Answer to Essential Question 5.7: Even in very low-friction conditions, it is safer to travel at the design speed than at a slower speed! If you go too slowly around a banked curve, there is a tendency for your vehicle to slide down the slope and run off the road on the inside of the curve.

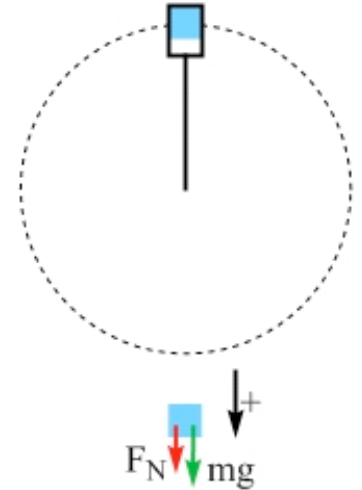## 5-8 Vertical Circular Motion

A common application of circular motion is an object moving in a vertical circle. Examples include roller coasters, cars on hilly roads, and a bucket of water on a string. The bucket and roller coaster turn completely upside down as they travel, so they differ a little from the situation of the car on the road, which (we hope) remains upright.

EXAMPLE 5.8A – Whirling a bucket of water
A bucket of water is being whirled in a vertical circle of constant radius r at a constant speed v. What is the minimum speed required for the water to remain in the bucket at the top of the circle?

SOLUTION
Let's apply the general method, starting with the diagram in Figure 5.20. We then draw a free-body diagram, although we have to decide whether to analyze the bucket or the water. If we consider the bucket, two forces act on it, the force of gravity and the tension in the string, both of which are directed down when the bucket is at the top of the circle. If we consider the water, there is a downward force of gravity, and a downward normal force from the bucket takes the place of the tension. The analysis is the same in both cases, so let's consider the water.

Next, we choose an appropriate coordinate system. It is generally best to choose the positive direction as the direction of the acceleration, which points toward the center of the circle. When the bucket is at the top of the path, the acceleration, and the positive direction, points down. We don't need to split any forces into components. Let's apply Newton's second law, $\sum \vec{F} = m\vec{a}$.

Figure 5.20: A bucket of water whirled in a vertical circle and a free-body diagram showing the forces acting on the water at the top of the loop.

Have a look at the free-body diagram to evaluate the left-hand side, and write the right-hand side in the usual circular-motion form. This gives:

$$+mg + F_N = +\frac{mv^2}{r} .$$

Solving for the normal force gives: $F_N = \frac{mv^2}{r} - mg$.

As long as the first term on the right exceeds the second term (in other words, as long as the normal force is positive), we're in no danger of having the water fall on us. Objects lose contact with one another (the water starts to fall out) when the normal force goes to zero. Setting the normal force to zero gives us the minimum safe speed of the bucket at the top of the circle:

$$0 = \frac{mv_{min}^2}{r} - mg , \qquad \text{which leads to } v_{min} = \sqrt{gr} .$$

Related End-of-Chapter Exercises: 12, 61.

EXAMPLE 5.8B – Apparent weight on a roller coaster
You are riding on a roller coaster that is going around a vertical circular loop. What is the expression for the normal force on you at the bottom of the circle?

SOLUTION
Once again, we apply the general method, starting with a diagram and a free-body diagram in Figure 5.21. We then draw a free-body diagram, which shows an upward normal force and a downward force of gravity. The system can be you or the car – the analysis is the same. Here we choose a coordinate system with a positive direction up, in the direction of the acceleration (toward the center of the circle). There is no need to split forces into components, so we can go straight to step 5 of the general method and apply Newton's Second Law:

$$\sum \vec{F} = m\vec{a} \ .$$

Have a look at the free-body diagram to evaluate the left-hand side, and remember that the right-hand side can be written in the usual circular-motion form. This gives:

$$+F_N - mg = +\frac{mv^2}{r} \ .$$

Solving for the normal force at the bottom of the circle gives:
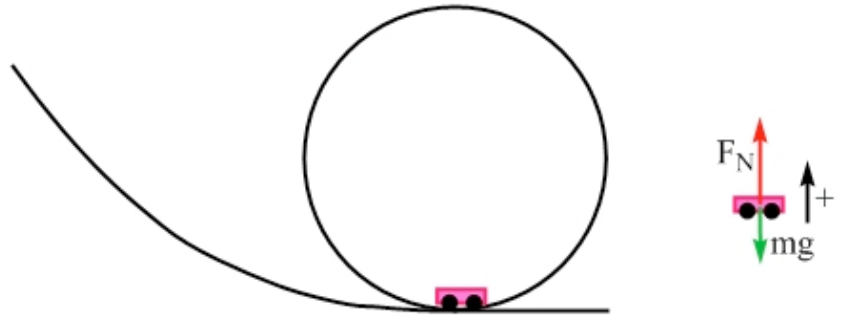
$$F_{N,bottom} = \frac{mv^2}{r} + mg \ .$$

Let's compare our expression for the normal force on the car (or you) at the bottom of the loop to the expression for the normal force when the car (or you) is at the top. We can use the expression that we derived for the bucket at the top, in Example 5.8A, because the free-body diagram is the same in the two situations at the top of the loop.

Figure 5.21: A car on a roller-coaster track (left), as well as (right) the free-body diagram when the car is at the bottom of the loop.

$$F_{N,top} = \frac{mv^2}{r} - mg \ .$$

Note that the normal force at the bottom is larger than it is at the top. This difference is enhanced by the fact that the speed of the roller coaster at the bottom of the loop is larger than the speed at the top. Does this change in the normal force match the experience of a rider, who feels that she is lighter than usual at the top of the loop and heavier than usual at the bottom? Yes, because the normal force is the rider's apparent weight. Roller coasters are generally designed to have non-zero but fairly small normal forces at the top, so a rider feels almost weightless. At the bottom of the loop, the apparent weight can be considerably larger than *mg*, so a rider feels much heavier than usual.

Related End-of-Chapter Exercises: 20, 62.

Essential Question 5.8: You are on a roller coaster that reaches a top speed of 120 km/h at the bottom of a circular loop of radius 30 m. If you have a mass of 50 kg (and therefore a weight of 490 N), what is your apparent weight at the bottom of the loop? If the roller coaster's speed at the top of the loop has dropped to 80 km/h, what is your apparent weight at the top of the loop?