Density: Methods of Measurement and Applications

Goal and Overview

The densities of aluminum (Al) and brass will be determined based on mass and volume measurements made on solid metal cylinders. The accuracy of the results will be analyzed by comparing experimental results to known values. Precision, which can be thought of as relating to exactness, will be investigated and assessed by using three methods (geometry; water displacement; and pycnometry) to determine the cylinders' volumes while keeping the precision of the mass measurements constant. To illustrate applications of density in experimentation, the volume of empty space (a void) within a hollow cylinder and the composition of a mixed (plugged) Al-brass cylinder will also be found.

Objectives and Science Skills

Use three methods to determine the volumes of solid aluminum and solid brass cylinders and assess the relative merits and limitations of each method.
Use mass-based pycnometry measurements to find the volume of a void in a hollow cylinder and the mass fractions of aluminum and brass in a plugged (mixed) metal cylinder.
Calculate results-based (values and uncertainties) experimental data, known mathematical relationships (e.g., between mass, volume, and density), and statistical methods (e.g., error analysis and propagation).
Evaluate the experimental methods and their outcomes in terms of parameters such as reliability, difficulty, accuracy, and precision.

Suggested Review and External Reading

Data analysis introduction, significant figure handout, textbook information on density

Introduction

Density, ρ, is an important physical parameter that can be used to characterize a sample of matter. It can be used in a wide range of applications, including the determination of sucrose in aqueous solutions and the alcohol content in liquid beverages, the quality of crude oil, the production of flavorings and fragrances, and the analysis of components such as nitrogen in products like fertilizers. Experimental procedures measuring and using density are often non-destructive, lowering environmental and economic burdens while still providing valuable information.

Density is the ratio of how much matter (mass, m) occupies a given amount of three-dimensional space (volume, V).

$$\rho = \frac{m}{V}$$

Experimentally determined masses and volumes, which reflect the care taken in designing and executing procedures and in collecting and analyzing data, should be as accurate and precise as possible. Accuracy can be checked through calibration, but there is always inherent uncertainty, σ, in experimental values that reflects the precision of the instruments, equipment, and processes used.

You may make single measurements of a physical quantity. For example, suppose you use an analytical balance to find the mass of a metal cylinder. The balance reads to four decimal places, and you assume that the uncertainty in the recorded mass is $\sigma = \pm 0.0001$ g.

You might also make repeated measurements of the same physical quantity to see how values differ trial to trial. For example, you might use the analytical balance to determine the mass of water in a pipet filled to its 10.00-mL mark. A single measurement has an uncertainty of ±0.0001 g, but you realize that the mass of water in the pipet depends on how precisely you are able to fill the pipet to precisely the same level every time you use it.

You perform four trials, noting that the mass values vary more than ±0.0001 g. You calculate the average mass of water in the pipet.

$$average\ mass = \overline{m} = \frac{m_1 + m_2 + m_3 + m_4}{4}$$

You also calculate the standard deviation, which is one way to quantify the variation in the individual masses. You realize that this value is representative of the uncertainty associated with filling the pipet to the mark (and the precision of the pipet).

$$standard\ deviation = \sigma_m = \pm\sqrt{\frac{(m_1 - \overline{m})^2 + (m_2 - \overline{m})^2 + (m_3 - \overline{m})^2 + (m_4 - \overline{m})^2}{3}}$$

Note: Uncertainty and standard deviation are *not* the same, and other courses will describe and emphasize their differences. In this class, however, you can think of them as representative of precision and use the symbol σ for both.

Individual data often must be mathematically combined to produce an overall result (or set of results) that is of greater interest to or has broader applications in the scientific community, but values based on experimental data cannot become more precise through arithmetic manipulation.

The uncertainty in the overall computed result must incorporate the individual uncertainties in the data used in the calculation. How this “error propagation” is carried out depends on the operations involved in a particular calculation (addition, subtraction, multiplication, division, logarithms, etc.). For example, mass is divided by volume to find density.

$$\rho = \frac{m}{V} \implies \sigma_\rho = \pm\rho\left(\frac{\sigma_m}{m} + \frac{\sigma_V}{V}\right)$$

Note: The statistical methods used in this course produce calculated uncertainties that tend to be slightly larger than other available treatment, allowing for marginally wider ranges of possible true values based on your measurements.

