

Answer to Essential Question 4.6 The result we found in Example 4.6, that the total time for the flight is $t_{\text{impact}} = 2v_{iy} / g$, is a special case that applies only when the projectile lands at the same height from which it was launched. In this special case, the time for the entire flight is twice the time to reach maximum height (thus, $t_{\text{max height}} = v_{iy} / g$). This result comes about because of the symmetry of the motion. The second half of the motion, when the projectile is coming down, is a mirror image of the first half of the motion, when the projectile is going up. These two halves take exactly the same time (neglecting air resistance). The projectile's speed on the way up at a particular height is equal to its speed on the way down as it passes through that same height.

4-7 Graphs for Projectile Motion

Graphs can give us a lot of information about a particular motion. Let's consider how to draw a set of graphs for the motion of the soccer ball in Example 4.6.

EXAMPLE 4.7 – Graphs for the ball chipped over the goalkeeper

Draw graphs showing, as a function of time, the x -component of position, x component of velocity, and x component of acceleration for the soccer ball in Example 4.6. Take x to be horizontal, with the positive x direction pointing from the launch point toward the goalkeeper and the net. Draw another set of graphs for the y component, taking the positive y direction to be up.

SOLUTION

A graph can be thought of as a picture of an equation. To graph these various parameters as a function of time, it can be helpful to write down the corresponding equation. Graphs for the x components are shown in Figure 4.16.

To graph the x component of the position as a function of time, take what we know about the ball and substitute these known quantities into Equation 4.3x (see Table 4.4 in Section 4.4):

$$x = x_i + v_{ix}t + \frac{1}{2}a_x t^2$$

$$x = 0 + (+5.00 \text{ m/s})t + 0.$$

$$x = +(5.00 \text{ m/s})t$$

This is a linear function starting at the origin and having a slope of $+5.00 \text{ m/s}$.

To graph the x component of the velocity as a function of time, take what we know about the ball and fill in equation 4.2x:

$$v_x = v_{ix} + a_x t$$

$$v_x = +5.00 \text{ m/s} + 0.$$

$$v_x = +5.00 \text{ m/s}$$

The x component of the velocity is constant, so the graph of this velocity component is a horizontal line, as shown in the middle graph in Figure 4.16.

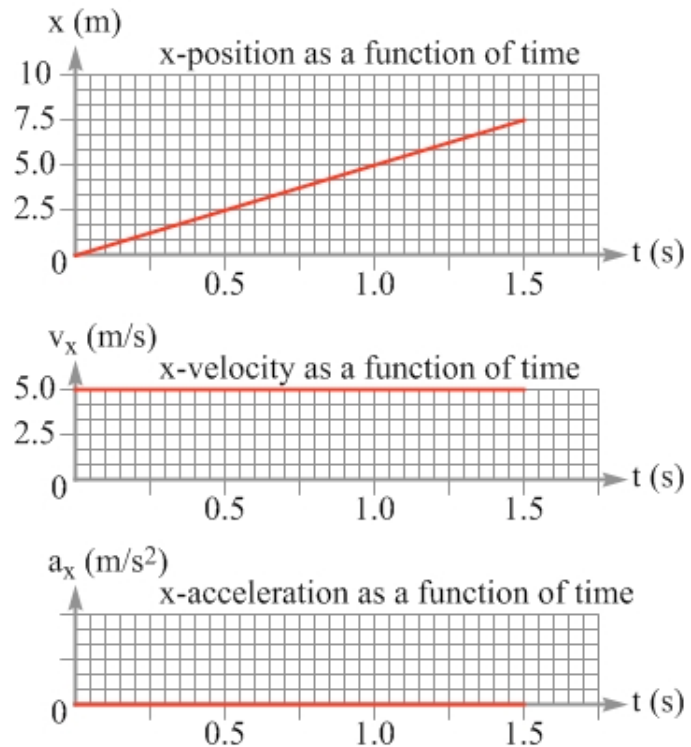


Figure 4.16: Graphs of the horizontal components of the ball's position, velocity, and acceleration.

