## Interference and Diffraction of Light

"No one has ever been able to define the difference between interference and diffraction satisfactorily. It is just a question of usage, and there is no specific, important physical difference between them. The best we can do, roughly speaking, is to say that when there are only a few sources, say two, interfering, then the result is usually called interference, but if there is a large number of them, it seems that the word diffraction is more often used." - Richard Feynman's Lectures on Physics, Vol. 1

## PURPOSE

- To observe the behavior of light passing through various configurations of slits.
- To investigate how the width of a slit and the wavelength of the light passing through it determine the diffraction of light.
- To determine the wavelength of laser light from the diffraction pattern produced when it passes through a single slit.
- To determine the wavelength of laser light from the interference pattern produced when it passes through a pair of slits.
- To measure the width of a narrow slit from the diffraction pattern produced when laser light passes through it.
- To investigate the role played by single slit diffraction in the variation in intensity of a double slit interference pattern.


## EQUIPMENT

Virtual Diffraction simulation
Pencil

## EXPLORE THE APPARATUS

Open the Virtual Diffraction simulation on the website.


Figure 1: Interference and Diffraction Apparatus

The Virtual Interference and Diffraction Apparatus is a very simple device with just a few controls. Monochromatic light from a fictitious multi-color laser passes through one or more vertical slits in an opaque slide. The light arrives at a Viewing Screen 1.00 m away from the slide where it produces an interference pattern. The laser is its own switch. Click it to turn it on.

We'll refer to the table with the Laser, Slits, and Viewing Screen as "the apparatus." Below the apparatus you'll find controls and display screens. In Figure 1, notice the Viewing Screen, Intensity Graph, and Fringe Display.

Be sure that your Laser is turned on and click the Scan Viewing Screen button. A Photodetector moves across the interference pattern on the Viewing Screen, measuring the intensity of the light as a function of horizontal position. The intensity vs. position data is displayed in two ways. The intensity is indicated by

- the height of the Intensity Graph, which is a plot of the intensity of the light vs. position on the Viewing Screen.
- the brightness of the bars of light, called fringes, on the small Viewing Screen on the apparatus, and in the enlarged replica of this interference pattern in the Fringe Display below the Intensity Graph.

The Contrast Color slider beneath the Fringe Display may be used to adjust the background to make the fringes more visible.

Four parameters can be adjusted with sliders or numeric steppers.

- The wavelength can be adjusted throughout the typical visible (human) range of 400 nm to 700 nm .
- The width of the slits can be adjusted from $20 \mu \mathrm{~m}$ to $160 \mu \mathrm{~m}$.
- The slit spacing, the distance between the centers of adjacent slits, can be adjusted from $250 \mu \mathrm{~m}$ to $500 \mu \mathrm{~m}$.
- The number of slits can be varied from 1 to 5 .

The first three parameters also have several preset unknown values. In your WebAssign assignment, you'll be given a random "unknown number" - the number which you'll use to select your unknowns. So if you are told to use your assigned unknown wavelength and your unknown number is 2 , you'll just select " 2 " with the wavelength stepper.

Two rulers are provided to measure the

- slit width and slit spacing. (Zooming in is recommended here.)
- position of a minimum or maximum point relative to the central maximum on the Intensity Graph.

Each ruler is calibrated in units appropriate to the size of the object being measured.
Any changes you make to the apparatus are immediately reflected on the various screens and displays. The only exception is the Intensity Graph, which doesn't change until you scan the Viewing Screen with the photodetector.

The first objective of this lab is for you to observe what happens when light passes through various configurations of slits. This is something that you almost certainly have not done in the classroom because of the nature and small size of the effects we'll be working with. But with this virtual apparatus, you'll be able to adjust all the variables that appear in the equations to see their actual roles in the phenomena of interference and diffraction.
"Perhaps some gentle persuasion with some sharply pointed questions will encourage you to do some preliminary investigation. We will turn on the laser now. And by we I mean you." - movie villain of your choice.

1 Drag the wavelength slider all the way to the left. We'll call this color violet. What's its wavelength?

2 Drag the wavelength slider all the way to the right. We'll call this color red. What's its wavelength?


Figure 2: (a) From Newton's Optiks; (b) From Pink Floyd's Optiks

For humans there's a range of wavelengths that could be called red, a range called orange, etc. We, like Newton, find that as we move through the colors from red to violet, we come upon about six or seven with widely agreed upon names and hues. His representation of the colors as a continuum doesn't correspond to any real behavior. And indigo might be a stretch for most of us.

3 It's useful in our work to know about the order of these colors in the spectrum and how they relate to wavelengths. So, let's get to know them. Select a wavelength in the range of each color in the sequence ROYGBIV. There are no exact answers. Just try again if your estimate is rejected and blame it on your monitor.
a Red
b Orange
c Yellow $\square$ (very narrow)
d Green $\qquad$
e Blue $\qquad$
f Indigo no way
g Violet $\qquad$ (very narrow)

With this apparatus, light from the laser passes through one or more slits and then hits a screen one meter away. In addition to the color of the light, you can adjust the width of the slits, the number of slits, and the spacing between the slits. Try each of these and notice how the enlarged view of the slits on the Fringe Display reflects your changes.

4 Set the laser color to a red, the number of slits to one, and the slit width to $40 \mu \mathrm{~m}$. Scan. (This means to click the Scan Viewing Screen button.) You should see a nice red "fringe" that takes up about half the width of the Fringe Display. Two other very dim fringes appear just at the edge of the Fringe Display. Adjust the background contrast as needed. Zoom in three times on the Viewing Screen on the apparatus. The Fringe Display just shows the central part of this full, but tiny, display. Zoom back out to $100 \%$.