Equipment List

Metal cylinders: brass, aluminum, mixed brass/aluminum, and hollow*
Vernier calipers
50-mL Erlenmeyer flask, 100-mL graduated cylinder, 400-mL beaker
Marker/sharpie
Pasteur (disposable) pipet
Thermometer
*If you work with a second pair of students, pair #1 must work with the solid Al and hollow cylinders and pair #2 must use the solid brass and mixed (plugged) cylinders. Your TA will check the cylinders out and in.

Procedure

This experiment has two main sections.

First, you will work with solid metal cylinders to find the densities of aluminum (Al; 2.70 g/cm^3) and brass (an alloy of copper and zinc with density dependent on the constituents' relative proportions; unless otherwise noted, assume 8.44 g/cm^3). You will use three methods to find the cylinders' volumes while maintaining the absolute precision of their mass values at ±0.0001 g. You will assess your results and analyze the three methods in terms of their relative accuracy, precision, and difficulty. Which way of determining volume produced the most accurate densities? Which gave the most precise results? Which was the most difficult experimentally to carry out?

Second, using your most *accurate* experimentally-determined densities for Al and brass, you will apply your conceptual understanding and your experimental know-how to non-destructively determine the size of a void in a hollow metal cylinder and the mass fractions of Al and brass in a mixed (or "plugged") metal cylinder.

As a scientist, you will frequently work with others. Please respect, help, and talk with your colleagues. If you and your partner are assigned to collaborate with another pair of students, it is very important that you share your data and record all of the other pair's values before leaving. You must have a complete data set in order to complete your assignments.

Parts 0 – 3. Density of Al and of brass based on mass and three methods of volume determination

Part 0. Masses of metal cylinders

1. Check out four metal cylinders – solid Al, solid brass, hollow, and mixed ("plugged").

If you are working with another pair of students, make sure you have **either solid Al and hollow OR solid brass and plugged**.

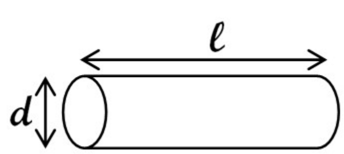
Please return the cylinders to your TA when you are finished with the experiment.

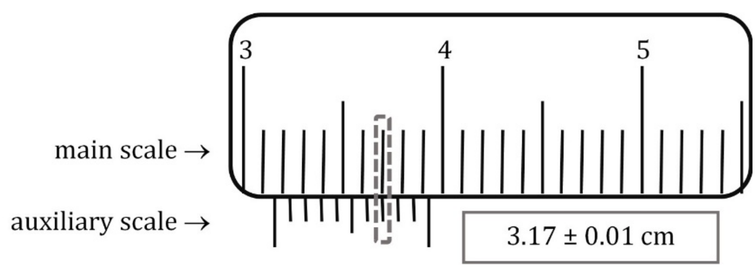
2. Record the cylinders' numbers (S##, S##, H##, P##).

3. Record the cylinders' masses to four decimal places using the analytical balance (σ = ±0.0001 g).

Part 1. Volume by Geometry – Solid Al and Solid Brass Cylinders

4. Measure the diameters and the lengths of the solid cylinders to two decimal places (σ = ±0.01 cm) using the Vernier calipers. The ones and tenths places are read from the main scale. The digit in the hundredths place corresponds to the line on the auxiliary scale that lines up best with a line above it on the main scale.





5. Calculate the density and uncertainty of each solid cylinder, $\rho_{Al} \pm \sigma_{\rho_{Al}}$ and $\rho_{brass} \pm \sigma_{\rho_{brass}}$ (unrounded then correctly rounded).

$$V_{cyl} = \pi r^2 l = \frac{\pi}{4} d^2 l \text{ so } \rho = \frac{m}{V} = \frac{4m}{\pi d^2 l} \text{ and } \sigma_\rho = \pm\rho\left(\frac{\sigma_m}{m} + \frac{\sigma_d}{d} + \frac{\sigma_d}{d} + \frac{\sigma_l}{l}\right)$$

In the correctly rounded values, the uncertainty should have one significant figure and the density should be rounded to that decimal place. For example, 2.648 ± 0.0753 g/cm^3 would be reported as 2.65 ± 0.08 g/cm^3.

