Appendix A: Significant Figures

1. DETERMINING THE NUMBER OF SIGNIFICANT FIGURES

Most measurements made in the laboratory involve the reading of some scale. The precision of the reading is limited by the gradation of the scale and by the width of the lines marking the boundaries. For every measurement made using an instrument, the last digit capable of being read is going to be an estimate.

For example, suppose a reading is taken using an ordinary meterstick whose smallest divisions are millimeters; the result recorded might be 10.42 cm. The first three digits are exactly precise but the last digit, 2, is only an estimate indicating that the reading is probably no more than 10.50 cm and no less than 10.40 cm. So even though the last digit cannot be read with certainty, it still provides relevant information about the measurement. It is important to note that in any reading one and only one digit (the last digit) should be estimated.

A significant figure, therefore, can be defined as a digit that provides useful information about the measurement and is known to be at least somewhat reliable. In the example above, the reading 10.42 cm has four significant figures.

In determining the number of significant figures for a reading, the location of the decimal point is irrelevant. For example, the number 10.42 cm could have been written as 104.2 mm or as 0.1042 m. In all three cases, the information provided by the reading is equally precise, regardless of the shift in the decimal point; each number has four significant figures. It is the total number of significant figures in a measurement reading and not the number of digits to the right of the decimal point that indicates the precision of the measuring devices.

When a reading is taken to the limit of the precision of the measuring instrument, all nonzero digits in the number are significant. A zero may or may not be significant depending on its position within the number. The following are some guidelines that can be used to determine the number of significant figures:

1 All nonzero digits are significant.

2 Zeros that fall between nonzero (significant) digits are always significant.

3 Since the placement of the decimal point within the number has no effect on the precision of the number, zeros used to indicate the decimal location are not significant. For example, the number 0.0000214 has eight digits but only three (nonzero digits) are significant figures. An easy way to recall this is to remember that placeholding zeros are not significant figures.

4 When a zero falls at the end of a number, it tends to be ambiguous. In some instances the presence of the zero digits at the end of a number is only necessary to properly position the decimal point. For example, if a measured distance is 34.1 km, the number has three significant figures. If the result is recorded in meters instead of in kilometers, then the result, written 34,100 m, still has only three significant figures; the zeros once again are present only to locate the decimal point. So zeros appearing at the end of a number are considered significant only if indicated by the measurement itself.

In such situations where zero digits appear at the end of a number, it is best to clearly distinguish which zeros are significant and which zeros are not. One method is to place a dash over the last significant figure that appears in the number. For the case above, the distance measurement would be written $34,\bar{1}00$ m. If the instrument could read to the nearest 10 meters, then the second to the last zero would also be significant and it would be written as $34,1\bar{0}0$ m. If the reading could be taken to the nearest meter, then both zeros would be significant and the distance would be recorded as $34,10\bar{0}$ m. In another method used to indicate the number of significant figures, the value is expressed by writing the correct number of significant figures multiplied by the proper power of 10. The three readings above would be expressed as 3.41×10^3 m, 3.410×10^3 m, 3.4100×10^3 m.

It is just as important to record a zero as it is to record any other significant figure. For example, in reading an ordinary meterstick graduated in millimeters, a reading of 50.00 cm would be appropriate, indicating that the measurement is accurate to the nearest tenth of a millimeter. This should not be recorded as 50 cm since this would imply that the reading could be taken only to the nearest centimeter.

2. SIGNIFICANT FIGURES IN CALCULATIONS

Although there is generally no difficulty in deciding how many significant figures (s. f.) a measurement should contain, problems can arise when calculations are made using these numbers. When calculators or computers are used to manipulate data, results often contain a great many digits that appear significant but are not. The results from a calculation cannot have a precision that is greater than that of the quantities used in it. So there are certain rules that should be followed for numerical calculations.

A. Addition and Subtraction

When adding or subtracting a set of numerical quantities, carry the result only as far as the first column that contains an estimated digit.

Example 1:

Consider the sum of the following numbers (the estimated digit in each number is in italics).

$$
\begin{array}{rrl}
 & 10.7\mathit{7} & \text{(4 s. f.)} \\
 & 0.031\mathit{4} & \text{(3 s. f.)} \\
 & 2.\mathit{1} & \text{(2 s. f.)} \\
+ & 0.0060\mathit{6} & \text{(3 s. f.)} \\
\hline
 & 12.\mathit{90756} & \\
 & 12.\mathit{9} & \text{(3 s. f.)}
\end{array}
$$

The presence of an estimated digit in a column means that the sum (or difference) of that column is also an estimate. In the example above, a calculator might record the result as 12.90756; however, according to the rule that only one estimated digit for a number can be retained, the real result would be reported as 12.9 cm, a number containing three significant figures. In dropping nonsignificant figures; the final result should be rounded off according to the following rule: if the digit being dropped is five or greater, the last digit being retained should be rounded up by one; if the digit being dropped is less than five then the last digit retained remains unchanged.