5 Slowly adjust the wavelength from red to violet. Scan.
As the wavelength decreases, the width of the central, bright fringe $\qquad$ . (increases or decreases)

6 Reset the color to red. Scan. The slit width should be set to $40 \mu \mathrm{~m}$. Slowly increase the slit width to its maximum value. Scan. What three significant changes do you observe as you increase the slit width? One involves the Intensity Graph.

7 Reset the slit width to $40 \mu \mathrm{~m}$. Scan.
Change the number of slits to two. Scan. Describe the changes in the Fringe Display and the Intensity Graph. Also comment on what stays the same.

8 Feel free to change the number of slits up to 5 and Scan if you like spiky things.

## THEORY

## A. Diffraction and Interference

While you're waiting for your opponent to arrive at the tennis court, you can warm up by hitting the ball against a wall. You can count on the ball to behave in a predictable manner. If you have good aim, the ball is always going to bounce back. If you hit it past either side you know right where to go to pick it up. So unless you hit the ball over the wall the area behind the wall is a tennis ball shadow.

If you do hit the ball over the wall and some kind person on the other side throws it back over the wall to you, you don't have to climb the wall or walk to the edge of the wall to shout your thanks. Not only can you just shout toward the wall, you can shout in almost any direction and still be heard. Clearly sound doesn't behave like a tennis ball. There is no sound shadow behind the wall.

There are actually two phenomena involved in this "hearing around corners" phenomenon. When the sound arrives at an edge of the wall, it bends around the wall. This "bending" of waves when they reach an opening or an edge is called diffraction. So how does the sound get behind the wall when you don't even shout in the direction of the wall? It diffracts when it exits your mouth. Differently shaped speakers for different situations and cheerleading megaphones suggest that there are ways of modifying the amount and direction of diffraction.

There's another factor involved in talking to someone behind the wall. If you or your listener were to move around a bit you'd find that the sound heard would be clearer and louder at some points and more garbled at others. This is because all this sound wrapping around and over the wall is recombining at your listener's ears to reproduce the pattern of compression and decompression of the air that you originally produced.

Sound coming by different paths will be out of sync to different degrees, so what's heard is a mash up of different parts of "Thank you very much for returning the ball." Something like:
"Thank you very much for returning the ball." $\rightarrow$
"Thank you very much for returning the ball." $\rightarrow 8$
"HUH? All I got was a bunch of noise ending with bbball."
"Thank you very much for returning the ball." $\rightarrow$

The merging together of these sound waves to produce one sequence of compressions and rarefactions that you're trying to interpret is called wave interference. Once the sounds split into several paths the geometry has to be just right to get them back in sync. It's no wonder that marching bands are impossible to hear clearly. The sound is diffracting in all directions when it leaves their instruments, and then it reflects off of a sloping wall of people and concrete to get to your ears!

In summary, multiple waves arriving at a given point combine to produce a resultant wave, which is the sum of the amplitudes of the various waves. This is the process of interference.

That seems pretty tidy. We have diffraction and interference. But, well, diffraction involves interference. As Dr. Feynman pointed out in the opening quote, there is not a clear distinction between interference and diffraction. Physicists have gotten accustomed to being very loose with these terms, and it seems to be beyond fixing.

We'll first look at cases of pure interference of waves and then explore interference resulting from diffraction. (See, there's a good example of the loose use of these terms. It's a typical and acceptable example of their use but confusing if you're trying to understand the difference in their definitions. See Dr. Feynman's opinion on that.) We'll also find that several factors can be important. An equally loud bee buzzing in front of the wall will not be as easy to hear behind the wall. Wavelength matters. You've already done the research to understand that one in the introduction. The high frequency (shorter wavelength) blue diffracts - bends around corners-less than the low frequency (longer wavelength) red light. Similarly, the high frequency bee would be harder to hear behind the wall.

## B. Interference

Let's look at the interference of a pair of waves on the surface of a tank of water. Figure 3 is a snapshot of a red marble jiggling up and down when partially submerged in a tank of water. A continuous circular surface wave is produced. The small bit of Styrofoam jiggles up and down with a slight delay due to the travel time of the wave.


Figure 3: Water Waves

In Figure (4a), the small red (top) and blue (bottom) dots at the left represent a pair of marbles jiggling similarly (overhead view). They are moving in phase. That is, they are at all times at the same height and moving in the same direction. Each time they strike the water a new trough is produced. Half a period, $T$, later the water will overshoot its equilibrium level and produce a crest. The circles represent wave crests. Troughs would be found halfway between adjacent crests. In Figure (4b), $\frac{r}{2}$ later, each wave has traveled a distance equal to half a wavelength. So each crest has reached a point previously occupied by a trough.

Each of the four circular viewports in each figure shows an edge-on snapshot of the water wave at the point it connects to. The top and bottom viewports, (i) and (iv), show regions where only one of the two waves has arrived. As the actual water waves move over time, the waves in these viewports would move left to right across the crosshairs. Notice how from Figure (4a) to Figure (4b), each of these snapshots show the wave moving a distance equal to half a wavelength. A crest is shown at the center of the crosshairs in Figure (4a) and the following trough is centered in Figure (4b).

