Answer to Essential Question 10.6: The correct ranking is 3>1>2. In the rotational inertia equation, the distance from the axis to the ball (the length of the rod) is squared, while the mass is not. Thus, changing the length by a factor of 2 changes the rotational inertia by a factor of 4, whereas changing the mass by a factor of 2 changes the rotational inertia by only a factor of 2.

10-7 An Example Problem Involving Rotational Inertia

Our measure of inertia for rotational motion is somewhat more complicated than inertia for straight-line motion, which is just mass. Consider the following example.

EXAMPLE 10.7 – Spinning the system.

Three balls are connected by light rods. The mass and location of each ball are:
Ball 1 has a mass M and is located at $x = 0$, $y = 0$.
Ball 2 has a mass of $2M$ and is located at $x = +3.0$ m, $y = +3.0$ m.
Ball 3 has a mass of $3M$ and is located at $x = +2.0$ m, $y = -2.0$ m.
Assume the radius of each ball is much smaller than 1 meter.

(a) Find the location of the system's center-of-mass.
(b) Find the system's rotational inertia about an axis perpendicular to the page that passes through the system's center-of-mass.
(c) Find the system's rotational inertia about an axis parallel to, and 2.0 m from, the axis through the center-of-mass.

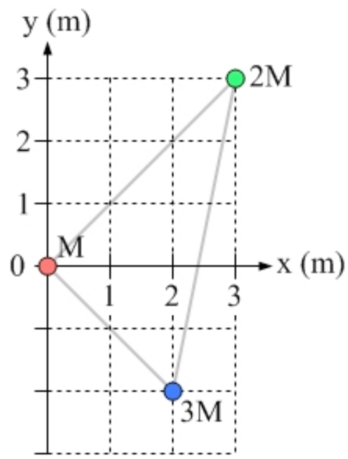

Figure 10.19: A diagram showing the location of the balls in the system described in Example 10.7.

SOLUTION

Let's begin, as usual, by drawing a diagram of the situation. The diagram is shown in Figure 10.19.

(a) To find the location of the system's center-of-mass, let's apply Equation 6.3. To find the x-coordinate of the system's center-of-mass:

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{(0)M + (+3.0\,\text{m})(2M) + (+2.0\,\text{m})(3M)}{M + 2M + 3M} = \frac{(+12.0\,\text{m})M}{6M} = +2.0\,\text{m}$$

The y-coordinate of the system's center-of-mass is given by:

$$Y_{CM} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3} = \frac{(0)M + (+3.0\,\text{m})(2M) + (-2.0\,\text{m})(3M)}{M + 2M + 3M} = \frac{(0)M}{6M} = 0\,.$$

(b) To find the system's rotational inertia about an axis through the center-of-mass we can find the rotational inertia for each ball separately, using $I = M\,L^2$, and then simply add them to find the total rotational inertia. Figure 10.20 is helpful for seeing where the different L values come from.

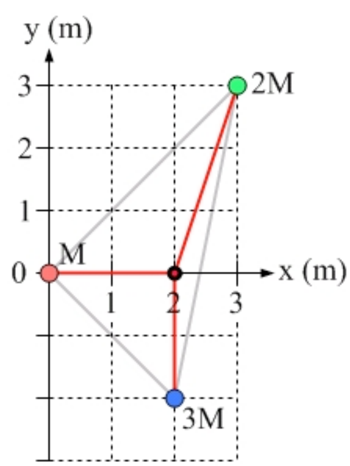

Figure 10.20: The center-of-mass of the system is marked at (+2 m, 0). The axis of rotation passes through that point. The dark lines show how far each ball is from the axis of rotation.

For ball 1, $L^2 = (2.0\,\text{m})^2 = 4.0\,\text{m}^2$ so $I_1 = M\,L^2 = \left(4.0\,\text{m}^2\right)M$.

For ball 2, $L^2 = 10\,\text{m}^2$ so $I_2 = 2M\,L^2 = \left(20\,\text{m}^2\right)M$.

For ball 3, $L^2 = 4.0\,\text{m}^2$ so $I_3 = 3M\,L^2 = \left(12\,\text{m}^2\right)M$.

The total rotational inertia is the sum of these three values, $(36\,\text{m}^2)M$.

(c) To find the rotational inertia through an axis parallel to the first axis and 2.0 m away from it, let's choose a point for this second axis to pass through. A convenient point is the origin, $x = 0$, $y = 0$. Figure 10.21 shows where the L values come from in this case.

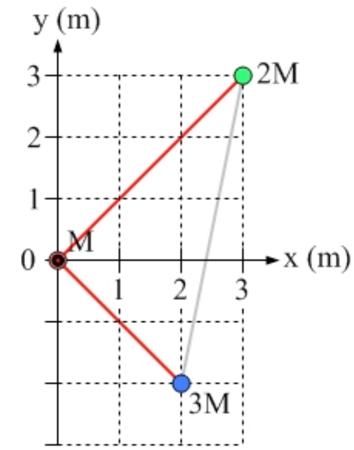

Figure 10.21: The axis of rotation now passes through the ball of mass M at the origin. The red lines show how far the other two balls are from the axis of rotation.

Repeating the process we followed in part (b) gives:

For ball 1, $L^2 = 0$ so $I_1' = 0$.

For ball 2, $L^2 = 18\,\text{m}^2$ so $I_2' = 2M\,L^2 = \left(36\,\text{m}^2\right)M$.

For ball 3, $L^2 = 8.0\,\text{m}^2$ so $I_3' = 3M\,L^2 = \left(24\,\text{m}^2\right)M$.

The total rotational inertia is the sum of these three values, $(60\,\text{m}^2)M$.

Related End-of-Chapter Exercises: 29, 31.

Does it matter which point the second axis passes through? What if we had used a different point, such as $x = +2.0$ m, $y = -2.0$ m, or any other point 2.0 m from the center-of-mass? Amazingly, it turns out that it doesn't matter. Any axis parallel to the axis through the center-of-mass and 2.0 m from it gives a rotational inertia of $(60\,\text{m}^2)M$. It turns out that the rotational inertia of a system is minimized when the axis goes through the center-of-mass, and the rotational inertia of the system about any parallel axis a distance h from the axis through the center-of-mass can be found from

$$I = I_{CM} + mh^2,$$ (Equation 10.11: **The parallel-axis theorem**)

where m is the total mass of the system.

Let's check the parallel-axis theorem using our results from (b) and (c). In part (b) we found that the rotational inertia about the axis through the center-of-mass is $I_{CM} = (36\,\text{m}^2)M$. The mass of the system is $m = 6M$ and the second axis is $h = 2.0$ m from the axis through the center-of-mass. This gives $I = (36\,\text{m}^2)M + 6M(2.0\,\text{m})^2 = (60\,\text{m}^2)M$, as we found above.

Essential Question 10.7: To find the total mass of a system of objects, we simply add up the masses of the individual objects. To find the total rotational inertia of a system of objects, can we follow a similar process, adding up the rotational inertias of the individual objects.