

Answer to Essential Question 22.2: (a) To find the wavelength, we can combine the equation $\lambda = v/f$ with the fact that the speed of light in air is 3.00×10^8 m/s. Thus, a frequency of 1×10^{18} Hz corresponds to a wavelength of 3×10^{-10} m, while a frequency of 90.9 MHz corresponds to a wavelength of 3.30 m. (b) Using Equation 22.2, with $c = 3.00 \times 10^8$ m/s, gives an amplitude of $B_{\max} = E_{\max}/c = (0.075 \text{ V/m})/(3.00 \times 10^8 \text{ m/s}) = 2.5 \times 10^{-10} \text{ T}$.

22-3 Energy, Momentum and Radiation Pressure

All waves carry energy, and electromagnetic waves are no exception. We often characterize the energy carried by a wave in terms of its intensity, which is the power per unit area. At a particular point in space that the wave is moving past, the intensity varies as the electric and magnetic fields at the point oscillate. It is generally most useful to focus on the average intensity, which is given by:

$$I_{\text{average}} = \frac{\text{average power}}{\text{area}} = \frac{E_{\max} B_{\max}}{2\mu_0} . \quad (\text{Eq. 22.3: The average intensity in an EM wave})$$

Note that Equations 22.2 and 22.3 can be combined, so the average intensity can be calculated using only the amplitude of the electric field or only the amplitude of the magnetic field.

Momentum and radiation pressure

As we will discuss later in the book, there is no mass associated with light, or with any EM wave. Despite this, an electromagnetic wave carries momentum. The momentum of an EM wave is the energy carried by the wave divided by the speed of light. If an EM wave is absorbed by an object, or it reflects from an object, the wave will transfer momentum to the object. The longer the wave is incident on the object, the more momentum is transferred. This time dependence complicates matters, though, so let's define something about this situation that does not depend on time, which is called radiation pressure, P .

When we looked at an analogous situation for a rubber ball bouncing off an object, in Chapter 7, the ball transfers twice as much momentum to the object when the collision causes the ball's velocity to be equal-and-opposite to what it was before the collision than it does when the ball is stopped completely by the collision. For electromagnetic waves, the pressure is twice as large when the wave reflects from a perfect reflector than when it is 100% absorbed.

$$P = \frac{2I}{c} . \quad (\text{Equation 22.4: Radiation pressure when a wave reflects 100\%})$$

$$P = \frac{I}{c} . \quad (\text{Equation 22.5: Radiation pressure when a wave is 100\% absorbed})$$

Simply shining a flashlight onto an object causes a pressure to be exerted on the object. For an ordinary flashlight, however, the pressure is so small that it is negligible. For comparison, atmospheric pressure is approximately 10^5 Pa. To exert that pressure with an electromagnetic wave that reflects 100% requires an electromagnetic wave with an intensity of 1.5×10^{13} W/m², which is about 10 orders of magnitude more intense than bright sunlight!

Related End-of-Chapter Exercises: 18, 20, 35, 45 – 49.

It has been proposed that spacecraft use radiation pressure for propulsion. The idea is that the craft would unfurl a low-mass large-area reflective sail, and sunlight reflecting from the sail would provide a force to accelerate the spacecraft. Such a spacecraft is known as a **solar sailboat**. Radiation pressure associated with sunlight striking solar panels is exploited on some satellites to make minor adjustments in their motions without needing to use the on-board power source.

EXPLORATION 22.3 – Designing a solar sailboat

Let’s design a solar sailboat that we can use to explore the solar system, making use of the following data. Mass of the Sun: $M = 2 \times 10^{30}$ kg; mass of the solar sailboat: $m = 1000$ kg; power emitted by the Sun in the form of electromagnetic waves: $power = 4 \times 10^{26}$ W.

Step 1 – Find an expression for the gravitational force exerted on the satellite by the Sun, if the satellite is a distance r from the Sun. Applying Newton’s Law of universal gravitation, which we covered in Chapter 8, we find

$$F_g = \frac{GmM}{r^2}, \text{ where the constant } G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2.$$

Step 2 – Find an expression for the intensity of sunlight reaching the spacecraft. For a source like the Sun, which emits waves uniformly in all directions, the intensity at a particular distance is the radiated power divided by the surface area of a sphere with a radius equal to that distance. So,

$$I = \frac{\text{power}}{4\pi r^2}.$$

Step 3 – Assuming the sail deployed by the spacecraft is perfectly reflective, find an expression for the force exerted on the sail by the reflecting sunlight. Assume also that the sails are oriented to reflect the sunlight straight back toward the Sun. The force associated with the radiation pressure is $F_{rad} = P_{rad} A$, where P_{rad} is the radiation pressure and A is the sail area.

Using Equation 22.4, along with the result from Step 2, to determine the radiation pressure, we get

$$F_{rad} = \frac{2I}{c} A = \frac{2 \times \text{power}}{c \times 4\pi r^2} A = \frac{A \times \text{power}}{2\pi r^2 c}.$$

Step 4 – Determine the sail area required to balance the gravitational force exerted on the spacecraft by the Sun. Setting the gravitational force equal to the force associated with the radiation pressure gives:

$$\frac{GmM}{r^2} = \frac{A \times \text{power}}{2\pi r^2 c} \Rightarrow A = \frac{2\pi c GmM}{\text{power}}.$$

Interestingly, the area required to balance the forces does not depend on the distance the spacecraft is from the Sun, because both forces are inversely proportional to r^2 . Plugging in the values for the various constants, and the values stated above, in this situation the sail area works out to 630000 m^2 , which, if the sail was square, would require a sail almost $800 \text{ m} \times 800 \text{ m}$.

Key ideas for solar sailboats: Radiation pressure from sunlight reflecting from a very light metal sail can be a propulsion mechanism for a spacecraft. Solar sailboats need no fuel, and thus can be much lighter than a conventional spacecraft. **Related End-of-Chapter Exercises: 21, 22, 44.**

Essential Question 22.3: Return to Exploration 22.3. Using a sail area 10% larger than that calculated in Step 4, determine the acceleration of the spacecraft if it is the same distance from the Sun that the Earth is ($r = 1.5 \times 10^{11}$ m). Use this acceleration to approximate the spacecraft’s speed one week after it starts from rest.