

Rotational Inertia

INTRODUCTION

When you were younger, you might have had toy cars that you rubbed quickly across the floor several times to “rev them up.” Then, when you put them down, they would go zipping along for some distance. Or perhaps you have seen an auto accident on television where the wheels of the overturned car continued to turn for a little while. Maybe you have watched a helicopter land, and have noticed that the blades continue to rotate after the pilot turns the engine off. All of these are examples of rotational inertia.¹ The toy car has a little wheel inside, called a flywheel, which is attached to the car’s wheels. When you “rev it up” the flywheel begins to spin. Then, when you let the car go, the flywheel continues to spin because of rotational inertia, and so turns the wheels. The wheels on the overturned car and the helicopter blades both continue to turn until friction overcomes their rotational inertia. You have already learned about inertia in linear systems; an object in motion will continue in motion as long as there is no net force acting on it. Rotational inertia is a similar concept applied to objects whose motion is rotational instead of linear. The rotational inertia of an object is that object’s resistance to a change in its angular velocity. That is, if an object is spinning, it has a tendency to keep spinning until a net torque acts upon it. This is just another manifestation of Newton’s second law.²

DISCUSSION OF PRINCIPLES

In Newton’s second law, $F = ma$ the mass m of an object is a measure of its inertia. Clearly the smaller the mass, the less force is required to change the object’s linear velocity. In rotational motion, it is the rotational inertia, often called the moment of inertia³ I that determines the torque τ , required to change an object’s angular velocity ω . The analogue of Newton’s second law for rotational motion is

$$\vec{\tau}_{\text{net}} = I\vec{\alpha} \quad (1)$$

where $\vec{\tau}_{\text{net}}$ is the *net* torque and $\vec{\alpha}$ is the angular acceleration. The moment of inertia of an object depends on the shape of the object and the distribution of its mass relative to the object’s axis of rotation. A uniform disk of mass m is not as hard to set into rotational motion as a “dumbbell” with the same mass and radius. For a symmetric, continuous body (like a solid disk) that is rotating about an axis of symmetry, e.g., an axle through the center and perpendicular to the disk, the moment of inertia is calculated by carrying out the integral

$$I = \int r^2 dm \quad (2)$$

where dm is the tiny bit of mass located a distance r from the axis of rotation. For a collection of

¹http://en.wikipedia.org/wiki/Rotation_around_a_fixed_axis

²http://en.wikipedia.org/wiki/Newton's_laws_of_motion

³http://en.wikipedia.org/wiki/Moment_of_inertia

n point masses, the moment of inertia is calculated by carrying out the sum

$$I = \sum_{i=1}^n m_i r_i^2 \quad (3)$$

where m_i and r_i are the mass and position, respectively of the i th particle. Thus, for the dumbbell $n = 2$ and with mass m at each end, separated by a distance d , the moment of inertia is

$$I = 2m \left(\frac{d}{2} \right)^2 = m \frac{d^2}{2} \quad (4)$$

For moments of inertia of different symmetrical solids see this list.⁴

In this experiment you will use a disk of nearly uniform mass and apply a torque by adding weight to a string attached to a step pulley at the center of the disk. See Fig. 1. Using your results, you will determine the moment of inertia of the disk.

