

Answer to Essential Question 27.3: (a) Blue light has a higher frequency than red light, and thus the photons in the blue light have a higher energy than the photons in the red light. We know that the photons in the red light have an energy larger than the metal's work function, because electrons are emitted, so the photons in the blue light have more than enough energy to cause electrons to be emitted, too. (b) We actually get fewer electrons emitted with the blue light. The energy per photon is larger for the blue light, so to achieve the same intensity there are fewer photons per second per unit area incident on the plate with the blue light. Fewer photons produce fewer electrons, although the electrons are emitted with, on average, more kinetic energy.

27-4 Photons Carry Momentum

In Chapter 6, we defined an object's momentum as the product of the object's mass and its velocity. Photons have no mass, so we might expect them to also have no momentum. However, we have already discussed the fact that light does carry momentum – recall the discussion of radiation pressure and solar sailboats in Chapter 22. Thus, photons do have momentum, but we need an equation that gives momentum for massless particles, keeping in mind that the units must be the familiar units of momentum that we have worked with throughout the book.

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}. \quad (\text{Equation 27.6: Momentum of a photon})$$

One of the key pieces of evidence supporting the photon model of light is an experiment involving light interacting with matter. When light of a particular frequency is incident on matter, the light can change both direction and frequency. The shift in frequency cannot be explained in terms of the wave model of light, but the particle (photon) model provides quite a straightforward explanation. The phenomenon is known as the Compton effect after its discoverer, the American physicist A. H. Compton (1892 – 1962), for which he won the 1927 Nobel Prize in Physics.

The explanation of the Compton effect is very similar to that of the photoelectric effect. In both cases, a single photon interacts with a single electron. With the Compton effect, the analysis is similar to our analysis of a two-dimensional collision in Section 7.6, except that, in this situation, one of the objects (the electron) is initially at rest. As always in a collision situation, momentum is conserved. In a collision like this, involving a photon and an electron, there is nothing to transfer energy out of the system, so energy is also conserved. A diagram of the collision is shown in Figure 27.6.

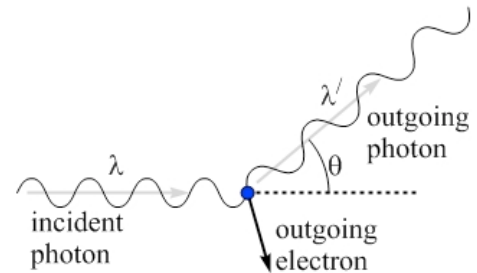


Figure 27.6: In the Compton effect, an incident photon of wavelength λ collides with an electron (the gray sphere). After the collision, the wavelength of the photon is λ' .

The photon is incident with a wavelength λ , and the photon transfers some of its energy to the electron. Thus, after the collision, the photon has lower energy, which means its frequency is lower while its wavelength λ' is higher. The photon's direction changes by an angle θ . Applying energy conservation to this situation gives one equation, while applying momentum conservation in two dimensions gives us two more equations. Combining these equations is somewhat involved, but it leads to a simple relationship relating the change in wavelength experienced by the photon to the angle through which the photon has been scattered.

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta), \quad (\text{Equation 27.7: The Compton Effect})$$

where m_e is the mass of the electron, and the quantity $h/(m_e c) = 2.43 \times 10^{-12} \text{ m}$ is known as the Compton wavelength. Because $\cos\theta$ varies from +1 to -1, the quantity $(1 - \cos\theta)$ varies from 0 to 2. Thus, the shift in the photon's wavelength from the Compton effect varies from 0 (for a scattering angle of 0, which is essentially no collision) to two Compton wavelengths, for a scattering angle of 180° .

EXAMPLE 27.4 – Working with the Compton effect

A photon with a wavelength of $4.80 \times 10^{-11} \text{ m}$ collides with an electron that is initially stationary. As shown in Figure 27.7, the photon emerges traveling in a direction that is at 120° to the direction of the incident photon.

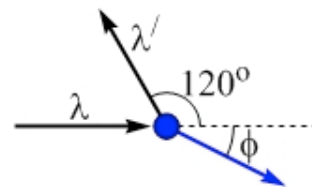


Figure 27.7: The geometry of the Compton effect situation described in Example 27.4.

- What is the wavelength of the photon after the collision with the electron?
- What is the magnitude of the momentum of the incident photon? What is the magnitude of the momentum of the outgoing photon?
- What is the magnitude of the electron's momentum after the collision?

SOLUTION

(a) To find the wavelength of the outgoing photon, we apply Equation 27.7. This gives:

$$\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos\theta) = (4.80 \times 10^{-11} \text{ m}) + (2.43 \times 10^{-12} \text{ m}) \times (1 + 0.5) = 5.16 \times 10^{-11} \text{ m}.$$

(b) To find a photon's momentum, we can apply Equation 27.6, $p = h/\lambda$.

$$\text{For the incident photon, } p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J s}}{4.80 \times 10^{-11} \text{ m}} = 1.380 \times 10^{-23} \text{ kg m/s}.$$

$$\text{For the outgoing photon, } p' = \frac{h}{\lambda'} = \frac{6.626 \times 10^{-34} \text{ J s}}{5.16 \times 10^{-11} \text{ m}} = 1.284 \times 10^{-23} \text{ kg m/s}$$

(c) Momentum is conserved in the collision, so we can apply momentum conservation to find the x and y components of the electron's momentum.

$$x\text{-direction: } p = p'_x + p_{e,x} \quad \text{so}$$

$$p_{e,x} = p - p'_x = (1.380 \times 10^{-23} \text{ kg m/s}) - (1.284 \times 10^{-23} \text{ kg m/s}) \cos(120^\circ) = 2.022 \times 10^{-23} \text{ kg m/s}.$$

$$y\text{-direction: } 0 = p'_y - p_{e,y} \quad \text{so}$$

$$p_{e,y} = p'_y = (1.284 \times 10^{-23} \text{ kg m/s}) \sin(120^\circ) = 1.112 \times 10^{-23} \text{ kg m/s}.$$

Applying the Pythagorean theorem, we can find the magnitude of the electron's momentum.

$$p_e = \sqrt{(p_{e,x})^2 + (p_{e,y})^2} = \sqrt{(2.022 \times 10^{-23} \text{ kg m/s})^2 + (1.112 \times 10^{-23} \text{ kg m/s})^2} = 2.308 \times 10^{-23} \text{ kg m/s}$$

Related End-of-Chapter Exercises: 6, 7, 17 – 20, 49 – 53.

Essential Question 27.4: Return to Example 27.4. (a) What is the electron's speed after the collision? (b) If the direction of the electron's velocity after the collision makes an angle ϕ with the direction of the incident photon, what is ϕ ?