B. Multiplication and Division

When multiplying or dividing a set of numerical quantities, the result should have as many significant figures as the number of significant figures in the least precise quantity in the data.

Example 1:

Suppose the dimensions of a wooden block are recorded as 7.812 cm, 2.415 cm, and 1.01 cm. The volume of the block is the product of these three numbers:

$$7.812 \times 2.415 \times 1.01 = 19.0546398 \text{ cm}^3 = 19.1 \text{ cm}^3.$$

The least precise factor in the calculation is the dimension 1.01 cm, having only three significant figures. Thus the result should have only three significant figures and the volume would be recorded as 19.1 cm^3.

If the calculations are done in a series of steps, then retain only one more figure in each intermediate result than occurs in the quantity having the least number of significant figures. The final result is then rounded off to match the least precise quantity.

Example 2:

If the mass of the block is also measured then the density can be found using $\rho = \dfrac{\text{mass}}{\text{volume}}$. In this case

$$\rho = \frac{24.219}{19.05} = 1.113852 \text{ gram/cm}^3 = 1.11 \text{ gram/cm}^3.$$

In order to reduce round-off error for the calculation, one more (nonsignificant) digit was retained for the volume. The volume, however, still has only three significant figures and since it is the least precise of the two factors, the final result for the density has three significant figures: $\rho = 1.11 \text{ gram/cm}^3$.

C. Constants

When a constant is present in a calculation involving measured values, the constant should be viewed as being infinitely precise. This means that a constant will be the most precise quantity in the calculation and so will not affect the number of significant figures in the final result.

Example 1:

If the radius, r, of a circle is 5.15 cm (3 s. f.), then the diameter of the circle would be reported to three significant figures, since the number 5.15 cm is the least precise quantity:

$$d = 2r = (2.00000...)(5.15) = 10.3 \text{ cm}.$$

For constants such as π or g, where the digits to the right of the decimal place are nonzero or non-repeating, the values used in the calculations should contain at least one more significant figure than the most precise quantity in order to avoid inaccuracies due to round-off errors.

Example 2:

The circumference of a circle of diameter 10.3 cm (3 s. f.) would be calculated using circumference, πd. In this case, π should contain at least four digits:

$$\text{circumference} = (3.142)(10.3) = 32.4 \text{ cm}.$$

D. Assorted Calculations

When the calculations involve both addition (subtraction) and multiplication (division), each intermediate step should comply with the rules for significant figures.

Example 1:

Suppose a hollow sphere has an inner radius of 3.550 cm (four significant figures) and an outer radius of 5.21 cm (three significant figures). Then the volume of the sphere is

$$V = \frac{4}{3}\pi(r_{\text{out}}^3 - r_{\text{in}}^3).$$

Applying the rules for multiplication, ${r_{\text{out}}}^3$ and ${r_{\text{in}}}^3$ are

$$\begin{aligned} r_{\text{in}}^3 &= 44.738875 \rightarrow 44.7\overline{4} \text{ cm}^3 \qquad (4 \text{ s. f.}) \\ r_{\text{out}}^3 &= 141.420761 \rightarrow 14\overline{1} \text{ cm}^3 \qquad (3 \text{ s. f.}). \end{aligned}$$

When calculating the intermediate results $(r_{\text{out}}^3 - r_{\text{in}}^3)$ use the rules for subtraction and retain one more nonsignificant digit to eliminate round-off error,

$$\begin{array}{rcll} (r_{\text{out}}^3 - r_{\text{in}}^3) & = & 14\overline{1}.4 & (3 \text{ s. f.}) \\ & - & 44.7\overline{3}9 & (4 \text{ s. f.}) \\ \hline & & 96.61 \rightarrow 9\overline{7} \text{ cm}^3 & (2 \text{ s. f.}). \end{array}$$

In calculating the final volume again retain an extra nonsignificant digit:

$$V = \tfrac{4}{3}(3.142)(9\overline{6}.6) = 405.1085 \rightarrow 4\overline{1}0 \text{ cm}^3 \quad (2 \text{ s. f.}).$$