Answer to Essential Question 18.9 is given below Figure 18.24.

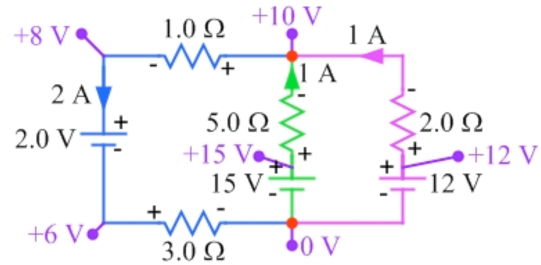

Figure 18.24: The solution to the circuit in Exploration 18.9, with the correct currents and with the potential labeled at various points.

Answer to Essential Question 18.9: Figure 18.24 shows the current in each branch, and the potential at various points relative to $V = 0$ at the lower junction. Labeling potential is like doing the loop rule. Starting at the lower junction and moving up the middle branch, the potential stays at $V = 0$ until we reach the 15-volt battery. Crossing the battery from the – terminal to the + terminal raises the potential by the battery emf to +15 V. The potential difference across the 5.0 Ω resistor is $IR = 5.0 \text{ V}$. As we move through the resistor in the same direction as the current the potential decreases, reaching +15 V – 5 V = +10 V at the upper end of that resistor, and at the upper junction. Thus, the upper junction has a potential 10 V higher than the lower junction.

Labeling the potential is a way to check the answer for the currents. Going from the lower junction to the upper junction via any branch gives the same answer for the potential of the upper junction. If the answer depended on the path, we would know something was wrong.

18-10 RC Circuits

In some circuits the current changes as time goes by. An example of this is an RC circuit, involving a resistor (R) and a capacitor (C).

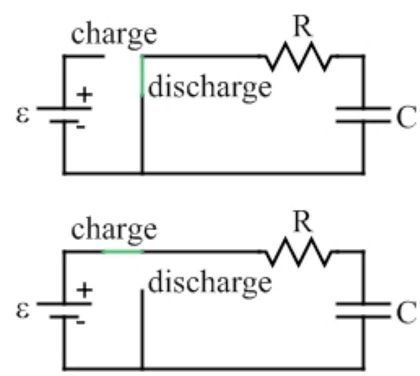

Figure 18.25: An RC circuit with a battery, resistor, capacitor, and switch.

EXPLORATION 18.10 – RC Circuits

In the RC circuit in Figure 18.25, the resistor and capacitor are in series with one another. There is also a battery of emf ε, and a switch that is initially in the "discharge" position. The capacitor is initially uncharged, so there is no current in the circuit.

Step 1 – ***What are the general equations for the potential difference across a resistor, and the potential difference across a capacitor?*** The potential difference across a resistor is given by Ohm's law, $\Delta V_R = IR$, while the potential difference across a capacitor is given by $\Delta V_C = Q/C$.

Step 2 – ***Use the loop rule to find the potential difference across the resistor, and across the capacitor, immediately after the switch is moved to the "charge" position.*** The capacitor is uncharged, so its potential difference is zero. The closed switch has no potential difference, so by the loop rule the potential difference across the resistor equals the emf of the battery.

Step 3 – ***What happens to the potential difference across the capacitor, the potential difference across the resistor, and the current in the circuit as time goes by?*** In this circuit, the battery pumps charge from one plate of the capacitor to the other. Because the charge is pumped through the resistor the rate of flow of charge is limited. As time goes by the charge on the capacitor increases, as does the potential difference across the capacitor (being proportional to the charge on the capacitor). By the loop rule, the potential difference across the resistor, and the current in the circuit, decreases as time goes by. Because the current (the rate of flow of charge) decreases, the rate at which the potential difference across the capacitor rises also decreases, slowing the rate at which the current decreases. This gives rise to the exponential relationships reflected in Figure 18.26, and characterized by the product of resistance and capacitance, which has units of time.

$$\tau = RC \quad . \qquad \text{(Equation 18.9: \textbf{Time constant for a series RC circuit})}$$