The graph of the acceleration in the x direction is also a horizontal line, being drawn along the time axis, because there is no acceleration in the x direction.

In the y direction, we can do something similar. The three y -component graphs are drawn in Figure 4.17. To graph the y component of the position as a function of time, take what we know about the ball and fill in equation 4.3y:

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$

$$y = 0 + (7.40 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2.$$

The graph representing this equation is a parabola curving downward. Note the symmetry of the graph, with the second half of the motion being a mirror-image of the first half. Equation 4.4y can be used to find the maximum height the ball rises above the ground, knowing that at maximum height the y component of the velocity is zero. From Equation 4.4y we get:

$$v_y^2 = v_{iy}^2 + 2a_y \Delta y,$$

$$\text{which gives } \Delta y = \frac{v_y^2 - v_{iy}^2}{2a_y} = \frac{0 - (7.40 \text{ m/s})^2}{-2(9.81 \text{ m/s}^2)} = \frac{54.76 \text{ m}^2/\text{s}^2}{19.62 \text{ m/s}^2} = 2.79 \text{ m}.$$

It is helpful to know the value of the maximum height when we graph the y component of the position.

To graph the y component of the velocity as a function of time, we take what we know about the ball and fill in Equation 4.2y:

$$v_y = v_{iy} + a_y t = +(7.40 \text{ m/s}) - (9.81 \text{ m/s}^2)t.$$

A graph of the y component of the velocity is a straight line with a y -intercept of $+7.40 \text{ m/s}$ and a slope of -9.81 m/s^2 . Note that we can tell when the ball reaches maximum height from both the y -position graph and the y -velocity graph.

The y component of the acceleration is constant at -9.81 m/s^2 , so that graph is a horizontal line.

Related End-of-Chapter Exercises: 47 – 49.

Essential Question 4.7 Compare the two sets of graphs in Example 4.7 to the graphs for motion with constant velocity, and motion with constant acceleration, that we drew in Chapter 2. What similarities are there?

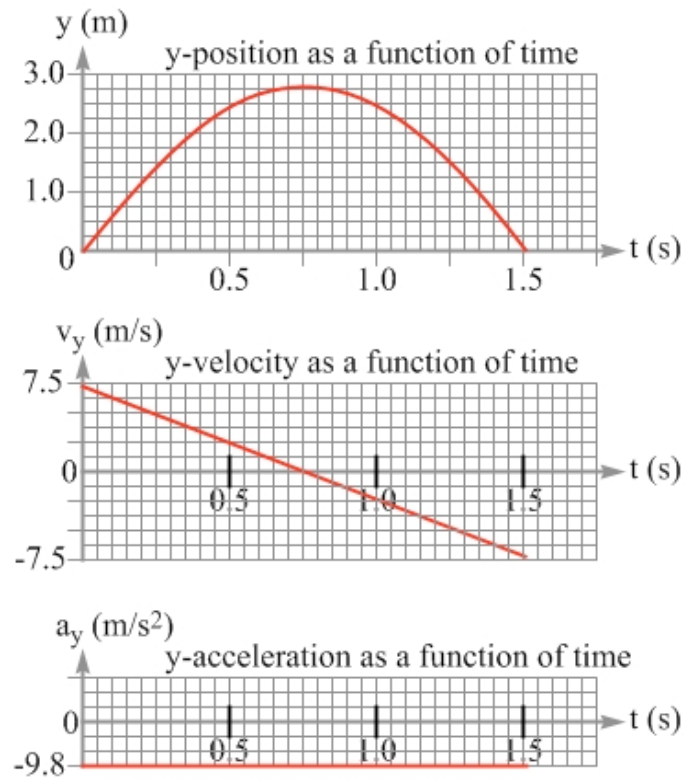


Figure 4.17: Graphs of the vertical components of the ball's position, velocity, and acceleration.