Significant Figures

SIGNIFICANT FIGURES IN CALCULATIONS

Values cannot become more precise by mathematical manipulation. The following are purely mathematical rules for significant figures *in the absence of experimental limitations on precision.*

Addition and subtraction

- The final answer is rounded to the least precise decimal place.

Multiplication and division

- The final answer is rounded to the least number of significant figures.

Two lengths, 22.80 cm and 14.5540 cm, are mathematically combined in the four basic ways (+, −, ×, ÷).

	Addition	Subtraction	Multiplication	Division
Unrounded	22.80 cm + 14.5540 cm = 37.3540 cm	22.80 cm − 14.5540 cm = 8.2460 cm	22.80 cm × 14.5540 cm = 331.8312 cm^2	22.80 cm ÷ 14.5540 cm = 1.5665796...
Report	37.35 cm	8.25 cm	331.8 cm^2	1.567

SIGNIFICANT FIGURES IN EXPERIMENTS

A measured quantity is usually reported as the value and its associated uncertainty. In this class, the following generic symbols are used: x for the value, $<x>$ for the average, and σ for the uncertainty or standard deviation.

Please note that uncertainty and standard deviation are technically not the same, but this idea will be covered in more advanced classes.

For a single measurement of one quantity, the uncertainty is generally determined by the instrument used.

The pan balance reads to 0.01 g, so an example data would be: 25.10 ± 0.01 g.

The analytical balance reads to 0.0001 g, so an example would be: 25.1000 ± 0.0001 g.

A 100-mL graduated cylinder might read to 0.5 mL, so examples are: 25.0 ± 0.5 mL or 31.0 ± 0.5 mL.

For multiple measurements of the same quantity, generally the average value is determined. The uncertainty (or standard deviation) can be calculated based on the random fluctuation of the individual data points around the average. For n measurements of x:

$$< x >= \frac{(x_1 + x_2 + \ldots + x_n)}{n} \tag{1a}$$

and

$$\sigma \equiv \sqrt{\frac{(x_1 - < x >)^2 + (x_2 - < x >)^2 + \ldots + (x_n - < x >)^2}{n-1}}. \tag{1b}$$

MATHEMATICAL MANIPULATIONS OF DATA AND ERROR PROPAGATION

For calculations involving the addition, subtraction, multiplication, and division of two numbers, $A \pm \sigma_a$ and $B \pm \sigma_a$, the following conventions are followed.

Addition and subtraction—uncertainties add.

$$\text{Addition: } (A \pm \sigma_a) + (B \pm \sigma_b) = (A + B) \pm (\sigma_a + \sigma_b) \tag{2}$$

$$\text{Subtraction: } (A \pm \sigma_a) - (B \pm \sigma_b) = (A - B) \pm (\sigma_a + \sigma_b) \tag{3}$$

Multiplication and division—use relative errors.

$$\text{Multiplication: } (A \pm \sigma_a)(B \pm \sigma_b) = (AB) \pm (AB) \left(\frac{\sigma_a}{A} + \frac{\sigma_b}{B}\right) \tag{4}$$

$$\text{Division: } \frac{(A \pm \sigma_a)}{(B \pm \sigma_b)} = \left(\frac{A}{B}\right) \pm \left(\frac{A}{B}\right) \left(\frac{\sigma_a}{A} + \frac{\sigma_b}{B}\right) \tag{5}$$

Relative errors in A and B are the following.

$$\frac{\sigma_a}{A} \text{ and } \frac{\sigma_b}{B} \tag{6}$$

Percent errors in A and B are the following.

$$\frac{\sigma_a}{A} \times 100\% \text{ and } \frac{\sigma_b}{B} \times 100\% \tag{7}$$

Uncertainty values should contain one significant figure. Final values are reported to that same decimal place.

For example, suppose two volumes are determined: 49.06 ± 0.05 mL and 12.74 ± 0.05 mL.

Addition:	Total volume	=	49.06 + 12.74	=	61.80 mL	*to least precise decimal place*
	Uncertainty	=	0.05 + 0.05 mL	=	0.1 mL	
	Report:		61.<u>8</u> ± 0.<u>1</u> mL			
Subtraction:	Total volume	=	49.06 − 12.74	=	36.32 mL	*to least precise decimal place*
	Uncertainty	=	0.05 + 0.05 mL	=	0.1 mL	
	Report:		36.<u>3</u> ± 0.<u>1</u> mL			
Multiplication:	Surface area	=	49.06 × 12.74	=	625.0 mL^2	*to least number of sig. figs.*
	Uncertainty	=	$\pm(625.0mL^2)\left(\frac{0.05}{49.06}+\frac{0.05}{12.74}\right)=\pm(625.0mL^2)(0.001019+0.003945)=\pm 3.090mL^2$			
	Report:		62<u>5</u> ± <u>3</u> mL			
Division:	Relative vol.	=	49.06 / 12.74	=	3.851	*to least number of sig. figs.*
	Uncertainty	=	$\pm(3.851)\left(\frac{0.05}{49.06}+\frac{0.05}{12.74}\right)=\pm(3.851)(0.001019+0.003945)=\pm 0.01904$			
	Report:		3.8<u>5</u> ± 0.0<u>2</u> mL			