The middle pairs of viewports, (ii) and (iii), show the waves at two points where they have already overlapped. In this region, at a given instant, the wave height at any point is the sum of the heights of the individual waves passing through that point, at that instant. The individual waves have lost their individual identities. Viewport (ii) in Figure (4a) shows that at the point it refers to the water is a double-depth trough. In Figure (4b), it indicates a double-height crest. These waves will always be in phase at this point, which means they will always be at the same point in their cycle of up and down motion. At this instant, their resultant wave has twice their individual amplitudes. This addition of amplitudes where both waves are displaced in the same up or down direction is called constructive interference. This effect is at its maximum at this instant since both waves are at maximum points in their cycle. The waves were created in sync and each has traveled 4.5 wavelengths to arrive at the fixed point where the snapshot is being taken.


Figure 4: Interference of Two Overlapping Circular Waves: (a) In Phase-Trough (b) In Phase - Crest (c) Nodal and Antinodal Lines

In viewport (iii), we see the opposite effect. Again the waves start out in phase, but in Figure (4a), the red (top source) wave has traveled 5 wavelengths but the blue (bottom source) wave has traveled only 3.5 wavelengths. And in Figure (4b), each has traveled another half wavelength but at the fixed point we're observing they are still exactly out of phase. Thus the waves are exactly out of phase and will remain that way over time. This out-of-phase addition is called destructive interference.

Further inspection shows that all points along the dashed centerline will experience double-height crests and troughs. In Figure (4c), the top and bottom dashed lines have been drawn passing through intersections of waves where they are in phase. All points along these lines will also experience alternating double-height crests and double-depth troughs.

The other pair of dashed lines connect points where troughs of one wave intersect with crests of the other wave. Again, the waves "cancel" leaving approximately still water. These lines of constructive and destructive interference alternate. You'll see later that the spacing, and hence the number of lines that appear, depends on the spacing of the wave sources and the wavelength.

Figure 5 shows an actual photograph of this wave pattern. If you were to sit in an inner tube anywhere along one of five thin antinodal lines you'd be subjected to large up and down motions as large crests and troughs arrive in succession.

Sitting in your inner tube anywhere along one of the four thick nodal lines is a fairly calm experience. The waves approximately cancel out to the level of still water. Hopefully you can clearly see how this behavior parallels the behavior of the light hitting our Viewing Screen. The bright points are equivalent to the ends of the antinodal lines in Figure 5.

What we've seen so far with water waves is also found with other types of waves such as sound and light waves. We'll consider the particular case of light from here on. Constructive interference will produce bright fringes on a screen while destructive interference will produce dark fringes.


Figure 5: Nodal and Antinodal Lines in Water; Bright and Dark Fringes on a Screen

You've already observed this pattern of bright and dark fringes in your initial exploration with the apparatus. The same geometry that produces these effects provides a framework for a mathematical description of this behavior.

## Equations for the location of nodal (dark) and antinodal (bright) fringes

We've found that points of constructive or destructive interference are due to the difference in distance, the path difference, $\Delta r$, between the sources and positions on a screen. (This assumes that the sources are in phase, which they'll always be with this apparatus.) Specifically, if light travels $\lambda, 2 \lambda$, $3 \lambda$, etc., farther from one source than from another, then constructive interference will occur. Similarly, destructive interference will occur if the path difference is $0.5 \lambda, 1.5 \lambda, 2.5 \lambda$, etc.

In Figure 6, light sources $S_{1}$ and $S_{2}$ produce identical waves in phase. Since $r_{1}$ and $r_{2}$ are equal distances, all waves will arrive at point P in phase. Crests will arrive together, as will troughs. So point P will be a bright fringe. Due to the symmetry of the arrangement, we would call it the central bright fringe. This corresponds to point (ii) in Figures (4a-c).


Figure 6: Central Bright Fringe

If we examined other points along the screen, we'd find a first pair of bright points equal distances above and below the original point P. Figure 7 shows the two new paths taken to reach one such point, P . Both paths are longer than before, but they are no longer equal. The path from $\mathrm{S}_{2}$ to P must be exactly $\lambda$ further than the path from $S_{1}$ to $P$.

Between the central bright fringe at O and the next bright fringe at P will be a dark fringe at a point where $r_{2}-r_{1}=\lambda / 2$.


Figure 7: First Order Bright Fringe

Imagine a screen much farther to the right. For $\theta$ to remain the same, $y$ would have to be proportionally larger. And at a great enough distance, $L$, the lines labeled $r_{1}$ and $r_{2}$ would be approximately parallel as shown in Figure 8. (This figure is enlarged. There is no change in the value of $d$.) In this case, where $L \gg d$, the mathematics is much simpler than what we see in Figure 7. We'll just deal with this case, which was developed by Joseph Fraunhofer. (See the "Cosmos: A Spacetime Odyssey" "Hiding in the Light" episode to see how we almost lost young Joseph. He was fortunate to have a house fall on top of him.)


Figure 8: First Order Bright Fringe, where $L \gg d$

For this case, the angle $\theta$ between $r_{1}$ or $r_{2}$ and $L$ will be the same as the angle $\theta$ in the right triangle shown in Figure 8.

For the first bright fringe, we found that the path difference $\Delta r=\lambda$. For the next bright fringe, at a larger angle, $\theta, \Delta r=2 \lambda$, and so on. Similarly, for dark fringes we'd have $\Delta r=\frac{1}{2} \lambda, \frac{3}{2} \lambda$, and so on.