Part 2. Volume by Water Displacement– Solid Al and Solid Brass Cylinders

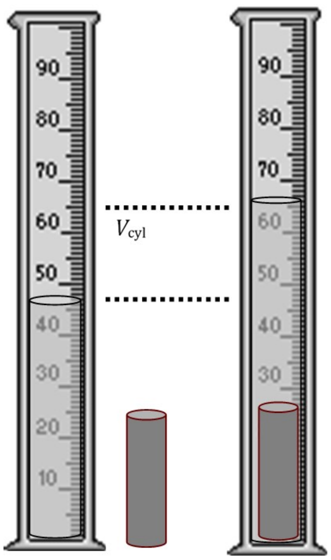
6. Put enough deionized water in a 100-mL graduated cylinder to cover the metal cylinder. Record the volume, V_{in}, based on where the bottom of the meniscus (curved surface of the water) falls on the scale. It is difficult to estimate the volume with an

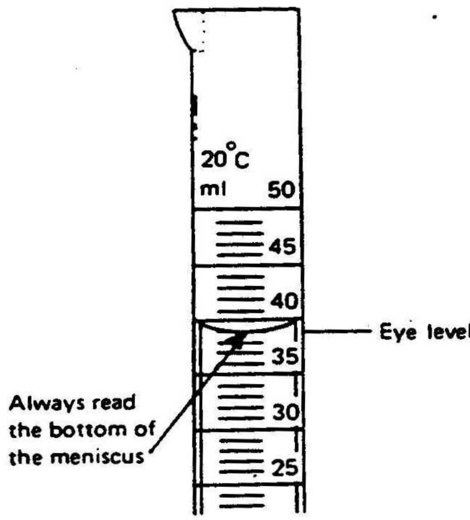
uncertainty better than ±0.5 mL; the bottom of the meniscus appears to be either on a line (##.0 mL) or halfway between two lines (##.5 mL).

7. Tilt the graduated cylinder and **carefully slide the metal cylinder down the side** into the water. With the metal cylinder completely submerged, record the new volume, V_f, to the nearest 0.5 mL.

8. The volume of water displaced equals the volume of the metal cylinder.

$V_{cyl} = V_f - V_{in}$ and $\sigma_V = \pm(\sigma_{V_f} + \sigma_{V_{in}}) = \pm(0.5 + 0.5)\text{mL} = \pm 1\text{ mL}$





9. Calculate the density and uncertainty of each solid cylinder (unrounded then correctly rounded).

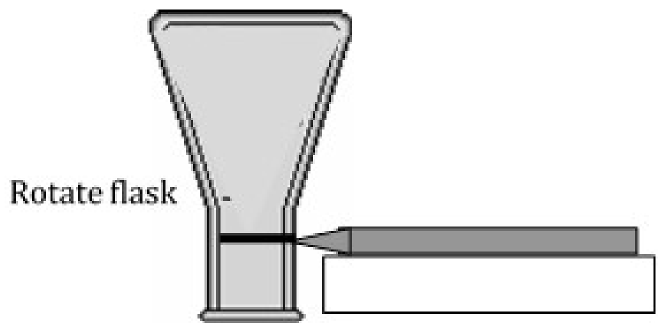
$\rho = \dfrac{m}{V}$ and $\sigma_\rho = \pm\rho\left(\dfrac{\sigma_m}{m} + \dfrac{\sigma_V}{V}\right)$

Part 3. Volume by Pycnometry – Solid Al and Solid Brass Cylinders

10. Fill a 400-mL beaker with deionized water and allow it to come to room temperature. Record this temperature to 1 decimal place.

11. Make your pycnometer.

Invert a 50-mL Erlenmeyer flask on the bench. Set a waterproof marker on something solid and hold it in place. Rotate the flask so that the marker makes a line around its neck.



12. Calibrate your pycnometer.

Your pycnometer's uncertainty is determined by how well you can fill the pycnometer with water to the exactly same level each time you use it.

The pycnometer filled with water to the mark is denoted as "A"; its mass is m_A.

Fill the pycnometer to the mark from the room temperature water in your beaker. Use a disposable pipet to adjust the water level so that the bottom of the meniscus falls on the top of the line. Record the mass, m_{A_1}, on the analytical balance.

Empty the water back into the beaker and repeat the process three additional times. Be as precise as possible when adjusting the water level.

You should have four similar m_A values, each with four decimal places. Calculate the average and standard deviation. Keep four decimal places in your average but round the standard deviation to one significant figure.