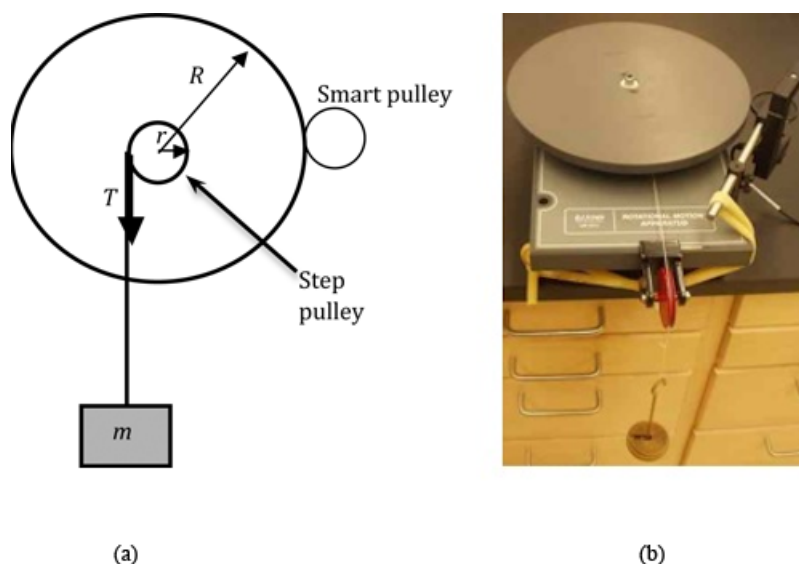


Figure 1: Sketch and photo of the apparatus

Even though the wheel is horizontal and the string passes over a second pulley, in Fig. 1a we show the wheel in a vertical position. As the hanging mass falls, the wheel rotates counterclockwise. This rotation is due primarily to the torque exerted by the tension force in the string, which arises from the weight of the hanging mass. The wheel has bearings at its axle, and there is some friction in their movement. The torque associated with that friction force f acts to oppose the torque produced by the tension force T in the string.

If we consider just the wheel, we can examine the two torques that are acting, that of the friction

⁴http://en.wikipedia.org/wiki/List_of_moments_of_inertia

τ_f and that of the tension τ_{tension} . The magnitude of the torque due to the tension is

$$\tau = rT \quad (5)$$

where r is the radius of the step pulley.

The net torque is the vector sum of these two torques

$$\tau_{\text{net}} = rT - \tau_f \quad (6)$$

Applying Eq. (1) we obtain

$$rT - \tau_f = I\alpha \quad (7)$$

Here we follow the sign convention that counterclockwise torques are positive and clockwise torques are negative.

Now let us consider the hanging mass m . Figure 2 shows the free body diagram for the hanging mass. We see that the tension is opposing the force of gravity.

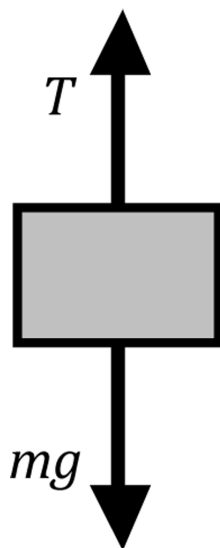


Figure 2: Free body diagram for the hanging mass

Newton's second law written for the hanging mass is then

$$mg - T = ma \quad (8)$$

where a is the acceleration of the hanging mass, and we have chosen the *downward* direction to be positive. Solving for T ,

$$T = m(g - a) \quad (9)$$

If we assume that the string doesn't slip, then a is also the tangential acceleration of a point on the step pulley. Similarly, a' is the tangential acceleration of a point at the edge of the wheel. This is where we will actually measure the acceleration with the *Smart Pulley*. The angular acceleration of the wheel is related to these tangential accelerations by

$$\alpha = \frac{a}{r} = \frac{a'}{R} \quad (10)$$

With Eqs. Eq. (9) and (10) we can rewrite Eq. (7) as

$$m(g - a)r - \tau_f = I \left(\frac{a}{r} \right) \quad (11)$$

Analysis by Linearization

There are two points to consider with regard to Eq. (11): it is not linear in m and a and we do not know τ_f . In this analysis, we will make an approximation that will result in Eq. (11) becoming linear.

We assume that $g \gg a$, and, therefore $T \approx mg$. As a consequence, Eq. (11) becomes

$$mgr - \tau_f = I \left(\frac{a}{r} \right) \quad (12)$$

Solving Eq. (12) for a gives

$$a = \left(\frac{gr^2}{I} \right) m + \left(\frac{-\tau_f r}{I} \right) \quad (13)$$

Thus, if we determine the acceleration a for various hanging masses m , and graph the acceleration versus mass, then the slope will be equal to gr^2/I and the y -intercept will be $\tau_f r/I$. *Note:* If we determined the accelerations, a_1 and a_2 , for only two masses m_1 and m_2 , then

$$\text{slope} = \frac{a_2 - a_1}{m_2 - m_1} = \frac{gr^2}{I} \quad (14)$$

and therefore

$$I = \frac{(m_2 - m_1)gr^2}{(a_2 - a_1)} \quad (15)$$

Analysis by the Exact Method

As in the note above, suppose the procedure is repeated with two different masses, m_1 and m_2 , resulting in accelerations a_1 and a_2 , respectively.

$$m_1(g - a_1)r - \tau_f = I \left(\frac{a_1}{r} \right) \quad (16)$$

$$m_2(g - a_2)r - \tau_f = I \left(\frac{a_2}{r} \right) \quad (17)$$

Now, τ_f can be mathematically eliminated. Solving the two simultaneous Eqs. (16) and (17) for I , we have

$$I = \frac{(m_2 - m_1)gr^2 + (m_1a_1 - m_2a_2)r^2}{(a_2 - a_1)} \quad (18)$$

Compare Eqs. (15) and (18).