For two sources, created in phase, at a distance $L$ from a screen:

$$
\begin{align*}
& \Delta r=m \lambda \quad \text { where } m=0, \pm 1, \pm 2, \ldots \text { for bright fringes (double slit) }  \tag{1}\\
& \Delta r=\left(m+\frac{1}{2}\right) \lambda \quad \text { where } m=0, \pm 1, \pm 2, \ldots \text { for dark fringes. (double slit) } \tag{2}
\end{align*}
$$

We use $\pm m$ values to distinguish fringes on either side of the central maximum.
Using right triangle $\Delta S_{1} S_{2} B$, in Figure 8, and noting that $d$ is the distance between $S_{1}$ and $S_{2}$, we can say

$$
\begin{equation*}
\sin \theta=\frac{\Delta r}{d} \tag{3}
\end{equation*}
$$

Substituting each expression for $\Delta r$ into equation 3 , we can solve to find

$$
\begin{align*}
& m \lambda=d \sin \theta \quad \text { where } m=0, \pm 1, \pm 2, \ldots \text { for bright fringes } \quad \text { (double slit) }  \tag{4}\\
& \left(m+\frac{1}{2}\right) \lambda=d \sin \theta \quad \text { where } m=0, \pm 1, \pm 2, \ldots \text { for dark fringes. (double slit) (5) } \tag{5}
\end{align*}
$$

One practical problem with equations 4 and 5 is that it's difficult to precisely measure these very small angles. The geometry of Figure 7 suggests an alternative. Again assuming that $L \gg d$, we can use the large triangle $\Delta \mathrm{AOP}$ to produce the equation

$$
\begin{equation*}
y=L \tan \theta \tag{6}
\end{equation*}
$$

In addition, we know that for small angles $\tan \theta \approx \sin \theta$. So we can solve equation 6 for $\tan \theta$ and substitute the result for the $\sin \theta$ term in equation 4 to get

$$
\begin{equation*}
y_{m}=\frac{m L \lambda}{d} \quad \text { where } m=0, \pm 1, \pm 2, \ldots \text { for bright fringes. (double slit) } \tag{7}
\end{equation*}
$$

## PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

## I. Double Slit Interference-Measuring an Unknown Wavelength

So how might we arrange for two sources of light to come together in phase, or out of phase, on a screen? You've probably read about a modified version of Thomas Young's famous experiment in 1801 that involved light passing through a single slit, then through a pair of closely spaced slits, and then being viewed on a screen in a darkened room. Using light from this single source to illuminate both slits insured that the light coming from the two slits was in phase.

A more convenient method unavailable to Thomas Young by about 150 years is to use laser light. Due to its method of production, laser light is always in phase. In Figure 9, laser light diffracts separately through each slit, providing us two in-phase sources of light to illuminate our screen.

We'll now set up our virtual apparatus in this two-slit mode, and use equations $1-7$ to determine the wavelength of our monochromatic laser light.


Figure 9: Monochromatic Laser Light Through Two Slits

## Trial 1-Measuring Wavelength using Double Slit Interference

Did you ever wonder how we're able to measure the wavelength of light? Or how we know that light is a wave to begin with? Experiments like Young's were the answer to both questions. The interference patterns on Young's screen indicated that light might be a wave; and the experiment we're about to do provided the wavelength. We'll first use equation 7 to calculate the wavelength of a shade of light in the green-yellow region and then repeat the process with your unknown wavelength. In each step below, you'll want to record your settings, measurements, and calculated values in Table 1 in your WebAssign assignment.

Be careful with units. The table conveniently uses four different units of length to correspond to the apparatus display units and ruler units. You'll inconveniently need to convert each of these units to meters when you do calculations. It could be worse. You could have a house fall on you.

1 Set the number of slits to two, and the slit spacing, $d$, to $250 \mu \mathrm{~m}$. ( $\mu$, micro $=10^{-6}$, n , nano $\left.=10^{-9}\right)$.

2 Set the slit width, $w$, to $40 \mu \mathrm{~m}$.
3 Turn on the laser. Adjust the wavelength slider to 540 nm . Notice how the apparatus-laser, slit, and screen-matches our assumptions about $d, L, \theta$, etc. Rays traveling from the two slits to a given point on the screen are very nearly parallel. In the following images, a red-orange wavelength will be used as an example. Your green-yellow wave $y$-values will be smaller than those shown. (The green doesn't print out very well on paper.)

4 Scan to record and display the intensity data. You'll see a set of yellow-green fringes of varying brightness. The bright fringes are maxima, and the minima are in between. The specific location of a maximum is at its brightest point. As you can see, this is hard to determine from the thick fringes.
The Intensity Graph takes care of that problem. The scanning photodetector measures the intensity of the light reaching the Viewing Screen at each horizontal position on the screen. This intensity data is plotted so that the height of the curve corresponds to the brightness of the light measured by the photodetector at a given point. This curve more precisely shows the variation in brightness of the light.


Figure 10: (a) Maxima and Minima (b) Position, $y_{m}$, of a Maximum (c) $-y_{2}$ to $+y_{2}$

We'll use the distance from the center line out to a peak as a measure of the position of the maximum. (Use the checkbox to turn on the mm ruler.) In Figure (10b), that would be about 2.5 mm for the first maximum and about 5.1 to the second maximum. Be sure to note that the smallest increments on the ruler are 0.2 mm apart. The position of a maximum is represented by $y_{m}$, where $m$ is the order of the maximum. The maximum at 2.5 mm in Figure 10 is $y_{1}$ since it's the first one-to-one side of the central maximum. There's another first order maximum equidistant at $-y_{1}$ on the left.
To improve our accuracy we'll take advantage of the equal spacing of the maxima. Figure (10c) shows the $m=1$ and 2 peak on either side of the central maximum. The ruler shown is aligned to measure the distance between $-y_{2}$ and $+y_{2}$. By zooming in a couple of times, you should be able to measure this distance to 0.1 mm . Since this is the span of four maxima, half of that distance would be $y_{2}$. If that total distance is 10.2 mm , then $y_{2}$ would be 5.1 mm .

