

Answer to Essential Question 22.3: With a sail area 10% larger than that needed to balance the forces, there is a net force on the spacecraft of 10% of F_{rad} from step 3. The acceleration, by Newton's second law, is thus

$$a = \frac{0.1 \times F_{rad}}{m} = \frac{0.1 \times A \times power}{c \times 2\pi r^2 m} = 6 \times 10^{-4} \text{ m/s}^2.$$

Using $v = v_i + at$, with $t = 604800$ s in one week, gives a speed of $v = 360$ m/s. Even though the acceleration is very small, after many weeks the spacecraft builds up a large speed. The acceleration decreases as the distance from the Sun increases, but the spacecraft continually picks up speed as it moves away from the Sun.

22-4 The Doppler Effect for EM Waves

In Chapter 21, we spent considerable effort in coming to understand the Doppler effect for sound. We looked at how, for instance, it is not simply a relative-velocity phenomenon. The sound waves travel through a medium, so what matters is how the source of the waves as well as the observer of the waves moves with respect to the medium.

Because electromagnetic waves do not need a medium, the Doppler effect for EM waves is simply a relative-velocity phenomenon. The shift in frequency observed by an observer depends only on the relative velocity, \bar{v} , between the source and the observer. If the source emits EM waves that have a frequency f , the observed frequency f' is given by

$$f' = f \left(1 \pm \frac{v}{c} \right), \quad (\text{Equation 22.6: The Doppler effect for electromagnetic waves})$$

where v is the magnitude of the relative velocity between the source and the observer. As with the Doppler effect for sound, we use the top (+) sign when the source and observer are moving toward one another, and the bottom (−) sign when the source and observer are moving farther apart.

Applications of the Doppler effect for EM waves

Figure 22.4 illustrates a common application in which the Doppler effect is exploited by astrophysicists to determine how fast, and in what direction, distant stars or galaxies are moving with respect to us here on Earth. At the bottom is the spectrum received on Earth from the Sun. The dark lines at specific wavelengths correspond to light that is absorbed by hydrogen atoms in the Sun. These same lines are seen in the spectrum received from a distant source, at the top, except all the lines are Doppler-shifted toward the red end (the right end) of the spectrum, indicating that the source is moving away from the Earth. It was data such as this that was used by Edwin Hubble (1889 – 1953) to show that the universe is expanding.

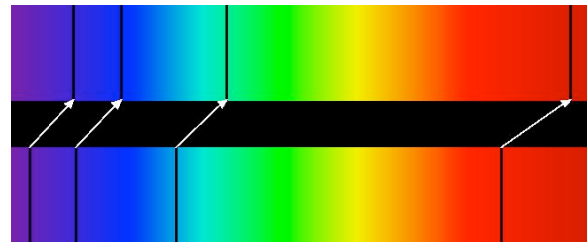


Figure 22.4: The spectrum on the bottom represents light coming to us from the Sun, which is essentially at rest with respect to the Earth. The four dark lines in the spectrum are caused by the absorption of light of particular wavelengths by hydrogen atoms in the Sun. The spectrum at the top represents light coming to us from a distant star, which is moving away from the Earth at 5% of the speed of light. The characteristic hydrogen absorption lines in the top spectrum have been shifted to the right, toward the red end of the spectrum. This is known as redshift.

A second application of the Doppler effect is Doppler radar, which is used in weather forecasting, as it can pick up rotating storm systems. Doppler radar has many sports applications, too, such as detecting the speed of a serve in tennis, or of a pitch in baseball. Doppler radar is also used by police to catch speeding motorists. Let's explore this last application in more detail.

EXPLORATION 22.4 – To catch a speeder

A police officer in a stationary police car aims a radar gun at a truck traveling directly toward the police car. The frequency of the radar gun is 10.525 GHz (10.525×10^9 Hz), and the frequency of the waves reflecting from the truck and returning to the radar gun is shifted from the emitted by frequency by 1600 Hz. If the speed limit on the road is 60 km/h, should the officer pull the truck over to give the driver a ticket? Let's work through the problem to decide.

Step 1 – Is the frequency of the waves coming back to the radar gun higher or lower than the frequency of the emitted waves? The relative velocity of the two vehicles brings them closer, which effectively lowers the wavelength of the waves, corresponding to a higher frequency.

Step 2 – If the truck is traveling at a speed v , write an expression for f' , the frequency of the waves received by the truck. To do this, we can simply apply Equation 22.6.

$$f' = f \left(1 + \frac{v}{c} \right), \text{ where } f \text{ is the emitted frequency. We use the plus sign in the equation}$$

because the truck is moving toward the police car.

Step 3 – Write an expression for f'' , the frequency of the waves that are picked up by the radar gun after reflecting from the truck. Your expression should be in terms of f , rather than f' .

We apply Equation 22.6 again, and use the result from step 2.

$$f'' = f' \left(1 + \frac{v}{c} \right) = f \left(1 + \frac{v}{c} \right)^2, \text{ where } f \text{ is the emitted frequency. Again, we use the plus}$$

sign in the equation because the truck is moving toward the police car.

Step 4 – Solve for the speed of the truck, and decide whether the truck driver should get a speeding ticket.

$$\text{Expanding the bracket in the expression from step 3 gives } f'' = f \left(1 + \frac{2v}{c} + \frac{v^2}{c^2} \right) \approx f \left(1 + \frac{2v}{c} \right).$$

The speed of light is orders of magnitude larger than the speed of the truck, so v^2/c^2 will be so much smaller than v/c that the v^2/c^2 term can be neglected. Writing the expression in terms of the known frequency shift, 1600 Hz, allows us to solve for the speed of the truck.

$$1600 \text{ Hz} = f'' - f = \frac{2fv}{c} \Rightarrow v = \frac{c \times 1600 \text{ Hz}}{2f} = \frac{(3 \times 10^8 \text{ m/s}) \times 1600 \text{ Hz}}{2 \times (10.525 \times 10^9 \text{ Hz})} = 22.8 \text{ m/s}.$$

Converting the speed to km/h gives a speed of $22.8 \text{ m/s} \times 0.001 \text{ km/m} \times 3600 \text{ s/h} = 82 \text{ km/h}$. This is well over the speed limit, so the driver should certainly get a speeding ticket.

Key idea for Doppler radar: In typical Doppler radar applications, the waves are emitted and detected at the same place, and thus the Doppler effect is applied twice to calculate the frequency shift between the emitted and detected waves. **Related End-of-Chapter Exercises: 3, 4, 50 – 53.**

Essential Question 22.4: Return to Exploration 22.4. If the truck is traveling away from the police car at 82 km/h instead of traveling toward it, would the frequency shift of the waves received by the radar gun still be 1600 Hz?