OBJECTIVE

In this experiment you will use a wheel with a step pulley to measure the acceleration of the rotating wheel and determine the moment of inertia of the wheel. You will also determine the moment of inertia of an extension that you will add to the wheel.

EQUIPMENT

Wheel with step pulley

Smart Pulley with photogate

String

Mass with hanger

DataStudio software

Caliper

Balance

Meter stick

Hoop

Plate

PROCEDURE

The apparatus (see Fig. 1) consists of a wheel with a small step pulley mounted at its center. A string wound around this pulley and attached to a hanging mass provides the necessary torque needed to set the wheel rotating. A small pulley with a photogate is mounted so it makes contact with the rim of the wheel. As the wheel rotates, this *Smart Pulley* will also rotate. The spokes of the rotating pulley will interrupt the infrared beam from the photogate and the photogate sends these signals to the computer. The computer uses the information to measure the speed at which the pulley turns. The computer measures the speed at which the pulley turns and calculates first the tangential velocity of the edge of the wheel, and then the tangential acceleration a' at the rim of the wheel. From this, you will calculate the tangential acceleration at the step pulley's rim using Eq. (10). An extension like a hoop or a rectangular plate can be added to the wheel and the moment of inertia of the combination can be measured. Because the extension has the same axis of rotation as the wheel, the moment of inertia of the combination will be

$$I_{\text{total}} = I_{\text{wheel}} + I_{\text{extension}}. \quad (19)$$

From this the moment of inertia of the extension can be determined.

- 1 Weigh the wheel and record this value on the worksheet. You do not need to include the hanger.
- 2 Use a meter stick to measure the diameter of the wheel. Once you have found the diameter, use it to find the radius R of the wheel, and record this value on the worksheet.
- 3 Calculate the accepted value for the Moment of Inertia for your wheel using

$$I_{\text{disk}} = (1/2)MR^2. \quad (20)$$

- 4 Use a caliper to measure the diameter of your step pulley. This is the smallest pulley that the string is wrapped around. See Appendix D.

Use the diameter to calculate the radius r of your step pulley, and record this value on the worksheet.

- 5 Open the *DataStudio* file associated with this lab. A screen similar to Fig. 3 is displayed.

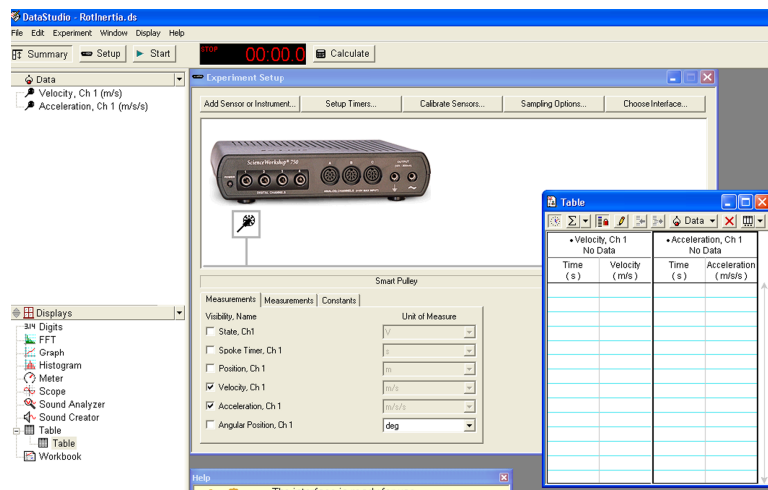


Figure 3: Start-up screen for rotational inertia lab

- 6 Use the balance to determine the mass m_1 of the hanger and record this value on the worksheet. This will be the mass for the first data point.
- 7 Wind the string onto the smallest diameter step pulley.

