Macroscopic and Microscopic Springs – Concepts

INTRODUCTION

Spring Forces

The magnitude of the force exerted by a spring on an object attached to it, is linearly proportional to the absolute value of the stretch of the spring. The spring constant, k_s, represents the stiffness of a particular spring, and has units of N/m. The stretch can be positive or negative, and is defined as the difference between the current length of the spring, L, and its original, relaxed length, L_0. Sometimes the symbol ΔL is used to represent the stretch: $\Delta L = L - L_0$.

Since we will be dealing with both a macroscopic and a microscopic stretch in this lab, we will refer to the macroscopic stretch as ΔL. This makes the force law for a spring:

$$|\overrightarrow{F}| = k_s|\Delta L|. \tag{1}$$

Young's Modulus

The stiffness of a wire depends on its length and thickness, so different wires made from the same metal will have different stiffnesses. It is useful to have a measure of the stiffness of a particular material (such as aluminum, copper, gold, carbon nanotubes), independent of the wire's dimensions.

Calculating Young's modulus is a way to measure the "springiness" of a material and factor out the size and shape of the particular wire. Young's modulus is the ratio of "stress" to "strain". Stress is the force exerted per square meter of cross-sectional area, F/A. Strain is the fractional stretch of the wire, $\Delta L/L_0$.

$$Y = \frac{(\text{"stress"})}{(\text{"strain"})} = \frac{(F/A)}{(\Delta L/L_0)} \tag{2}$$

Interatomic Spring Stiffness

Our simple model of a solid object is a bunch of tiny balls (atoms) that are held together by springs (chemical bonds). The relaxed length of the little spring between two atoms (the interatomic bond) is just the distance from the center of one atom to the center of the other atom, d, which for our model is just twice the radius of one of the atoms since the electron cloud of the atom fills in the extra space. A simple cubic arrangement of the atoms would mean that the volume that each atom fills would be $d \times d \times d$ and would have a cross-sectional area of $d \times d$.

Since the interatomic bonds are modeled as springs, they have a stiffness, $k_{s,i}$, that relates the interatomic force to the stretch of the interatomic bonds. We use s for the **microscopic** stretch.

$$|F| = k_{s,i}|s| \tag{3}$$