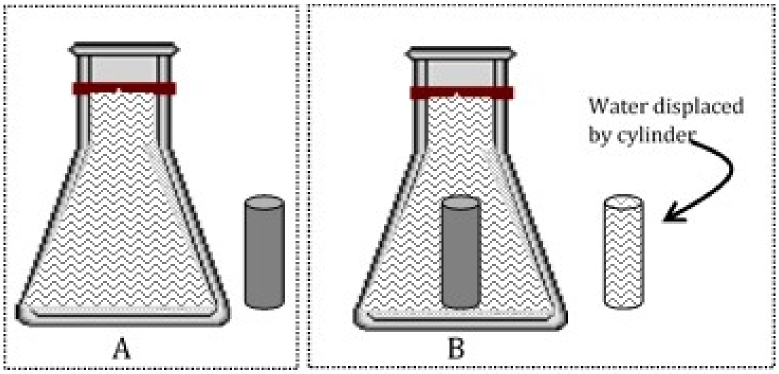
Note: if you are working with another pair of students, make sure to record their four m_A values. Label which pycnometer data is to be used with which metal cylinders.

Solid Al and hollow: pycnometer #1 with $\overline{m_{A_{\#1}}} \pm \sigma_{m_{A_{\#1}}}$
Solid brass and plugged: pycnometer #2 with $\overline{m_{A_{\#2}}} \pm \sigma_{m_{A_{\#2}}}$

13. Use your pycnometer to determine $\rho_{Al} \pm \sigma_{\rho_{Al}}$ and $\rho_{brass} \pm \sigma_{\rho_{brass}}$.

The pycnometer containing the metal cylinder and filled with water to the mark is denoted as "B"; its mass is m_B.

Carefully insert the metal cylinder into the pycnometer and then fill to the mark with water from your beaker. Adjust the water level as you did during calibration. Record the mass, m_B, on the analytical balance to four decimal places.



$$\overline{m_{A\,\#1or\#2}} + m_{cyl} = m_B + m_{displaced\ water}$$

Calculate the volume of the metal cylinder and its uncertainty based on your mass measurements.

In B, the metal cylinder occupies volume that had been taken up by water. The cylinder's volume equals that of the displaced water (the volume of water that is excluded when the metal cylinder is in the filled pycnometer).

$$m_{displaced\ water} = \overline{m_{A\#1\ or\ \#2}} + m_{cyl} - m_B$$

$$\sigma_{m_{displaced\,water}} = \pm\left(\sigma_{m_{A\#1or\#2}} + \sigma_{m_{cyl}} + \sigma_{m_B}\right) = \pm\left(\sigma_{m_{A\#1or\#2}} + \sigma_{m_{cyl}} + \sigma_{m_{A\#1or\#2}}\right)$$

$$V_{displaced\ water} = \frac{m_{displaced\ water}}{\rho_{water\ at\ temp\ T}} = V_{cyl}$$

$$\sigma_{V_{displaced\,water}} = \pm\left(\frac{\sigma_{m_{displaced\,water}}}{\rho_{water\ at\ temp\ T}}\right) = \sigma_{V_{cyl}}$$

Note: The density of water is assumed to contribute negligible uncertainty because ρ_{water} as a function of temperature is known quite precisely.

Use the correct value for ρ_{water} (to six decimal places) from the table provided at the end of this procedure. The values are temperature dependent. Whole degrees are listed down the left hand side of the table1, while tenths of a degree are listed across the top. For example, to find the density of water at 5.4°C, look for the whole degree by searching down the left-hand column to '5'. Then, slide across that row to '0.4'. The density of water at 5.4°C is 0.999957 g/mL.

14. Calculate the density and uncertainty of each solid cylinder (unrounded then correctly rounded).

$$\rho = \frac{m}{V} \text{ and } \sigma_\rho = \pm\rho\left(\frac{\sigma_m}{m} + \frac{\sigma_V}{V}\right)$$

You can calculate the percent error associated with each of your densities.

$$\% \; error = \left| \frac{literature\ value - experimental\ value}{literature\ value} \right| \times 100\%$$

Parts 4 – 5. Volume of a void in a hollow metal cylinder and the mass fractions of Al and brass in a mixed (plugged) metal cylinder by pycnometry

In parts 4 and 5, you will use experimental values for ρ_{Al} and ρ_{brass} (to two decimal places). Choose the values that are closest to the literature values (2.70 g/cm^3 for Al or 8.44 g/cm^3 for brass). If two experimental densities differ from the literature value by the same amount, use the higher density.

