Conservation of Mechanical Energy – Procedure – Alternate Lab

OBJECTIVE

In this experiment, you will roll a ball down a ramp and off the table, measuring horizontal and vertical distances associated with the motion in order to determine the speed of the ball at the time it leaves the table. You will use two different methods for this experiment.

1 Kinematics, treating the ball as a point mass in free fall

2 Energy, looking at conservation of the ball's energy

These methods are discussed in more detail below.

EQUIPMENT

Plastic track and stand

Metal ball

Meterstick

Modeling clay

White paper

Tape

BACKGROUND

The objective of this experiment is to determine if conservation of mechanical energy can predict the velocity of a rolling sphere after rolling down a ramp. You will test your hypothesis by placing the sphere at various heights and measuring the velocity at the end of the ramp. You will compare the measured velocity with the predicted velocity using conservation of mechanical energy.

If we let the sphere roll down a ramp, its vertical height will decrease while its translational and rotational velocity will increase. Potential energy is lost and kinetic energy is gained. Assuming that the sphere rolls without slipping and there is no friction or air drag, the loss of potential energy will equal the gain in kinetic energy.

Based on conservation of mechanical energy, if the sphere is placed at height h_1, as shown in Figure 1, you can show that the final speed v_2 at the end of the ramp is

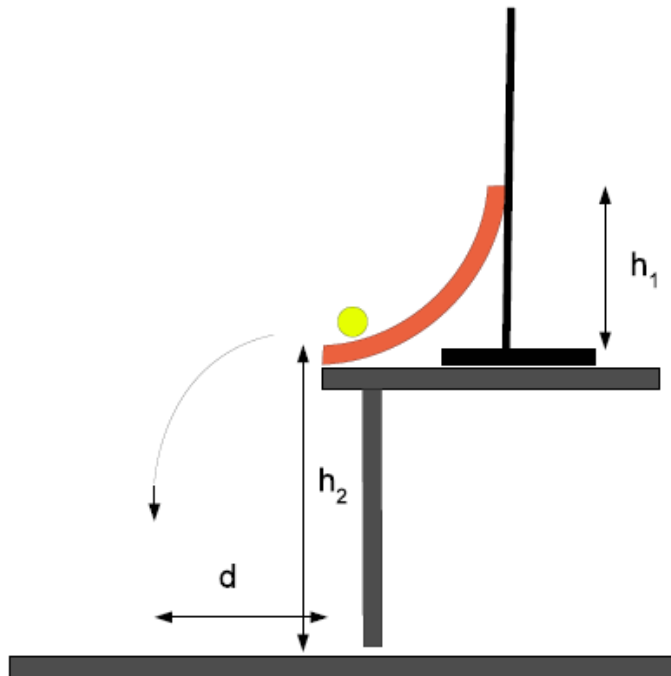$$v_2 = \sqrt{\frac{10gh_1}{7}}. \tag{1}$$

Figure 1: Sphere rolling down a ramp

Equation 1 tells us that we can predict the final translational speed v_2 of the sphere at the bottom of the inclined portion of the ramp if we know the initial height h_1.

In the experiment, you will measure the translational speed of the sphere at the bottom of the ramp and compare it to the speed predicted by conservation of mechanical energy. If these two (experimental and predicted) values of the translational speed agree to within experimental uncertainties, you can say that you have verified conservation of mechanical energy.

EXPERIMENTAL DESIGN

Let us consider several ways in which the translational speed of a small sphere can be measured.

1 **Use a police radar speed gun:** A **radar gun** or **speed gun** is a small Doppler radar[1] unit used to detect the speed of objects, especially trucks and automobiles for the purpose of regulating traffic, as well as pitched baseballs, runners, or other moving objects in sports. It relies on the Doppler effect[2] applied to a radar[3] beam to measure the speed of objects at which it is pointed. Radar guns may be hand-held or vehicle-mounted; see "Radar gun[4]" or "Police RADAR[5]."

A radar speed gun measures the instantaneous speed of the sphere. As the sphere rolls down the ramp, its speed changes. It would be difficult to read the radar gun at the precise moment when the small sphere reaches the bottom of the ramp.

