The Pendulum

Objectives

- to determine what characteristics of the pendulum change the period of oscillations
- to calculate the experimental mass of Earth

Equipment

set of 6 identical washers that will act as the simple pendulum, support stand with a photogate connected to the Science Workshop Interface, meter stick, protractor
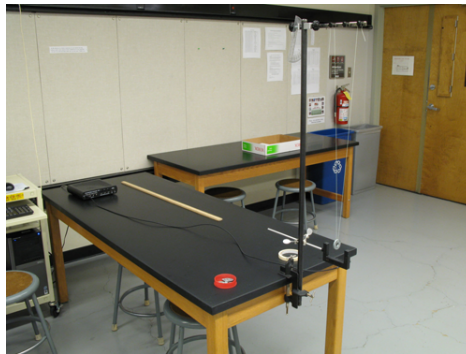

Figure 1

Introduction and Theory

A simple pendulum consists of a point mass and a string. The size of the bob should be at least 10 times smaller than the length of the string that is holding it. The point mass is suspended at one end of the string, and the other end of the string is held fixed. The motion of a pendulum takes place in a vertical plane as illustrated in Fig. 2, where the angle θ is measured from the vertical line to the string. This initial angle is called the amplitude.
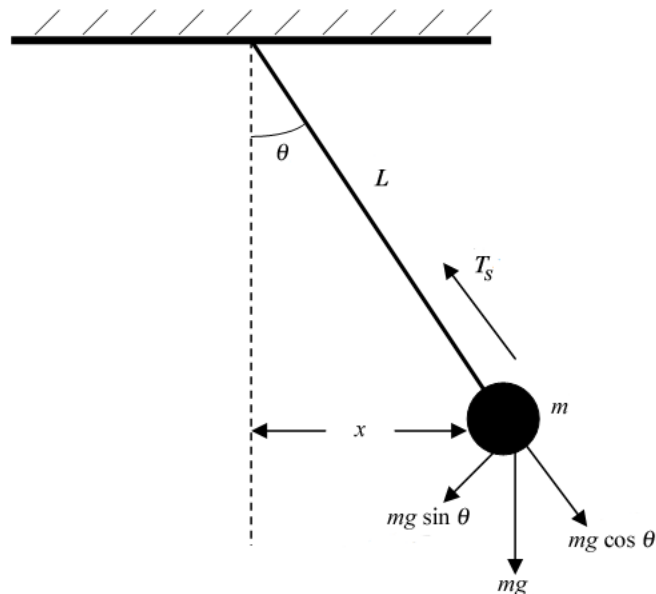

Figure 2: The force diagram for a pendulum

The gravitational force exerted upon the mass m can be decomposed into two perpendicular components. The sum of the forces in the direction along the string is

$$T_{\mathrm{s}} - mg\ \cos(\theta) = ma_{\mathrm{r}}, \tag{1}$$

where $a_{\mathrm{r}} = v^2/r$ is the radial acceleration. In the other direction, the sum of the forces is

$$-mg\ \sin(\theta) = ma_{\mathrm{t}}, \tag{2}$$

where a_{t} is the tangential acceleration and is equal to $-g\ \sin(\theta)$. This acceleration is tangent to the path of the pendulum at every point.

If the angle θ is sufficiently small, then $\sin(\theta) \approx \tan(\theta) \approx x/L$, and equation 2 becomes

$$F_{\mathrm{net}} = -mg\ \frac{x}{L} = -\left(\frac{mg}{L}\right) x. \tag{3}$$

Equation 3 is the equation for simple harmonic motion, and from this, the period of the pendulum can be determined to be

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{4}$$

The motion of the pendulum is periodic in time, which is the basis for pendulum clocks.

If the angle θ is not small, $\sin(\theta)$ has to be expanded into an infinite series, and the result for the period becomes

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{4}\ \sin^2\left(\frac{\theta}{2}\right) + \frac{9}{64}\ \sin^4\left(\frac{\theta}{2}\right) + ...\right). \tag{5}$$

Note that equation 5 can be used to estimate the error in the period when θ is not small. For example, when $\theta = 10°$,

$$\frac{\Delta T}{T} = \frac{1}{4}\ \sin^2\left(\frac{\theta}{2}\right) + \frac{9}{64}\ \sin^4\left(\frac{\theta}{2}\right) = 1.907 \times 10^{-3}, \tag{6}$$

which is approximately a 0.2% error. Additional information about the pendulum can be found in your textbook.

