Conservation of Mechanical Energy – Concepts

INTRODUCTION

When a body moves, some things—such as its position, velocity, and momentum—change. It is interesting and useful to consider things that *do not* change. The total energy is a quantity that does not change; we say that it is *conserved* during the motion. There are several forms of energy[1] with which you may be familiar, such as solar, nuclear, electrical, and thermal energies. In this experiment, you will investigate two kinds of mechanical energy[2]: kinetic energy[3] and potential energy[4]. You will carry out an experiment that demonstrates the conservation of the total mechanical energy[5] of a system.

DISCUSSION OF PRINCIPLES

Conservation of Mechanical Energy

The total mechanical energy E of a system is defined as the sum of the kinetic energy K and potential energy U of the system.

$$E = K + U \tag{1}$$

In the absence of nonconservative forces, such as friction or air drag, the total mechanical energy remains a constant and we say that mechanical energy is conserved. If K_1, U_1, K_2, and U_2 are the kinetic and potential energies at two different locations 1 and 2 respectively, then conservation of mechanical energy leads to the following mathematical expression.

$$K_1 + U_1 = K_2 + U_2 \tag{2}$$

This can also be expressed in terms of the changes in kinetic and potential energies

$$\Delta K + \Delta U = 0 \tag{3}$$

where $\Delta K = K_2 - K_1$ and $\Delta U = U_2 - U_1$.

Conservation of Mechanical Energy is one of the fundamental laws of physics that is also a very powerful tool for solving complex problems in mechanics.

[1] http://en.wikipedia.org/wiki/Energy
[2] http://en.wikipedia.org/wiki/Mechanical_energy
[3] http://en.wikipedia.org/wiki/Kinetic_energy
[4] http://en.wikipedia.org/wiki/Potential_energy
[5] http://en.wikipedia.org/wiki/Conservation_of_mechanical_energy

Kinetic Energy

Kinetic energy K is the energy a body has because it is in *motion*. When work[6] is done on an object, the result is a change in the kinetic energy of the object. Energy of motion can be translational kinetic energy[7] and/or rotational kinetic energy[8].

Translational kinetic energy K_{T} is the energy an object has because it is moving from place to place, regardless of whether it is also rotating; K_{T} is related to the mass m and velocity or speed v of the object by

$$K_{\mathrm{T}} = \frac{1}{2}mv^2. \tag{4}$$

Rotational kinetic energy K_{R} is the energy an object has because it is rotating, regardless of whether the body as a whole is moving from place to place. K_{R} is related to the moment of inertia[9], I, and angular velocity[10], ω, of the object by

$$K_{\mathrm{R}} = \frac{1}{2}I\omega^2. \tag{5}$$

The angular velocity of a rolling sphere that is not slipping is the velocity (relative to the center of the sphere) of a point on the circumference, divided by the radius R of the sphere.

$$\omega = \frac{v}{R} \tag{6}$$

Just as the mass m of a body is a measure of its resistance to a change in its (translational) velocity, the moment of inertia I of an object is a measure of that object's resistance to a change in its angular velocity. The moment of inertia of a solid sphere (of uniform density) is given by

$$I = \frac{2}{5}MR^2 \tag{7}$$

where M and R are the mass and radius of the sphere, respectively.

The *total kinetic energy* K_{total} of the rolling sphere used in this experiment is the sum of the translational and rotational kinetic energies.

$$K_{\mathrm{total}} = K_{\mathrm{T}} + K_{\mathrm{R}} \tag{8}$$

[6] http://en.wikipedia.org/wiki/Work_(physics)
[7] http://en.wikipedia.org/wiki/Translational_kinetic_energy
[8] http://en.wikipedia.org/wiki/Rotational_kinetic_energy
[9] http://en.wikipedia.org/wiki/Moment_of_inertia
[10] http://en.wikipedia.org/wiki/Angular_velocity

Potential Energy

An object might have energy by virtue of its *position* on account of the work done to put it there. The object is said to have potential energy U. Gravitational potential energy[11], which we will be concerned with in this experiment, depends on the mass of the object, the acceleration due to gravity, and its location. It is important to remember that potential energy is only defined relative to a location, which can be chosen arbitrarily (that is, for convenience) without affecting the body's subsequent motion. For example, if the rolling sphere is held 1/2 meter above a table top, and the table top is 1 meter above the floor, the potential energy of the sphere has one value relative to the table top and a larger value relative to the floor.

The potential energy of an object near the surface of the Earth is given by

$$U = mgh \tag{9}$$

where g is the acceleration due to gravity, m is the mass of the object, and h is the height above the chosen reference level (the end of the ramp, where the potential energy is zero by choice).

If we drop an object, it will lose height and gain velocity as it falls in accordance with equation 2.

[11] http://en.wikipedia.org/wiki/Gravitational_potential_energy#Gravitational_potential_energy