Then pass the string over the second pulley and attach the hanger to the end of the string so that it hangs over the edge of the table.

- 8 Click the **Start** button once and allow the wheel to start rotating. Click **Stop** after the mass hits the floor or when you run out of string and the wheel begins to turn the opposite way.

It is not necessary that you try to release the wheel and click the mouse at exactly the same time, you might want to click the mouse about half a second before you release the wheel.

Be sure to stop the wheel from rotating when it runs out of string.

The table of times, velocities, and accelerations like the one shown in Fig. 4 is automatically completed.

Velocity, Ch 1 Run #1		Acceleration, Ch 1 Run #1	
Time (s)	Velocity (m/s)	Time (s)	Acceleration (m/s/s)
2.5519500	1.00671	2.5593875	0.2279
2.5668250	1.01010	2.5742500	0.1529
2.5816750	1.01237	2.5890875	0.0768
2.5965000	1.01351	2.6038875	0.0774
2.6112750	1.01466	2.6186375	0.2344
2.6260000	1.01810	2.6333375	0.3951
2.6406750	1.02389	2.6479750	0.4820
2.6552750	1.03093	2.6625250	0.4921
2.6697750	1.03806	2.6770000	0.3322
2.6842250	1.04287	2.6914125	0.2528
2.6986000	1.04651	2.7057250	0.3444
2.7128500	1.05140	2.7199625	0.4343
2.7270750	1.05758	2.7341750	0.3505
2.7412750	1.06257	2.7483375	0.0888

Figure 4: Data table for a sample run

- 9 Scroll through the data to choose ten successive accelerations near the middle of the run, not those at the very beginning or the very end.

Record these ten values in the data table on the worksheet.

- 10 Using the *summation function* located in the top left corner of the data table window, determine the average of these ten acceleration measurements and enter the result on the worksheet.

You might need to repeat the run if the numbers are very erratic.

- 11 Repeat steps 7 through 9 three more times by adding mass to the hanger in increments of 50 grams.

Note: You might need to increase the precision for the accelerations as you add more runs of data. You can do this by clicking on **Increase Precision** in the menu at the top right hand corner of the data table window. See Fig. 4.

CHECKPOINT 1: Ask your TA to check your data and calculations before proceeding.

Procedure A: Linearization Method

- 12 Using Excel, create a graph of acceleration vs. hanging mass. See Appendix G.
- 13 Using the Linest function, determine the slope and intercept. Record these values on the worksheet. See Appendix J.
- 14 From the slope and intercept values calculate the moment of inertia and frictional torque using Eq. (13). Record these values on the worksheet.
- 15 Calculate the percent difference between the moment of inertia that you just calculated and the moment of inertia you calculated in step 3. See Appendix B.

CHECKPOINT 2: Ask your TA to check your data and calculations before proceeding.

Procedure B: Exact Method

- 16 Use the data from the first and last trial to calculate the moment of inertia using Eq. (18).
- 17 Calculate τ_f using Eq. (17).
- 18 Calculate the percent difference between the moment of inertia that you just calculated and the moment of inertia you calculated in step 3.

CHECKPOINT 3: Ask your TA to check your calculations before proceeding.
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Procedure C: Determining the Moment of Inertia of the Extension

19 Choose either the hoop or the plate as the extension to add to the wheel.

Measure the dimensions and the mass of the extension and record these values on the worksheet.

20 Calculate the accepted value of the moment of inertia for the extension (use the formulas provided on the worksheet).

21 Algebraically solve Eq. (11) for the moment of inertia. Record this formula on the worksheet.

22 Mount the extension on the wheel and perform steps 7 through 9 with a hanging mass of 200 g.

23 Find the acceleration a' of the wheel, and the acceleration a of the step pulley. Record these values on the worksheet.

24 Using the frictional torque τ_f determined from step 17, and the formula from step 21, calculate the total moment of inertia of the extension and the wheel. Record this value on the worksheet.

25 Using the moment of inertia of the wheel obtained by the exact method from step 16, and Eq. (19) find the moment of inertia of the extension, by itself. Record this value on the worksheet.

26 Find the percent difference between this value and the accepted value you calculated in step 20. Record this value on the worksheet.

CHECKPOINT 4: Ask your TA to check your calculations.
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