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[1]http://en.wikipedia.org/wiki/Doppler_radar
[2]http://en.wikipedia.org/wiki/Doppler_Effect
[3]http://en.wikipedia.org/wiki/Radar
[4]http://en.wikipedia.org/wiki/Radar_gun
[5]http://hyperphysics.phy-astr.gsu.edu/hbase/sound/radar.html

2 Use an ultrasonic motion detector: The sonic motion detector transmits a burst of ultrasonic pulses. The ultrasonic pulses reflect off a target and return to the face of the sensor. The target indicator flashes when the transducer detects an echo. The sensor measures the time between the trigger rising edge and the echo rising edge. It uses this time and the speed of sound to calculate the distance to the object. To determine velocity, it uses consecutive position measurements to calculate the rate of change of position.

The speed of a rapidly changing body is difficult to measure this way. One has to be sure exactly where the sound reflected to get the velocity. In general, one is limited by the wavelength of sound in terms of this positional accuracy, and audible sound waves have wavelengths between 17 m and 17 mm. Even for shorter waves, we would have to know where the sound was making contact, and we would need the position to not be changing much over the duration of each wave. The reflection intensity goes down as the area gets smaller, so this can make the experiment difficult.

3 Use a photogate: A photogate is a beam of light and a detector that monitors the motion of objects that break the beam. If the size of the object is known, by measuring the time an object blocks the gate, you can determine the velocity.

The size of the infrared beam from a typical photogate is not small compared to the diameter of our spheres. Thus, you could not make a very accurate determination of the length passing through the infrared beam.

4 Use kinematics: Once the sphere leaves the ramp, it is under free fall. By measuring the vertical and horizontal components of the sphere's displacement after it leaves the ramp, you can determine the sphere's speed at the end of the ramp. Measuring the horizontal and vertical displacements of the sphere does not involve the use of motion detectors or photogates and is straightforward. You will use this method for the experiment.

Finding the Speed Using Kinematics

You will make your ramp out of a provided sheet of flexible plastic. The shape of the ramp actually does not matter, since we have only used conservation laws for the energy and the rotation rate is fixed by its translation rate as long as it does not slip on the track. We do want the ball to launch horizontally, so you should make it sit flat at the bottom (use the level provided to make sure this is true). Use your stand and the tape or clamp provided to make a setup as in Figure 1.

Once the sphere leaves the ramp, there is no force acting on it to change its horizontal velocity (assuming there is no air resistance). Gravity pulls the sphere down the instant it leaves the ramp. We will consider the equations of motion for the vertical and horizontal motions of the sphere.

The vertical distance h_2 that the sphere falls in time ΔT is given by

$$h_2 = \frac{1}{2}g(\Delta T)^2 \tag{2}$$

where $g = 9.81$ m/s^2 is the acceleration due to gravity. The initial vertical component of the velocity of the sphere as it leaves the ramp is zero.

The horizontal distance, d, the sphere moves from the end of the ramp to the point where it touches the table is given by

$$d = v_2(\Delta T). \tag{3}$$

Using equation 2 and equation 3, we can show that

$$v_2^{\text{kinematics}} = d\sqrt{\frac{g}{2h_2}}. \tag{4}$$

PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

Kinematics

The first method will apply the principles of uniformly accelerated motion to treat the ball as a projectile. Measuring d and h_2 as illustrated in Figure 1 will be enough to calculate the velocity of the ball at the time it leaves the ramp.

As a reminder, the uniformly accelerated motion equations are reproduced below. For more details on this method, refer to the Concepts[6] document.

$$
\begin{array}{rclcrcl}
v_x & = & v_{x,0} + a_x t & \qquad & v_y & = & v_{y,0} + a_y t \\
x & = & x_0 + v_{x,0}t + \frac{1}{2}a_x t^2 & \qquad & y & = & y_0 + v_{y,0}t + \frac{1}{2}a_y t^2 \\
v_x^2 & = & v_{x,0}^2 + 2a_x(x - x_0) & \qquad & v_y^2 & = & v_{y,0}^2 + 2a_y(y - y_0)
\end{array}
\tag{5}
$$

1 Arrange the track and stand as illustrated in Figures 1 and 2. Important notes:

- Make sure the bottom part of the track is flat and ends at the edge of the table.

- Make sure the top part of the track is angled, not vertical.