Part 4. Volume of a void (empty space) in a hollow cylinder

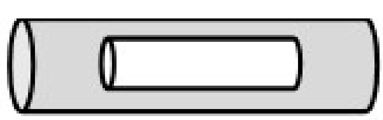
15. Record whether the hollow cylinder is Al (1) or brass (2).

16. Determine the volume of the hollow cylinder, V_{hollow}, by pycnometry (pycnometer #1 with $\overline{m_{A\#1}}$).

Make sure to record $m_{B,hollow}$ to four decimal places. You can keep four decimal places in V_{hollow} because it is an “intermediate” value in your calculations.

17. Calculate V_{void}.

V_{hollow} is the sum of the volume occupied by metal, V_{metal}, and the volume of the empty space inside it, V_{void}. V_{hollow} is found from your pycnometry data, and V_{metal} can be calculated from the cylinder’s mass and the metal’s density.



$$V_{hollow} = V_{metal} + V_{void}$$

$$V_{void} = V_{hollow} - V_{metal} = \frac{\left(\overline{m_{A\#1}} + m_{hollow} - m_{B,hollow}\right)}{\rho_{water\ at\ temp\ T}} - \frac{m_{hollow}}{\rho_{metal}}$$

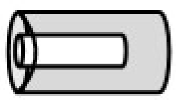
Check the significant figures in V_{void}, but you will likely report it to two decimal places. No error propagation is required.

Part 5. Mass fractions of Al and brass in a mixed metal (plugged) cylinder

18. Determine the density of the mixed cylinder, ρ_{mixed}, by pycnometry (pycnometer #2 with $\overline{m_{A\#2}}$).

Make sure to record $m_{B,mixed}$ to four decimal places. You can keep extra decimal places in ρ_{mixed} because it is an "intermediate" value in your calculations.

$$V_{mixed} = \frac{\left(\overline{m_{A\#2}} + m_{mixed} - m_{B,mixed}\right)}{\rho_{water\ at\ temp\ T}} \quad \text{and} \quad \rho_{mixed} = \frac{m_{mixed}}{V_{mixed}}$$



χ_{Al} and $\chi_{brass} = 1 - \chi_{Al}$

19. Calculate the mass fraction of Al, χ_{Al}.

The volume of the mixed cylinder is the sum of the volumes occupied by the two metals. The cylinder's mass is due to the masses of the two metals in it. Let χ_{Al} represent the fraction of the cylinder's mass that is Al; 1 – χ_{Al} is the fraction of the mass due to brass (your significant figures depend on your experimental values).

$$V_{mixed} = V_{Al} + V_{brass} = \frac{m_{Al}}{\rho_{Al}} + \frac{m_{brass}}{\rho_{brass}} = \frac{\chi_{Al} \cdot m_{mixed}}{\rho_{Al}} + \frac{(1 - \chi_{Al}) \cdot m_{mixed}}{\rho_{brass}}$$

Divide through by m_{mixed}, replace V_{mixed}/m_{mixed} with $1/\rho_{mixed}$, and solve for χ_{Al} and 1 – χ_{Al}. Report the mass fractions to the correct number of significant figures (probably two decimal places). No error propagation is required.

$$\chi_{Al} = mass\ fraction\ of\ Al = \frac{\frac{1}{\rho_{mixed}} - \frac{1}{\rho_{brass}}}{\frac{1}{\rho_{Al}} - \frac{1}{\rho_{brass}}} \quad \text{and} \quad 1 - \chi_{Al} = mass\ fraction\ of\ brass$$

Please return the metal cylinders to your TA.
Dispose of the Pasteur pipet by carefully putting it in a broken glass box (not the trash).
Clean up your work space and return anything that you borrowed from the reagent bench.

Results / Sample Calculations

Complete the online inlab assignment or write a lab report as directed by your TA.

Solid metal cylinders – masses, volumes, and densities by each method, including uncertainties

%errors relative to literature values of ρ_{Al} = 2.70 g/cm^3 and ρ_{brass} = 8.44 g/cm^3 (depends on the alloy's composition)
Void volume in the hollow cylinder
Mass fractions of Al and brass in the mixed cylinder

Discussion Questions and Review Topics

What did you find (refer to results tables) and how did you do it for all 5 parts?
What were the major experimental sources of error?
Compare the three methods used to determine volume - which method was the most accurate and why? Which was most precise and why? Did your results match what you predicted based on the procedure? What could be done to improve the precision in any or all of the methods?
How does uncertainty associated with an instrument compare to uncertainty based on standard deviation?