## NOTE:

Any time you make a change in the parameters-wavelength, slit width, etc.-you'll need to Scan to reset the Intensity Graph.

5 Measure the distance between $-y_{3}$ on the left and $+y_{3}$ on the right.
6 Calculate $y_{3}$ by dividing the $-y_{3}$ to $+y_{3}$ distance by 2 .

7 Calculate $\theta_{3}$, the angle for the third maximum, using equation 6. For your results to be valid, $\theta$ must be less than a few degrees. You should get a very tiny angle. (Be careful to use meters for all distances in your calculations!)

8 Use equation 7 to determine the experimental value of the wavelength of your laser light.
9 Compare your experimental value to the theoretical value by calculating the percentage error.
10 Show your calculations of $\theta_{3}, \lambda$, and your percentage error.
Trial 2-Determining your Unknown Wavelength using Double Slit Interference
Repeat the data collection and calculations above to determine your assigned unknown wavelength.
11 Adjust the laser to your assigned wavelength by setting the numeric stepper to your unknown number.

12 Collect all the data and calculate all the values you need to fill in the trial 2 row of Table 1.
13 Show your calculations of $\theta_{3}$ and your wavelength.
Trial 3-The Effect of Slit Spacing, $d$, on the Position of the Fringes
Let's have another look at the geometry that produced our interference patterns. In Figure 11, the small triangle has two vertices located at the centers of the slits. For the first maximum to be located at the point shown, the third vertex must be located one wavelength along ray $r_{2}$ from the center of the bottom slit. Note that the angle is $90^{\circ}$ at this vertex. Draw a tiny wavelength $(\sim)$ in the figure to show the path difference of $1 \lambda$.


Figure 11: The Geometry of Interference Patterns

14 In Figure 12, the first maximum you just produced is shown as $y$-original, $y_{0}$. But the slits have been adjusted to be twice as far apart. The first maximum will now be at a new point which you need to find using the diagram. What effect would doubling the slit spacing have on the location of this first maximum? Add the missing lines from Figure 11 to this figure taking into account the new positions of the centers of the slits. It takes a bit of trial and error. Fortunately you're using a pencil.


Figure 12: The Effect of Slit Spacing on the Location of Maxima

15 You should now be able to make an estimate of what would happen to your trial 2 data if you doubled the slit spacing.
a What was your value that you recorded for the distance " $-y_{3}$ to $+y_{3}$ " from trial 2?
b What do you predict will be the value for " $-y_{3}$ to $+y_{3}$ " if you double the slit spacing?
16 Now test your prediction with your assigned wavelength. Double the slit spacing from 250 to $500 \mu \mathrm{~m}$. Scan. Measure $y_{3}$ with the new spacing. Enter your data as trial 3 in Table 1.

Let's summarize some of the basic ideas that you've learned so far. For light passing through a pair of narrow slits, how is the spacing of the fringes affected by
a the spacing of the slits?
b the wavelength of the light?
c the distance from the slits to the screen (you weren't able to test this with the apparatus, but geometry helps)?

## II. Single Slit Diffraction

When we looked at the interference of a pair of circular water waves in Part B of the theory, we found alternating maxima and minima just as we've found with a pair of slits. But there's something else going on that we haven't addressed. The fringe brightness drops off with increasing $y$-values. But then it increases at larger $y$-values! There's nothing about our water wave model that would suggest this behavior. Back to the lab!


Figure 13: Brightness vs. $y$

1 Reset your apparatus to the arrangement we used in trial 1 in Part I. That is, two slits, $\lambda=540 \mathrm{~nm}$, slit width, $w=40 \mu \mathrm{~m}$, slit spacing, $d=250 \mu \mathrm{~m}$. Scan.

2 In Figure 14 draw a simple sketch of what you observe on the Intensity Graph and Fringe Display.


Figure 14: Double Slit Intensity and Fringe Pattern

3 Take a Screenshot of the bottom portion of the lab screen - the part below the apparatus. Upload it as "interference_doubleslits.png".

4 Decrease the number of slits to one. Scan. What a difference a slit makes! You commented on this in your initial exploration. There are some obvious as well as some subtle things that occur when you switch from two slits to one. The multiple individual fringes are replaced by a wide central fringe and two very dim fringes on either end. Also, the varying brightness (intensity) of the original, double slit fringes is reflected in the varying brightness of the three single slit fringes at corresponding positions. If you didn't notice that, switch back and forth between one and two slits. Be sure to Scan after each switch. Also, the shape of the intensity curve of the single slit fringe pattern is identical to the shape of what we call the envelope of the double slit intensity curve.
To be clear about what we're observing, the intensity of the double slit curve is very spiky. But if you draw a smooth curve connecting the maxima of all the spikes, you get what's called the envelope of the intensity graph. That envelope looks just like the intensity graph of a single slit. (We'll address the intensity scale differences later.)
Why is this happening? Perhaps the variation in brightness with position on the Fringe Display is some sort of individual slit behavior and the number of fringes is a multi-slit behavior. To
understand why this occurs we need to observe the behavior of light passing through just one slit.

5 In Figure 15 draw a simple sketch of what you observe on the Intensity Graph and Fringe Display for the one-slit case.


Figure 15: Single Slit Intensity and Fringe Pattern

6 Take a Screenshot of the bottom portion of the lab screen - the part below the apparatus. Upload it as "interference_singleslit.png".

