2.11 Derivation of Kepler's Laws from Newton's Laws

PRE-LECTURE READING 2.11

- *Astronomy Today,* 8^{th} Edition (Chaisson & McMillan)
- *Astronomy Today,* 7^{th} Edition (Chaisson & McMillan)
- *Astronomy Today,* 6^{th} Edition (Chaisson & McMillan)

VIDEO LECTURE

- Derivation of Kepler's Laws from Newton's Laws[1] (23:56)

SUPPLEMENTARY NOTES

Derivation of Kepler's Laws from Newton's Laws

- Kepler's Laws (KI, KII, and KIII) can be derived from Newton's Laws using calculus (which Newton also invented).
- For the **special case** of an object of mass, m, in a **circular** orbit around an object of mass, M, where $\boldsymbol{m}$ **is negligible compared to** $\boldsymbol{M}$, KIII can be derived from Newton's Laws using only algebra.
- Additionally, KIII's constant of proportionality, and why it varies from central object to central object, can be determined.

Newton's Form of KI

> The orbital path of one world around another is elliptical (not circular), with the system's center of mass at one focus.

- This reduces to KI, with the central mass, M, at this focus, when the orbiting mass, m, is negligible compared to M.

Newton's Form of KII

> An imaginary line connecting the center of mass to either world sweeps out equal areas of that world's ellipse in equal intervals of time.

[1] http://youtu.be/zM2fqGsvwFQ

- This reduces to KII, with the imaginary line connecting the central mass, M, to the orbiting mass, m, when m is negligible compared to M.

Newton's Form of KIII

$$P^2 = \left[\frac{4\pi^2}{(M+m)}\right] \times a^3 \tag{14}$$

- This reduces to KIII, except with the constant of proportionality determined, when the orbiting mass, m, is negligible compared to the central mass, M:

$$P^2 \approx \left(\frac{4\pi^2}{M}\right) \times a^3 \tag{15}$$

- The constant of proportionality depends on the central mass, M, which explains why Kepler found different constants of proportionality for the naked-eye planets orbiting the sun, for the Galilean moons orbiting Jupiter, and for Earth's moon orbiting Earth.

Circular Speed

$$v_c = \left(\frac{GM}{r}\right)^{1/2} \tag{16}$$

- This is the speed that an object must attain to enter into a circular orbit around a mass, M, at a distance, r, from it.

Escape Speed

$$v_{\text{esc}} = \left(\frac{2GM}{r}\right)^{1/2} \tag{17}$$

- This is the speed that an object must attain to escape the gravitational field of a mass, M, by the time that it is a distance, r, from it.
- If escaping from the surface of a spherical world of radius, R, and if the escape speed can be attained quickly, $r \approx R$.
- v_{esc} is only 41% greater than v_c.

LAB LINK

Material presented in this unit is related to material presented in Lab 3 of *Astronomy 101 Laboratory: Our Place in Space*[2].

[2] http://skynet.unc.edu/introastro/ourplaceinspace/

In *Lab 3: Galilean Revolution,* we:

- Measure a moon's orbit around a planet.
- Use this information to measure the mass of the planet.
- Measure the phase and angular diameter of Venus.
- Use this information to distinguish between the geocentric and heliocentric models of the universe.

Video Lab Summary

- Galilean Revolution[3] (23:12)

EXERCISE

- Experiment with UNL's Earth Orbit Plot[4].

[3] http://youtu.be/rolUwut4pX0
[4] http://astro.unl.edu/classaction/animations/renaissance/earthorbitplot.html