Answer to Essential Question 13.1: We'll address this issue further in the next section, but a prediction that the cavity volume decreases because of the expansion of the glass is consistent with an alcohol level higher than it would be if only the alcohol expanded. Increasing the cavity volume corresponds to an alcohol level lower than it would be if only the alcohol expanded.

13-2 Thermal Expansion

When an object's temperature changes, we assume the change in length ΔL experienced by each dimension of the object is proportional to its change in temperature ΔT. This model is valid as long as ΔT is not too large. We can express this linear relationship as an equation:

$\Delta L = L_i \alpha \Delta T$, (Equation 13.4: **Length change from thermal expansion**)

where L_i is the original length and α is known as the linear thermal expansion coefficient, which depends on the material (see Table 13.1).

Material	**α (× 10^-6 /°C)**
Aluminum	23
Brass	19
Copper	17
Diamond	1
Glass	8.5
Gold	14
Iron or steel	12
Lead	29
Stainless steel	17

Table 13.1: Values of α, the linear thermal expansion coefficient, for various materials at 20°C.

Another way to express the linear nature of the model is to relate the initial length L_i to the new length L. Because $L = L_i + \Delta L$ we can express the new length as:

$L = L_i + L_i \alpha \Delta T = L_i (1 + \alpha \Delta T)$. (Equation 13.5)

Consider now a rectangular object with an area A_i that is the product of its height H_i and its width W_i. Both the height and width change with temperature, so the new area is:

$$A = HW = H_i(1+\alpha \Delta T)W_i(1+\alpha \Delta T) = H_i W_i(1+\alpha \Delta T)(1+\alpha \Delta T) = A_i\left[1 + 2\alpha \Delta T + (\alpha \Delta T)^2\right]$$

Because values of α are small, for small temperature changes $\alpha \Delta T$ is significantly less than 1. Squaring a number less than 1 makes it even smaller, so the $(\alpha \Delta T)^2$ term in the previous equation is negligible compared to the $2\alpha \Delta T$ term. Thus we can write the area equation as:

$A = A_i (1 + 2\alpha \Delta T)$. (Equation 13.6: **Area thermal expansion**)

A similar process can be followed to show that the equation for the volume V resulting from imposing a temperature change ΔT on an object of original volume V_i is:

$V = V_i (1 + 3\alpha \Delta T)$. (Equation 13.7: **Volume thermal expansion**)

EXAMPLE 13.2 – Shrink to fit

You are building a plane, but the stainless steel rivets you are using will not fit through the holes in the skin of the plane. At 20°C, each rivet has a diameter of 12.020 mm, while the diameter of a hole is 12.000 mm. To what temperature should you cool a rivet so it fits in a hole?

SOLUTION

We need to find the temperature change required so the diameter of the rivet equals the diameter of the hole. To find this we can use Equation 13.5, $L = L_0(1+\alpha\,\Delta T)$, with the thermal expansion coefficient from Table 13.1 for stainless steel, $\alpha = 17\times10^{-6}\,/°\text{C}$. Solving for ΔT gives:

$$\Delta T = \frac{L - L_0}{L_0\,\alpha} = \frac{12.000\,\text{mm} - 12.020\,\text{mm}}{(12.020\,\text{mm})(17\times10^{-6}\,/°\text{C})} = \frac{-0.020\ \text{mm}}{204.34\times10^{-6}\ \text{mm}/°\text{C}} = -98°\text{C}\,.$$

Because the initial temperature is 20°C, we get a final temperature of –78°C. Note that the equation involves ΔT, so we can work entirely in Celsius. Also, there is no reason to convert the lengths to meters because the length units cancel out. One way to cool the rivets this much would be to immerse them in liquid nitrogen, which has a temperature of about –196°C, or 77K.

EXPLORATION 13.2 – What do holes do?

Imagine trying to fit a brass ball through a brass ring that has an inner radius just a little smaller than the ring. Should we heat the ring or cool it so that the ball can pass through?

Step 1 – ***Take a solid disk with a radius equal to the outer radius of the ring. Draw a circle on this disk so the radius of the circle matches the inner radius of the ring. What happens when the temperature of the disk increases?*** A picture of this situation is shown in Figure 13.2. The disk expands when its temperature increases, and so does the circle drawn on the disk. Removing the material inside the circle leaves a ring.

Step 2 – ***Reverse the order of the actions. First remove the inner part of the circle and then increase the temperature. What is the result?*** Figure 13.2 shows that changing the order of the actions makes no difference. The ring is the same size in both cases – a hole expands or contracts as if it were solid.

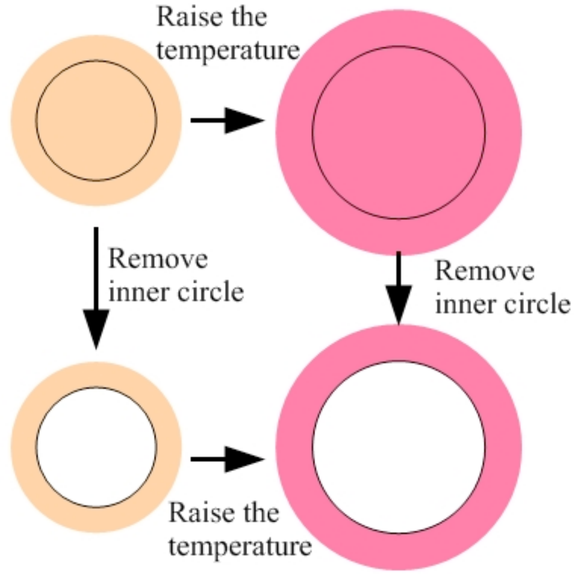

Figure 13.2: When the temperature of a disk increases, the entire disk expands, including the circle that was drawn on the disk. Removing the area within the circle leaves a ring with an inside diameter larger than the original diameter of the circle. Doing this in reverse order, removing the inside of the circle and then raising the temperature, has the same result. Holes expand as if they are filled with the surrounding material.

Key idea for holes: Holes and cavities in materials expand and contract as if they were made from the surrounding material.
Related End-of-Chapter Exercises: 17 – 19.

Increasing the temperature of a solid makes its atoms vibrate more energetically, so each atom effectively needs more space. If we imagine a circle of atoms around the inside diameter of the ring, as in Figure 13.3, the atoms spread out when heated, making the hole in the ring larger.

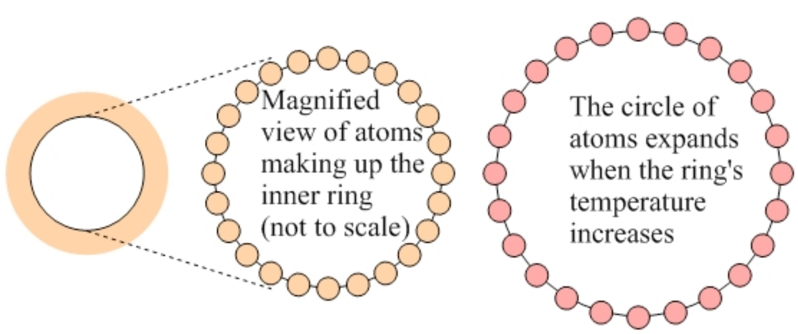

Figure 13.3: An object expands when its temperature increases because its atoms vibrate more energetically. This causes the circle of atoms along the inside diameter of a ring to expand when the temperature increases.

Essential Question 13.2: An iron ring will not quite fit over an aluminum cylinder. Can you fit the ring over the cylinder by increasing or decreasing the temperature of both objects at the same time? Explain.