## Slit Diffraction/Interference

In Figure 4, we saw the interference pattern produced by of a pair of point sources, which produced circular waves in all directions. We then created a similar effect with light by using a pair of slits. But we also found an unexpected feature - the variation in brightness-something that our water wave model would not have predicted. The difference is due to the finite (as opposed to extremely narrow) size of our slits. Our slits are not really point sources.

To understand why this is important, we first need to consider how waves actually travel through our slits. Christian Huygens developed a principle that explains how a wave front propagates, that is, how a wave front at one instant can produce a new wave front at a new position a short time later. He suggested that each point along a wave front acts as the source of tiny circular wavelets
that move forward at the wave's speed. The wave front a moment later is the tangent to all these individual wavelets.

In Figure (16a), we see a section of a long, straight wave traveling toward a narrow slit. The second wave from the left is shown producing three Huygens wavelets which result in a new straight wave. A wave produced three periods earlier is nearing the slit.

In Figure (16b), just a tiny part of the wave has passed through the slit, leaving just the circular wave from that point-like section of the original wave to continue on. This is the behavior of waves passing through a very narrow slit - the waves act almost as if they're all coming from the same point, much like our water waves! As mentioned before, this bending into what would be expected to be shadow is called diffraction. Figures (16a) and (16b) demonstrate a case of strongly diffracted waves.


Figure 16: Diffraction by a Narrow Slit

In Figure 17, a wave passes through a slit with a width of several wavelengths. The center part of the wave exits the slit without much change. But the parts of the wave near the edges of the slit diffract somewhat into the expected shadow region. But the intensity of the diffracted waves is much weaker in this case.


Figure 17: Diffraction by a Wider Slit

With our light waves we have something more extreme than what's illustrated in Figure 17. The slit width in Figure 17 would need to be maybe twenty times as wide to illustrate the relative
values of the light wavelengths and slit widths we've been working with. In other words, our slits are many wavelengths wide. And this results in much less diffraction.

7 Let's watch this happen with our apparatus, still with one slit. All of our work is being done with slit widths of at least $40 \mu \mathrm{~m}$. Use that width and $\lambda=540 \mathrm{~nm}$. Look at the Viewing Screen on the apparatus. You should see a very tiny bright fringe at the center of the screen. Zoom in several times. You should see the central bright fringe, then the faint $m= \pm 1$ fringes and then some still fainter $m= \pm 2$ fringes. The light passing through a $40 \mu \mathrm{~m}$ wide slit has expanded so that the central maximum is about 27 mm wide. The angle of this wedge of light is only about $1.6^{\circ}$. Now zoom back out.

8 Gradually slide the slit width slider to the left to $20 \mu \mathrm{~m}$ while watching the Fringe Display. Scan.

The central bright fringe fills the whole Intensity Graph. This is approaching, but still far away from what Figures (16a) and (16b) are illustrating-point source behavior. It's also closer to what the individual water waves did in Figures (4a) and (4b). Their amplitude seemed to be unrelated to $\theta$. You should be able to picture the result of further reduction in slit width.

9 There's another factor that influences the amount of diffraction. Return the slit width to $40 \mu \mathrm{~m}$. Drag the wavelength slider back and forth and notice the change in width of the diffraction pattern. Obviously the wavelength plays a key role in diffraction, too.

10 When light passes through a narrow slit, the amount of diffraction $\qquad$ with wavelength and $\qquad$ with the width of the slit. (increases or decreases) So diffraction is proportional to $\lambda / w$.

What's physically happening to cause the $\lambda / w$ effect and the related fall and rise in brightness with a single slit? We've already noticed that the brightness decreases and then increases with screen position, $y$. So this is not just the behavior of a single source fanning out uniformly. Look back at your drawing in Figure 14. We found very large increases and decreases in intensity with double slits. But without a second slit, how can we produce interference? The individual wavelets have to do it somehow with no outside help.

In Figure (18a), a segment of a long wave front, on the left, approaches a slit. Because it's part of a long wave front, it maintains its shape. Two periods later it's within the slit and only part of it can pass through. Note that the slit is hugely magnified so that we can see the wavelets. We'll use five wavelets to examine what will happen next. Specifically, we'll predict how much light arrives at various points on the screen.

Each wavelet will produce light in all forward directions perpendicular to the wavelet as indicated by the arrows in Figure (18a). But the distance traveled by each wavelet to a given point on a screen will be determined by the geometry. The differing distances traveled will determine whether or not the light arrives in phase, producing a bright spot, totally out of phase, producing a dark spot, or somewhere in between.


Figure 18: (a) Light Produced by Wavelets (b) Central Bright Fringe

Consider the light arriving at the center of the screen as shown in Figure (18b). Because of symmetry, light from the top wavelet ( $\# 1$ ) will arrive at the center of the screen in phase with light from the bottom wavelet ( $\# 5$ ), thus producing constructive interference. A similar pairing exists for $\# 2$ and $\# 4$, and every other wavelet pair. Thus, we expect a very bright central point.

How about a point just above the center of the screen? Our wavelet pairs will be somewhat out of phase due to the difference in distance traveled. And they'll be gradually more out of phase at increasing distance from the central point. Thus, the brightness drops off with increasing distance from the center, $y$.


