^2) \tag{2}$$

where M is the mass of the ring, R_1 is the inner radius of the ring, and R_2 is the outer radius of the ring. The rotational inertia of a solid disk of uniform density is given by

$$I = \frac{1}{2}MR^2 \tag{3}$$

where M is the mass of the disk, and R is the radius of the disk. To find the rotational inertia of the ring and disk experimentally, apply a known torque to the ring and disk and measure the resulting angular acceleration. Since $\tau = I\alpha$, we have

$$I = \frac{\tau}{\alpha} \tag{4}$$

where α is the angular acceleration and τ is the torque. Torque depends on the force applied and the distance from the pivot point of the rotating object to the point where the force is applied, or

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{5}$$

where $\vec{r}$ is the distance from the center of the ring or disk to the point where a force is applied (the 'lever arm'), and $\vec{F}$ is the applied force. The value of $\vec{r} \times \vec{F}$ is $r F \sin \phi$ where ϕ is the angle between $\vec{r}$ and the direction of $\vec{F}$, the applied force. The torque is maximum when $\vec{r}$ and $\vec{F}$ are perpendicular.

In this case, the applied force is the tension (T) in a thread that is attached to a part of a rotational apparatus. Gravity pulls a hanging mass m that is attached to the thread. The value of r is the radius of the step pulley on the apparatus. The radius is perpendicular to the applied force (Tension). Therefore, the torque is the following equation.

$$\tau = rT \quad (6)$$

The following solution is derived from the convention that up is positive and down is negative, counterclockwise is positive and clockwise is negative. Applying Newton's second law for the hanging mass, m , results in the following.

$$\Sigma F = T - mg = m(-a) \quad (7)$$

Solving for the tension in the string gives the following equation.

$$T = m(g - a) \quad (8)$$

The torque is given below.

$$\tau = rT = rm(g - a) \quad (9)$$

The linear acceleration a of the hanging mass is the tangential acceleration, a_T , of the rotating apparatus. The angular acceleration is related to the tangential acceleration as follows.

$$\alpha = \frac{a_T}{r} \quad (10)$$

Substituting the equations into each other gives the following.

$$I = \frac{\tau}{\alpha} = \frac{rm(g - a)}{\frac{a_T}{r}} = rm(g - a)\frac{r}{a_T} = \frac{mgr^2}{a_T} - mr^2 = mr^2 \left(\frac{g}{a_T} - 1 \right) \quad (11)$$

The system's rotational inertia, I , can be calculated from the tangential acceleration, $\vec{a}_T$.