Appendix

USING EXCEL TO FIT DATA TO A LINEAR EQUATION

1 In Excel, put values to be plotted along the x-axis in the rows of column A.

2 Put the corresponding values that are to be plotted along the y-axis into column B.

3 Next, highlight both columns. In the Toolbar, click on **Insert**. Then click on **Scatter**. In the drop-down box, click on the version without lines (the one in the upper left-hand corner). The graph will appear with the plot of the values you entered in the two columns.

4 Right-click one of the data points. In the drop-down menu, select **Add Trendline**. A Format Trendline box will open. Make sure **Linear** is selected (it should be selected already, as the default). And at the bottom of the box, select **Display Equation on Chart**. Click on **Close**. The trendline and its equation will be added to the graph.

THEORY OF ERRORS

The measurement of a physical quantity can never be made with perfect accuracy; there will always be some error or percent uncertainty present. For any measurement there are a number of factors that can cause a value obtained experimentally to deviate from the true (theoretical) value. Most of these factors have a negligible effect on the outcome of an experiment and can usually be ignored. However, some effects can cause a significant alteration, or error, in the experimental result. If a measurement is to be useful, it is necessary to have some quantitative idea of the magnitude of the errors. So when experimental results are reported, they are accompanied by an estimate of the experimental error, called the uncertainty. This uncertainty indicates how reliable the experimenter believes the results to be.

Types of Errors

In order to determine the uncertainty for a measurement, the nature of the errors affecting the experiment must be examined. There are many different types of errors that can occur in an experiment, but they will generally fall into one of two categories: random errors or systematic errors.

Random Errors

Random errors usually result from human errors and from accidental errors. Accidental errors are brought about by changing experimental conditions that are beyond the control of the experimenter; examples are vibrations in the equipment, changes in the humidity, fluctuating temperature, etc. Human errors involve such things as miscalculations in analyzing data, the incorrect reading of an instrument, or a personal bias in assuming that particular readings are more reliable than others. By their very nature, random errors cannot be quantified exactly since the magnitude of the random errors and their effect on the experimental values is different for every repetition of the experiment. Therefore, statistical methods are usually used to obtain an estimate of the random errors in the experiment.

Systematic Errors

A systematic error is an error that will occur consistently in only one direction each time the experiment is performed, i.e., the value of the measurement will always be greater (or lesser) than the real value. Systematic errors most commonly arise from defects in the instrumentation or from using improper measuring techniques. For example, measuring a distance using the worn end of a meter stick, using an instrument that is not calibrated, or incorrectly neglecting the effects of viscosity, air resistance, and friction are all factors that can result in a systematic shift of the experimental outcome. Although the nature and the magnitude of systematic errors are difficult to predict in practice, some attempt should be made to quantify their effect whenever possible.

In any experiment, care should be taken to eliminate as many of the systematic and random errors as possible. Proper calibration and adjustment of the equipment will help reduce the systematic error, leaving only the accidental and human errors to cause any spread in the data. Although there are statistical methods that will permit the reduction of random errors, there is little use in reducing the random errors below the limit of the precision of the measuring instrument.

Statistical Methods

When several independent measurements of a quantity are made, an appropriate result to report for that quantity is the average of the measurements. For a set of experimental data, S, containing N elements (or measurements) given by S_1, S_2, S_3, ..., S_N, the average, $\overline{S}$, is calculated using the formula

$$\overline{S} = \frac{S_1 + S_2 + S_3 + ... + S_N}{N}. \tag{1}$$

This average represents the closest approximation that is available to the true value of the quantity being measured and is sometimes referred to as the best estimate of the true value. The data S_1, S_2, S_3, ..., S_N are dispersed around the mean (average); a measure of this dispersion is called the standard deviation and is given by

$$\Delta S = \sqrt{\frac{(S_1 - \overline{S})^2 + (S_2 - \overline{S})^2 + ... + (S_N - \overline{S})^2}{N-1}}. \tag{2}$$

If there is a large number of normally distributed points, statistical analysis shows that about 68.3% of them will fall within the interval $\overline{S} - \Delta S$ to $\overline{S} + \Delta S$.

If the systematic errors have been reduced as far as possible, the random errors will dominate and hence will limit the accuracy of the final result. Clearly two ways to reduce the effect of random errors and to improve the accuracy of an experimental result are (1) to eliminate the majority of the random errors inherent in the experiment and (2) to obtain as many data points as is reasonable (thereby increasing N and reducing ΔS).

Percent Errors

The standard deviation is a measure of the precision of an experiment: the smaller the ΔS, the greater the precision of the best estimate. One way to report the precision of the experimental value is through the use of the percent standard deviation given by

$$\% \text{ standard deviation} = \frac{\text{standard deviation}}{\text{average}} \times 100\%. \tag{3}$$

Unfortunately, the average and the standard deviation indicate nothing about the accuracy of the measurement, i.e., how close the experimental or average value is to the true value. In other words, an experiment can yield extremely precise, consistent values without generating a result that is close to the true value. This type of result often occurs when the equipment has not been zeroed or calibrated properly and when other systematic errors have not been properly reduced. In many experiments, it is desirable to indicate the overall accuracy of the final experimental value by reporting some type of percent error.

If the quantity measured has a standard or true (theoretical) value that is known, then the accuracy of the experimental value is given by the ratio of the error to the true value:

$$\% \text{ error} = \frac{\text{experimental value} - \text{true value}}{\text{true value}}. \tag{4}$$

For many experiments, the true value of the quantity being measured is unknown. In this situation, it is often useful to compare two results obtained by different methods so that a percent difference can be obtained:

$$\% \text{ difference} = \frac{\text{difference in values}}{\text{average of values}} \times 100\%. \tag{5}$$

If an experiment is performed properly, with care taken to reduce the random and systematic errors as much as possible, then the percent errors will be correspondingly small. The magnitude of the percent errors will depend heavily on the overall precision of the measuring instruments. This means that while in some cases an error of 5% or 10% might be acceptable, in other cases such an error would indicate a very poorly run experiment. Thus the success of an experiment (in terms of percent error) can only be judged when the method and instrumentation of the experiment are taken into consideration.

Propagation of Errors

In many experiments, the quantities measured are not the quantities of final interest. Since all measurements have uncertainties associated with them, clearly any calculated quantity will have an uncertainty that is related to the uncertainties of the direct measurements. The procedure used to estimate the error for the calculated quantities is called propagation of errors.

Addition and Subtraction

If

$$U = A \pm B \pm C \pm D \pm ..., \tag{6}$$

then

$$\Delta U = \sqrt{(\Delta A)^2 + (\Delta B)^2 + (\Delta C)^2 + (\Delta D)^2 + ...}. \tag{7}$$

Multiplication and Division

If

$$U = \frac{ABC...}{XY...}, \tag{8}$$

then

$$\frac{\Delta U}{\overline{U}} = \sqrt{\left(\frac{\Delta A}{\overline{A}}\right)^2 + \left(\frac{\Delta B}{\overline{B}}\right)^2 + \left(\frac{\Delta C}{\overline{C}}\right)^2 + ... + \left(\frac{\Delta X}{\overline{X}}\right)^2 + \left(\frac{\Delta Y}{\overline{Y}}\right)^2 + ...}. \tag{9}$$

Exponents and Roots

If

$$U = X^n, \tag{10}$$

then

$$\frac{\Delta U}{\overline{U}} = n\frac{\Delta X}{\overline{X}}. \tag{11}$$