Figure 19: First Dark Fringe

With single slit diffraction, we generally focus on the dark fringes rather than the bright ones. In Figure 19, a dark fringe is illustrated. Remember, the slit is actually very tiny relative to what's shown in the figure, and the screen is far away, so the rays are essentially parallel as shown. You can see this very dim fringe with your apparatus. There's one equidistant on either side of the central maximum. At these points there is almost no light. Figure 20 shows the geometry that produces this first dark fringe.


Figure 20: Production of First Dark Fringe

At some particular angle, $\theta$, wavelet 5 travels exactly $\frac{1}{2} \lambda$ farther to get to the screen than wavelet 3. As a result they are exactly out of phase, and they interfere destructively. Similarly wavelet 4 and wavelet 2 interfere destructively at this angle. And wavelet 3 interferes destructively with wavelet 1 .

So for every wavelet exiting the slit, there is another wavelet that will interfere destructively at this point. The result is the dark fringe on the screen.

In between the angles $0^{\circ}$ and this angle $\theta$, there is neither fully constructive interference nor fully destructive interference. This produces the gradual drop off in brightness to approximately zero. Then, as $\theta$ continues to increase, constructive and destructive interference alternate.

In short, it's possible to have interference with just one slit because light from one part of the slit can interfere with light from another part!

Since the angle $\theta$ is associated with the location of a dark fringe, it's a good measure of the amount of bending of the light, the diffraction. You found in your exploration that the amount of diffraction is proportional to $\lambda / w$. Use Figure 20 to see why this is true. You'll want to think of point 1 as a pivot point and consider what happens to $\theta$ as changes are made to $\lambda$, or $w$.

11 For the first dark fringe on a screen and slit width $w$, as the wavelength, $\lambda$, increases, the angle $\theta$ $\qquad$ .

12 For the first dark fringe on a screen and wavelength $\lambda$, as the slit width, $w$, increases, the angle $\theta$ $\qquad$ _ .

## So now we know why diffraction is proportional to $\lambda / w$.

The geometry that produces the dark fringes is fairly straightforward. From the largest triangle shown in Figure 20, we can see that for the production of a dark fringe, the path difference between the top and bottom points in the slit must be one wavelength. For the angle $\theta$ in the triangle formed between points 1 and 5 , the side opposite $\theta$ is $\lambda$ and the hypotenuse is $w$. Thus $\sin \theta=\lambda / w$ and

$$
\begin{equation*}
w \sin \theta=\lambda \quad \text { for the first dark fringe produced by a single slit. } \tag{8}
\end{equation*}
$$

Equation 8 was produced by dividing the slit into two sections so that each wavelet in the top half had a matching wavelet in the bottom half that would destructively interfere with it. If we divide the slit into four sections, the top two will destructively interfere and the bottom two will destructively interfere. This will work with any even number of divisions.

For example, for the second dark fringe, the wave from point 5 must travel $\lambda$ farther than the wave from point 3 and point 3 must travel $\lambda$ farther than point 1 . Thus, equation 8 becomes $w \sin \theta=2 \lambda$. So $\sin \theta$ will be twice what it was for the first dark fringe. In general, for all dark fringes produced by a single slit,

$$
\begin{equation*}
m \lambda=w \sin \theta \quad \text { where } m=1,2,3, \ldots \quad \text { (single slit) } \tag{9}
\end{equation*}
$$

Again, for small angles we can also use the small angle approximation, $\sin \theta=\tan \theta=y_{n} / L$ to produce

$$
\begin{equation*}
y_{m}=\frac{m L \lambda}{w} \quad \text { where } m=1,2,3, \ldots \text { for dark fringes. (single slit) } \tag{10}
\end{equation*}
$$

## EXAMPLE:

Set the wavelength to 550 nm and the slit width to $100 \mu \mathrm{~m}$. Scan. We'll solve for the slit width, $w$, for our example. You should see one central peak and three full shorter peaks on either side. But we're interested in the dark fringes, not the bright ones. (You can see how they're easier to measure.) We have four dark fringes on each side. So $m=4$ and the distance between these dark fringes, referred to in the table as $y_{-4}$ to $y_{+4}$, is about 43.9 mm . So $y_{4}=(0.0439 \mathrm{~m}) / 2$. Solving equation 10 for $w$ and entering our data gives a value for $w$ of about

$$
w=\frac{m L \lambda}{y_{m}}=\frac{4 \times 1 \mathrm{~m} \times 550 \mathrm{~nm}}{(0.0439 \mathrm{~m}) / 2}=100 \mu \mathrm{~m}
$$

which is the $100 \mu \mathrm{~m}$ we expected.
This would be a handy technique for anyone needing to measure the width of small openings. We live in a nano-world.

## Trial 1-Measuring Wavelength Using Single Slit Diffraction

13 You've found your assigned unknown wavelength previously using the double slit equations. Let's find that value again with our single slit equation. Use a slit width of $40 \mu \mathrm{~m}$. Take whatever data you need and enter it and your results as trial 1 in Table 2.

14 Calculate the percent difference between the values you determined for your unknown wavelength with the single slit and the double slit calculations.

15 Show your calculations for the experimental value of your wavelength.

## Trial 2-Measuring the Width of a Small Slit Using Single Slit Diffraction

One important use of the single slit diffraction is to find the size of a tiny opening. We're accustomed to using macroscopic tools such as meter sticks to measure the sizes of objects or spaces between objects. But for very small objects or apertures, these clumsy tools are not so useful. Building a micrometer scale measuring stick is difficult enough, but reading it is also made difficult by the same behavior of light at small scales that we're investigating. Light's wave nature really stands out when it interacts with very small objects and openings.