Video

View the video online prior to beginning your lab.

A video or simulation is available online.

Procedure

Please print the worksheet for this lab.

CHECKPOINT:

Be sure to have your TA sign your lab worksheet, printed Inlab, and all printed graphs after each part is completed. Be sure the data can be seen on the graphs.

Open the preset experiment file: Desktop/pirt inst labs/PHY 113/PreSet Up Labs/Pendulum.
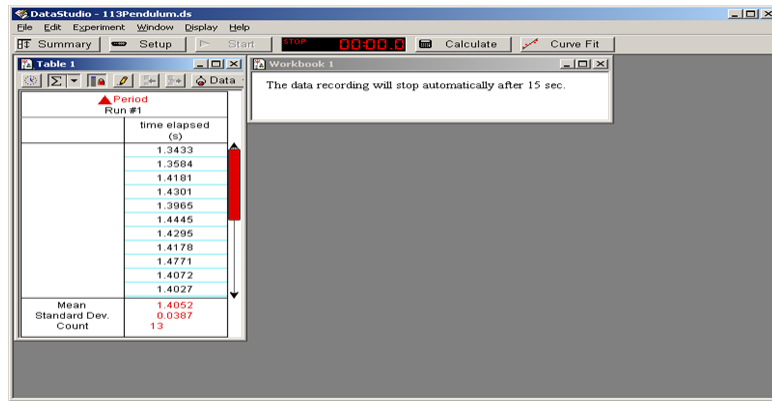

Figure 3: A sample of the DataStudio file for the pendulum exercise

We will use a photogate timer to make measurements of the period T of a simple pendulum. A photogate consists of an infrared diode, called the source diode, which emits an invisible infrared light beam. This beam is detected by another diode, the detector diode. When the mass m passes between the source and the detector, the infrared light beam is interrupted, producing an electronic signal that is used to start or stop an electronic timer (see Appendix B).

The photogate is set up to measure the period of oscillations of the pendulum (i.e., the time it takes for the pendulum to go all the way out and all the way back). In other words, the pendulum has to pass through the gate twice for one full swing. The photogate measures the time between the consecutive interruptions of its beam. See Fig. 4.
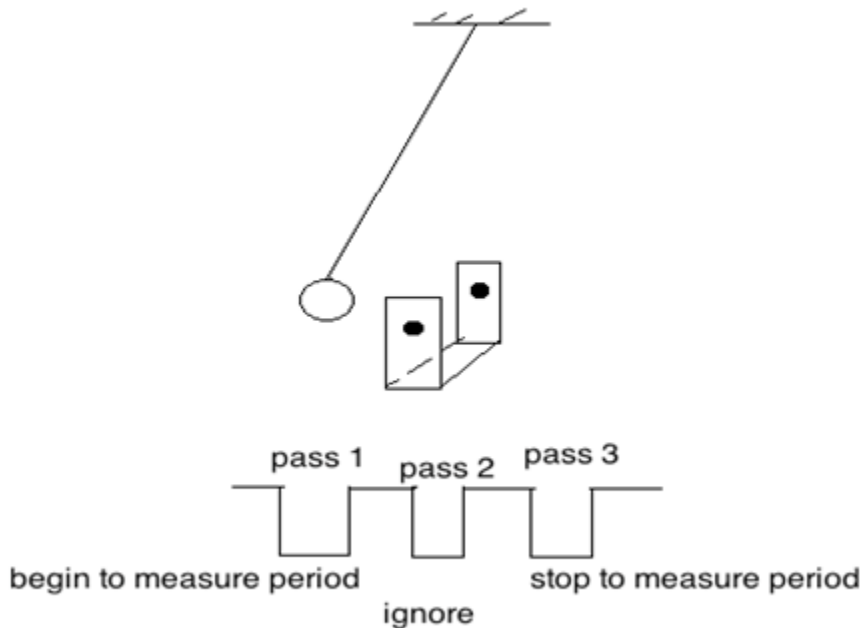

Figure 4

Check that the timer is working properly by passing a pencil three times through the photogate. The timer should start on the first pass, ignore the second pass, and stop on the third pass.

