Answer to Essential Question 10.4: Quite a number of tools and gadgets exploit torque, in the sense that they enable you to apply a small force at a relatively large distance from an axis, and the tool converts that into a large force acting at a relatively small distance from an axis. Examples include scissors, bottle openers, can openers, nutcrackers, screwdrivers, crowbars, wrenches, wheelbarrows, and bicycles.

10-5 Three Equivalent Methods of Finding Torque

EXPLORATION 10.5 – Three ways to find torque

A rod of length L is attached to a wall by a hinge. The rod is held in a horizontal position by a string that is tied to the wall and attached to the end of the rod, as shown in Figure 10.12.

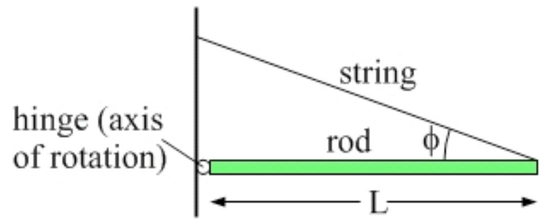

Figure 10.12: A rod attached to a wall at one end by a hinge, and held horizontal by a string.

Step 1 – ***In what direction is the torque applied by the string to the rod, about an axis that passes through the hinge and is perpendicular to the page?*** As we did in previous chapters, it's a good idea to draw a free-body diagram of the rod (or at least part of a free-body diagram, as in Figure 10.13) to help visualize what is happening. For now the only force we'll include on the free-body diagram is the force of tension applied by the string (we'll go on to look at all the forces applied to the rod in Exploration 10.8). Try placing your pen over the picture of the rod. Hold the pen where the hinge is and push on the pen, at the point where the string is tied to the rod, in the direction of the force of tension. You should see the pen rotate counterclockwise. Thus, we can say that the torque applied by the string, about the axis through the hinge, is in a counterclockwise direction.

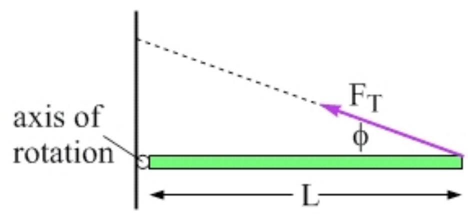

Figure 10.13: A partial free-body diagram for the rod, showing the force of tension applied to the rod by the string.

Note that we are dealing with direction for torque much as we did for angular velocity. The true direction of the torque can be found by curling your fingers on your right hand counterclockwise and placing your hand, little finger down, on the page. When you stick out your thumb it points up, out of the page. This is the true direction of the torque, but for simplicity we can state directions as either clockwise or, as in this case, counterclockwise.

Now we know the direction of the torque, relative to an axis through the hinge, applied by the string, let's focus on determining its magnitude.

Step 2 – ***Measuring the distance r in Equation 10.9 along the bar, apply Equation 10.9 to find the magnitude of the torque applied by the string on the rod, with respect to the axis passing through the hinge perpendicular to the page.***

Finding the magnitude of the torque means identifying the three variables, r, F, and θ, in Equation 10.9. In this case we can see from Figure 10.13 that the distance r is the length of the rod, L; the force $\vec{F}$ is the force of tension, $\vec{F}_T$; and the angle θ is the angle between the line of the force (i.e., the string) and the line the distance r is measured along (the rod), so θ is the angle ϕ in Figure 10.13. In this case, then, applying Equation 10.9 tells us that the magnitude of the torque is $\tau = L F_T \sin\phi$.

Step 3 – ***Now, determine the torque, about the axis through the hinge that is perpendicular to the page, by first splitting the force of tension into components, and then applying Equation 10.9.*** Which set of axes should we use when splitting the force into components? The most sensible coordinate system is one aligned parallel to the rod and perpendicular to the rod, giving the two components shown in Figure 10.14. Because the force component that is parallel to the rod is directed at the hinge, where the axis goes through, that component gives a torque of zero (it's like trying to open a door by pushing on the door with a force directed at the line passing through the hinges). Another way to prove that the force is zero is to apply Equation 10.9 with an angle of 180˚, which means multiplying by a factor of sin(180˚), which is zero.

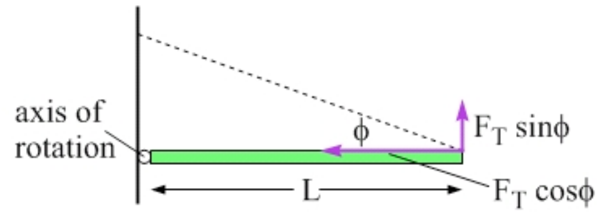

Figure 10.14: Splitting the force of tension into a component parallel to the rod, and a component perpendicular to the rod.

The torque from the force of tension is associated entirely with the perpendicular component of the force of tension. Now, identifying the three pieces of Equation 10.9 gives a force magnitude of $F = F_T \sin\phi$; a distance measured along the rod of $r = L$, and an angle of $\theta = 90^\circ$ between the line of the perpendicular force component and the line we measured *r* along. Because $\sin(90^\circ) = 1$, applying Equation 10.9 tells us that the magnitude of the torque from the tension, with respect to our axis through the hinge, is $\tau = L(F_T \sin\phi)\sin(90^\circ) = L F_T \sin\phi$. This agrees with our calculation in Step 2.

Step 4 – ***Instead of measuring r along the rod, draw a line from the hinge that meets the string (the line of the force of tension) at a 90˚ angle. Apply Equation 10.9 to find the magnitude of the torque applied by the string on the rod, with respect to the axis passing through the hinge, by measuring r along this line.***

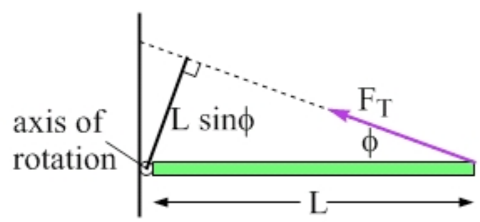

Figure 10.15: A diagram showing the lever arm, in which the distance used to find torque is measured from the axis along a line perpendicular to the line of the force.

As we can see from Figure 10.15, the *r* in this case is not *L*, the length of the rod, but is instead $L\sin\phi$. This result comes from applying the geometry of right-angled triangles. The magnitude of the force, F , is F_T , the magnitude of the full force of tension, and the angle between the line we measure *r* along and the line of the force is 90˚. This is known as the **lever-arm method** of calculating torque, where the lever-arm is the perpendicular distance from the axis of rotation to the force. Applying Equation 10.9 gives the magnitude of the torque as $\tau = (L\sin\phi) F_T \sin(90^\circ) = L F_T \sin\phi$, agreeing with the other two methods discussed above.

Key idea for torque: We can find torque in three equivalent ways. It can be found using the whole force and the most obvious distance; after splitting the force into components; or by using the lever-arm method in which the distance from the axis is measured along the line perpendicular to the force. Use whichever method is most convenient in a particular situation. **Related End-of-Chapter Exercises: 7, 23, 50.**

Essential Question 10.5: Torque can be calculated with respect to any axis. In Exploration 10.5, what is the torque, due to the force of tension, with respect to an axis passing through the point where the string is tied to the wall? In each case, assume the axis is perpendicular to the page.