Similarly diffraction of light around solid objects can be used to measure the widths of objects such as hairs or spider web silk. You'll now use equation 10 to measure the width of one of the six slits of unknown width.

16 Use the numeric steppers beside the wavelength and the slit width sliders to select your unknown number for each of these. Leave the slit spacing at $250 \mu \mathrm{~m}$ and set the number of slits to one.

17 Enter your just-determined experimental value of your unknown wavelength under the $\lambda$ ( nm ) column.

18 Take your $y$-data to determine your slit width.
19 Show your calculations of the unknown slit width in trial 2. You'll need to show your calculation of $y_{m}$ and $w$.

If you run into trouble, try some known values to test your math.

## III. Two Slit Interference with Single Slit Diffraction

We've mostly been dealing with slits that are less point-like than our water waves, and as a result, they each produce the diffraction patterns we've seen with single slits. We'll now examine their role in producing the interference patterns we've been observing with double slits.

1 Set up your apparatus with wavelength $\# 2$ and slit width $\# 1$. Leave the slit spacing $d=250 \mu \mathrm{~m}$.
At the start of your investigation of single slits in Section II, you switched back and forth between one and two slits and produced figures similar to the ones in Figure 21. We can now explain the results.


Figure 21: Double Slit Interference with Single Slit Diffraction

The multiple peak double slit interference pattern reflects the interference between the pair of slits. This results in the relatively large number of fringes and their matching peaks on the intensity curve. Their locations $y_{m(d s)}$ are given by equation 7 .

But the individual slits providing the light are wide enough to produce interference from point to point across their finite widths. It is this light, that varies in intensity with position on the screen, that takes part in the double slit interference. Thus this variation in intensity modulates the intensity of the double slit fringes. And not surprisingly both graphs have matching envelopes with similar shapes.

Notice the captions added to the double slit figure and how they relate to equations 7 and 10 below. You'll now explore these and other features of these curves. Feel free to use your apparatus to verify your answers. After each verification reset the apparatus to the initial settings in \#1 above.

$$
\begin{align*}
& y_{m}=\frac{m L \lambda}{d} \quad \text { where } m=0,1,2, \ldots \text { for bright fringes } \quad \text { (double slit) }  \tag{7}\\
& y_{m}=\frac{m L \lambda}{w} \quad \text { where } m=1,2,3, \ldots \text { for dark fringes } \quad \text { (single slit) } \tag{10}
\end{align*}
$$

2 Increasing the slit spacing, $d$, will $\qquad$ the number of bright fringes in Figure 21. (increase, decrease, not affect)

3 It will $\qquad$ the widths of the peaks of the envelope. (increase, decrease, not affect)

4 Replacing the blue light with $\qquad$ will reduce the distance between the two first order $(m=1)$ dark fringes of the double slit pattern. (violet, red)

5 Thus we can say the spacing of the bright fringes is $\qquad$ proportional to wavelength. (directly, inversely)

6 $\qquad$ the widths of the slits will reduce the distance between the two first order $(m=1)$ dark fringes of the double slit pattern. (Increasing, Decreasing)

7 Thus we can say that the spacing of the dark fringes is $\qquad$ proportional to slit width. (directly, inversely)

## Intensity of Single and Multiple Slits

There is one final observation that we've been skirting up to this point - the intensity of the light for single and multiple slits.

8 Reset your apparatus to two slits, $\lambda=540 \mathrm{~nm}$, slit width $w=40 \mu \mathrm{~m}$, slit spacing $d=250 \mu \mathrm{~m}$. Scan. (As before we'll use a better color for printing in our figures.)
We want to keep an eye on one arbitrary location on the screen. Let's use the $y_{3}$ fringe on the left. Notice the brightness of that fringe. Also notice the corresponding height of the Intensity Graph at that location.


Figure 22: $y_{3}$ Intensity

9 Switch back and forth between one and two slits. Don't Scan. Just compare the brightness of the light and height of the graph at the $y_{3}$ point for each case. They seem to be exactly the same. Is that correct? Let's look at the dark fringes on either side of $y_{3}$ first.
Set the number of slits to 2 . The dark bands on either side of $y_{3}$ were caused by destructive interference of light coming from each of the two slits as given by equation 5 . That works. They're both providing almost no light at that location.

10 What about the $y_{3}$ maximum? If one slit provides light of a certain brightness at that point, what brightness should we expect if light from a pair of slits interferes constructively at that point? Twice as much?

11 Click the "Number of Slits" stepper to change the number to 1 . Notice the label to the right of the peak of the graph. This establishes the intensity scale for the graph. If you move your pointer across the graph, this label will disappear. Moving your pointer over "Intensity vs. Position" will always bring it back.


Figure 23: Single Slit Scale

12 Now for two slits. If we were right about the intensity being twice as much, we'd expect the graph to be twice as high. Right? But we don't have room. So we'll adjust the scale instead. Change to two slits and note the scaling label. What gives?
Actually, all the intensity values will be four times as large. Here's why. When light waves interfere - add to produce a new single wave - the new wave has an amplitude equal to their sum. That would mean twice the amplitude of one wave in our case. But the intensity is proportional to the square of the amplitude. The actual Intensity Graph has the same shape, but should be four times as tall. We've just scaled it to fit on the screen.


Figure 24: Double Slit Scale

13 Try 3,4 , and 5 slits. The bright fringes are much narrower, but their intensity is much greater. There are also some new smaller secondary maxima which occur where light from only some of the slits interfere constructively. Our full-brightness fringes are called principle maxima. At these locations, the light from all the slits interferes constructively.