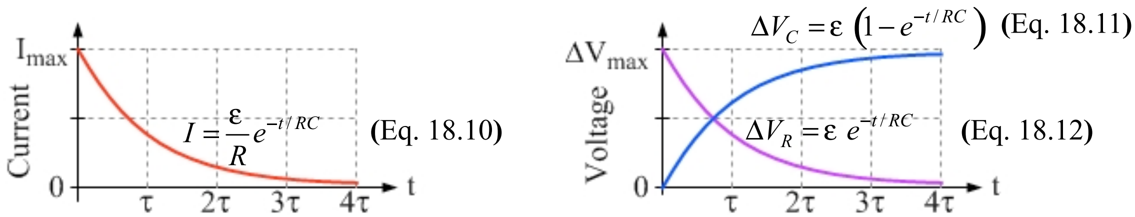

Figure 18.26: Plots of the current, resistor voltage, and capacitor voltage, as a function of time as the capacitor charges.

Step 4 – ***If we wanted the capacitor voltage to increase more quickly, could we change the resistance? If so, how? Could we accomplish this by changing the capacitance? If so, how?*** To change the capacitor voltage more quickly we could change the resistance or the capacitance. Decreasing the resistance increases the current, so charge flows to the capacitor more quickly. Decreasing the capacitance gives a larger potential difference across the capacitor with the same amount of charge, so that also works. This is consistent with the definition of the time constant, the product *RC*, which is a measure of how quickly the current, and potential differences, change in the circuit. Decreasing the time constant means that these quantities change more quickly.

Step 5 – ***When the switch has been in the "Charge" position for a long time, the circuit approaches a steady state, in which the current and the resistor voltage both approach zero, and the capacitor voltage approaches the battery emf. If the switch is now moved to the "Discharge" position, what happens to the potential difference across the capacitor, the potential difference across the resistor, and the current in the circuit as time goes by?*** Now the capacitor discharges through the resistor, so the current is in the opposite direction as it is when the capacitor is charging. The magnitude of the current decreases as time goes by because the potential difference across the resistor, which is the negative of the capacitor voltage by of the loop rule, decreases as time goes by. This gives rise to the relationships shown in Figure 18.27.

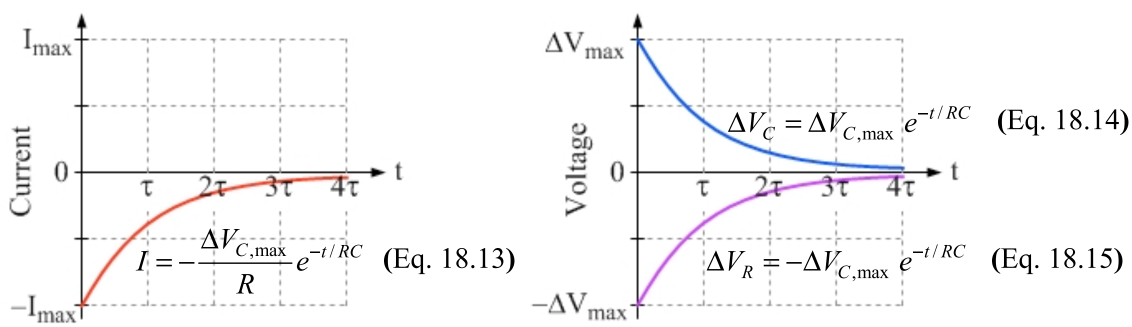

Figure 18.27: Plots of the current, resistor voltage, and capacitor voltage, as a function of time as the capacitor discharges.

Key ideas for RC Circuits: The current in an RC circuit changes as time goes by. In general, the current decreases exponentially with time. The expressions for the potential differences across the resistor and capacitor also involve negative exponentials of a quantity proportional to time, but the loop rule is satisfied at all times. **Related End-of-Chapter Exercises: 11, 12, 63, 64.**

Essential Question 18.10: A particular RC circuit is connected like that in Figure 18.25. The battery emf is 12 V, and the resistor has a resistance of 47 Ω. When the switch is placed in the "Charge" position, it takes 2.5 ms for the capacitor voltage to increase from 0 V to 3.0 V. What is the capacitance?