Answer to Essential Question 4.4 Assuming that we can neglect air resistance, the relative mass of the balls is completely irrelevant. If *B*'s mass was double *A*'s mass, for instance, the force of gravity on *B* would be twice that on *A*, but both balls would still have an acceleration of $\vec{g}$, and the two balls would still hit the ground simultaneously.

4-5 The Independence of x and y

A key to understanding projectile motion is the independence of *x* and *y*, the fact that the horizontal (*x*-direction) motion is completely independent of the vertical (*y*-direction) motion. Let's exploit this concept to continue our analysis of the race from Exploration 4.4.

EXPLORATION 4.5 – Analyzing the race

Step 1 – ***Find the acceleration of each ball.*** The free-body diagram in Figure 4.11, combined with Newton's second law, tells us that the acceleration of each object is simply the acceleration due to gravity, $\vec{g}$. This comes from:

$$\vec{a} = \frac{\Sigma\vec{F}}{m} = \frac{m\vec{g}}{m} = \vec{g}\,.$$

Step 2 – ***Use the general method to find the time it takes the balls to reach the ground.*** Because the motion is directed right and down, let's choose positive directions as $+x$ to the right and $+y$ down, and set the origin for each ball to be the point from which it is released. Choosing up as positive, with an origin at ground level, would also work well.

Let's say both balls fall through a vertical distance of *h*, and that the initial velocity of ball B is directed horizontally with a velocity of v_i. Table 4.5 shows how we organize the data. Note how the data for the *x*-direction (horizontal) motion for ball *B* are kept separate from the data for the *y*-direction (vertical) motion.

Component	**Ball *B*, *x* direction**	**Ball *B*, *y* direction**	**Ball *A*, *y* direction**
Initial position	$x_{iB} = 0$	$y_{iB} = 0$	$y_{iA} = 0$
Final position	$x_B = ?$	$y_B = +h$	$y_A = +h$
Initial velocity	$v_{ixB} = +v_i$	$v_{iyB} = 0$	$v_{iyA} = 0$
Final velocity	$v_{xB} = +v_i$	$v_{yB} = ?$	$v_{yA} = ?$
Acceleration	$a_{xB} = 0$	$a_{yB} = +g$	$a_{yA} = +g$

Table 4.5: Organizing the data for ball *A* (dropped from rest) and ball *B* (with an initial velocity that is horizontal). Note that everything is the same for the two balls in the *y*-direction, which is vertical.

One of the most common errors in analyzing a projectile-motion situation is to mix up information from the *x* and *y* directions, such as by using the acceleration due to gravity as the acceleration in the horizontal direction. Organizing the data into a table like the one above makes such errors far less likely. Including the appropriate sign on all vectors is another way to reduce errors, because it reminds us to think about which sign is correct and whether we really want a + or a –. A statement like $y_B = +h$ tells us that the final vertical position of ball *B* is a distance *h* from the origin in the positive *y*-direction.

Can we use the data from Table 4.5 to justify the conclusion from Exploration 4.4 that the two balls reach the ground at the same time? Absolutely. The appropriate motion diagrams are shown in Figure 4.12. Analyzing the y-direction subproblem for ball B, we can use equation 4.3y to find an expression for the time to reach the ground.

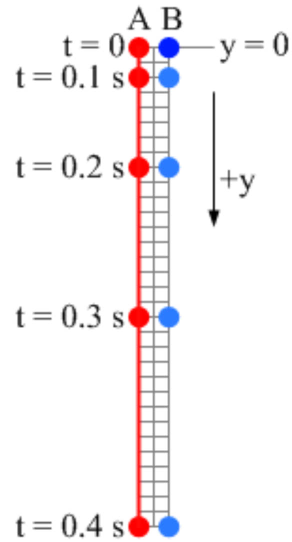

Figure 4.12: A motion diagram for the vertical components of the motion for the balls.

$$\vec{y} = \vec{y}_i + \vec{v}_{iy}t + \frac{1}{2}\vec{a}_y t^2 ,$$

$$+h = 0 + 0 + \frac{1}{2}gt^2 .$$

This gives $t^2 = +\frac{2h}{g}$.

Therefore, the time for ball B to reach the ground is $t = \sqrt{\frac{2h}{g}}$.

Does the answer make any sense? First, it does have the right units. Second, it says that if we increase h the ball takes longer to reach the ground, which makes sense. Third, it says that the larger the acceleration due to gravity the smaller the time the ball takes to reach the ground, which also sounds right. Note that if we solve for the time ball A takes to reach the ground we get exactly the same result, because A has the same initial position, final position, initial vertical velocity, and vertical acceleration as B.

Step 3 – ***Find an expression for the horizontal distance traveled by ball B before it reaches the ground.*** Even though we're dealing with the x subproblem, we can use the time from the y subproblem – that is often key to solving projectile motion problems. The motion diagram for the x-direction motion is shown in Figure 4.13. One way to find the horizontal distance that ball B travels is to use Equation 4.3x (see Table 4.4 in Section 4.4 for the equations).

$$x = x_i + v_{ix}t + \frac{1}{2}a_x t^2 ,$$

$$x = 0 + v_i t + 0 = +v_i\sqrt{\frac{2h}{g}} .$$

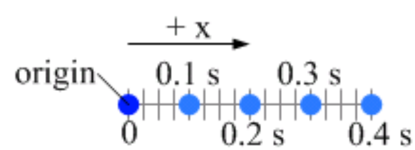

Figure 4.13: A motion diagram for the horizontal component of ball B's motion.

Again, be careful not to mix the x information with the y information. Here, for instance, we can use Table 4.5 to remind us that the acceleration in the x direction is zero. The motion diagram in Figure 4.13, showing constant-velocity motion, confirms that the acceleration in the x direction is zero.

Key idea for projectile motion: One way to solve a projectile-motion problem is to break the two-dimensional problem into two independent one-dimensional problems, ***linked by the time***, and apply the one-dimensional constant-acceleration methods from Chapter 2.
Related End-of-Chapter Exercises: 44, 45.

Essential Question 4.5 When a sailboat is at rest, a beanbag you release from the top of the mast lands in a bucket that is on the deck at the base of the mast. Will the beanbag still land in the bucket if you release the beanbag from rest when the sailboat is moving with a constant velocity?