Part 1: Measure the period of oscillations for pendulum bobs of different masses while keeping the length of the pendulum and amplitude fixed.

Allow the mass to swing back and forth a couple of times before you start making measurements to let the pendulum "settle." To start the data acquisition, press the **Start** button on the tool bar menu. Use small amplitudes (angles) of about 5°.

1   Put one washer at the end of the string. In GA, create columns to record the mean values of periods (in seconds) and the number of washers. All of the washers have the same mass, so the number of washers is proportional to the mass of the pendulum bob, and mass will be measured in number of washers. Start the timer in DataStudio. It will stop automatically in 15 s. In GA, record the mean value of the period shown in DataStudio.

2   Continue changing the mass of the pendulum by adding one washer at a time. Measure the period of oscillations for each mass. Keep recording the mean value of the period and the number of washers in GA.

3   In GA, make a graph of the pendulum period (the mean value) vs. the mass (the number of washers).

4   Record the slope and its uncertainty of the period vs. mass graph in the Inlab in WebAssign and answer the questions.

Part 2: Measure the period of oscillations for pendulum bobs at different amplitudes while keeping length and mass fixed.

1   Place **two** washers on the string. Start the timer in DataStudio. It will stop automatically in 15 s. Record the mean value of the period in GA that corresponds to each amplitude.

2   Repeat measurements of the period of oscillations for angles 3°, 5°, 8°, 10°, 12°, 15°, 18°, 20°, 25°, 30°, and any angle between 40° and 70°. In GA, create columns to record the mean value of the period and the starting amplitudes (angles). Make a bar graph of period vs. amplitude.

3   What conclusion can you make about the relationship between the period of oscillations and amplitude (maximum angle) for a simple pendulum?

Part 3: Measure the period of oscillations for pendulum bobs while varying length.

Keep the initial angle (i.e., 3°) and mass (two washers) constant. The length of a simple pendulum is the distance from the pivot point to the center of mass of the pendulum bob.

1   Measure the period of oscillations for 6 different lengths of the pendulum.

2   Enter your measured values of the period T (the mean value) and corresponding length L into GA. Create a new calculated column for the value of $\sqrt{L}$ (DATA → NEW CALCULATED COLUMN → sqrt "variable"). Plot T vs. $\sqrt{L}$ and apply "Linear Fit" from the "Analyze" pull-down menu. The slope of this graph and its error will allow you to determine the acceleration due to gravity g and the error. Record the slope of the graph and its error in the Inlab. Take a screenshot of the graph to upload in the Inlab. Use the slope value to calculate the value of g.

From equation 4:

$$T = \frac{2\pi}{\sqrt{g}} \cdot \sqrt{L} = \text{slope} \cdot \sqrt{L} \tag{7a}$$

$$\text{slope} = \frac{2\pi}{\sqrt{g}} \tag{7b}$$

3 Calculate the percent discrepancy between your experimental and theoretical values of gravitational acceleration.

Calculating the Mass of Earth

Since the factor of g occurs in the period of a pendulum, it is possible to use the measurements taken above to determine the mass of the Earth. The acceleration due to gravity near the surface of the Earth is defined as

$$g = \frac{GM_{\text{E}}}{r_{\text{E}}^2}. \tag{8}$$

Since G, M_{E} and r_{E} have fixed values, all objects near the surface of the Earth, regardless of their mass, are subject to the same acceleration.

Substituting this expression for g into equation 4 for the period of the pendulum we get

$$T = 2\pi \sqrt{\frac{Lr_{\text{E}}^2}{GM_{\text{E}}}}. \tag{9}$$

For one of your trials above, you might want to use the measured length of the pendulum and the average period as determined by DataStudio to compute the mass of the Earth. Compare it with the accepted value of 5.97×10^{24} kg by calculating the relative discrepancy.

Adapted from PASCO scientific at pasco.com.