## Uniform Circular Motion - Concepts

## DISCUSSION OF PRINCIPLES

## Part 1: Experimental Determination of Mass

The principle behind this experiment is that the weight of the hanging mass provides a tension that will provide the centripetal force for the swinging mass. Thus, for a mass moving in a horizontal arc, the weight of the hanging mass, $m_{\mathrm{h}} g$, is equal to the centripetal force on the stopper, $M_{\mathrm{s}} \frac{v^{2}}{r}$.

The speed of the moving mass can be related to the period of rotation and the radius of the path. Since speed is distance over time for constant speed, the distance of a circular path (the circumference, $2 \pi r$ ) divided by the time to travel the path (the period, $T$ ) gives the speed:

$$
\begin{equation*}
v=\frac{2 \pi r}{T} . \tag{1}
\end{equation*}
$$

So the centripetal acceleration is

$$
\begin{equation*}
a_{\mathrm{c}}=\frac{v^{2}}{r}=\frac{4 \pi^{2} r}{T^{2}} . \tag{2}
\end{equation*}
$$

In this case, the radius of the path will be $L$, the length of string between the end of the tube and the stopper.

Note that the centripetal force is being provided by the tension in the string necessary to support the weight of $m_{\mathrm{h}}$. That is,

$$
\begin{equation*}
F_{\mathrm{c}}=F_{\mathrm{T}}=m_{\mathrm{h}} g, \tag{3}
\end{equation*}
$$

which yields

$$
\begin{equation*}
m_{\mathrm{h}} g=M_{\mathrm{s}} a_{\mathrm{c}} . \tag{4}
\end{equation*}
$$

This will be useful for calculating $M_{\mathrm{s}}$ from the slope of the trend line.

## Part 2: An Alternate Method

In Part 1, an approximation is being made that the only force acting on the stopper is the tension in the string, and so the tension equals the centripetal force. In reality, gravity plays a role
in the process as well.
The string will actually be at an angle, as shown in Figure 1, rather than purely horizontal. In this case, the tension force (calculated with $m_{\mathrm{h}} g$ ), is the hypotenuse of a right triangle, where the vertical side is the gravitational force on the stopper, $M_{\mathrm{s}} g$, and the horizontal component is the centripetal force, $M_{\mathrm{s}} \frac{v^{2}}{r}$.


Figure 1: Including the force of gravity on the stopper

This means that when you include gravity, the centripetal force term, $M_{\mathrm{s}} a_{\mathrm{c}}$, is actually equal to only the horizontal component of the tension, $m_{\mathrm{h}} g \cos (\theta)$.

$$
\begin{equation*}
M_{\mathrm{s}} a_{\mathrm{c}}=m_{\mathrm{h}} g \cos (\theta) \tag{5}
\end{equation*}
$$

To measure the angle, $\theta$, you could use several methods. There are protractors around the room, so one person could stand back from the motion, hold a protractor in front of them at eye level, and try to line it up and estimate the angle (this doesn't have to be perfect; just shoot for being within the nearest $5^{\circ}$ or so). An easy way might be to take a video of it with a smart phone or laptop camera, pause the video at a point where the angle is clearly visible, and hold a small protractor up to the screen.