- Make the track as stable as possible using the provided clamps, tape, and anything else that seems handy.
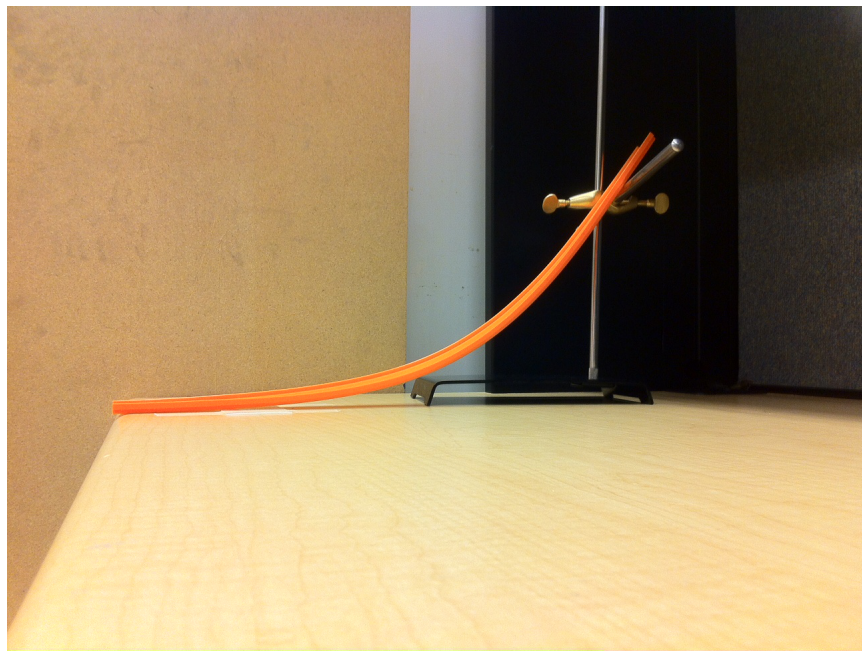
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[6]../lab_8_1/manual.html

Figure 2

2 Hold the ball steady at a point on the track and record the vertical distance from the table to the ball. (*Hint: Should you measure to the top, middle, or bottom of the ball?*) This will be h_1 as illustrated in Figure 1.

3 Release the ball and record where it lands. You can use video capture to record the impact, make a target with the modeling clay, or use any other method of your devising. If you use clay, you will be looking for the impression left by the ball where it lands. However, if you use that method, please put the clay on a piece of paper. **Do not stick clay directly to the floor.** Also, hold or tape the paper still so it doesn't slide during the impact.

The horizontal distance from the edge of the ramp to the impact point will be d as illustrated in Figure 1. The vertical distance from the table top to the floor will be h_2.

4 In order to improve the accuracy of the data, you will perform this measurement two more times using the same starting height (that is, the same h_1), finding the average value for d over the trials. Then for further accuracy, you will repeat the experiment with different starting locations, performing three trials for each different h_1.

5 Finally, use the kinematics equations and the collected data to determine the velocity of the ball at the instant it left the ramp.

Energy

Next, you will calculate the speed of the ball as it leaves the ramp again, this time using conservation of energy methods. During the ball's roll down the ramp, we will assume that it is rolling without slipping. This would mean that friction, while present, is not doing any work. If we also ignore the very small contribution due to air resistance, the conclusion is that the ball has no nonconservative force doing work on it and thus its mechanical energy is conserved.

As a reminder, here are the expressions for various types of mechanical energy.

$$K_{\text{trans}} = \frac{1}{2}mv^2 \tag{6}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \tag{7}$$

$$U_{\text{g}} = mgy \tag{8}$$

Note that for a solid sphere, $I = \frac{2}{5}MR^2$.

1 Using the data you collected in the Kinematics section, calculate the velocity at the bottom of the ramp for each starting height. Use the starting position (at the top of h_1) and the end of the ramp as your initial and final points. Should these calculations yield the same results as the kinematics calculations?

Conclusion

1 What are some of the sources of uncertainty in this lab that could have contributed to a discrepancy in the two data sets or to one or both of the calculations being too high or low? Specify which calculation method each source of uncertainty would contribute to, and whether it would tend to make the calculation too low or too high.

2 Considering all the sources of uncertainty you identified above, which method of calculation do you feel is likely to give a more accurate result? Consider how many sources of uncertainty each method might have and the magnitude of those uncertainties in your conclusion.