Chapter 19

Sinusoidal Functions

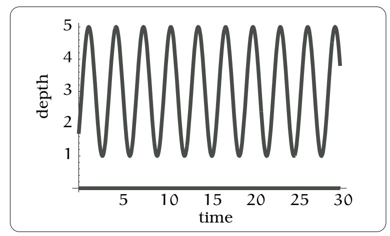

Figure 19.1: The depth of a salmon as a function of time.

A migrating salmon is heading up a portion of the Columbia River. It's depth $d(t)$ *(in feet) below the water surface is measured and plotted over a 30 minute period, as a function of time* t *(minutes). What is the formula for* $d(t)$*?*

In order to answer the question, we need to introduce an important new family of functions called the *sinusoidal functions*. These functions will play a central role in modeling any kind of periodic phenomena. The amazing fact is that almost any function you will encounter can be approximated by a sum of sinusoidal functions; a result that has far-reaching implications in all of our lives.

19.1 A special class of functions

Beginning with the trigonometric function $y = \sin(x)$*, what is the most general function we can build using the graphical techniques of shifting and stretching?*

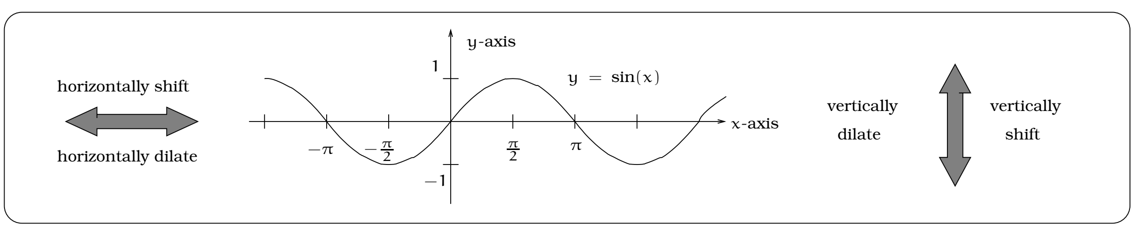

Figure 19.2: Visualizing the geometric operations available for curve sketching.

The graph of $y = \sin(x)$ can be manipulated in four basic ways: horizontally shift, vertically shift, horizontally dilate or vertically dilate. Each of these "geometric operations" corresponds to a simple change in the "symbolic formula" for the function, as discussed in Chapter 13.

If we vertically shift the graph by D units upward, the resulting curve would be the graph of the function $y = \sin(x) + D$; see Facts 13.3.1. Recall, the effect of the sign of D: If D is negative, the effect of shifting D units upward is the same as shifting $|D|$ units downward. Notice, the function $y = \sin(x) + D$ is still a periodic function, having the same period 2π as $y = \sin(x)$. Notice, whereas the graph of the function $y = \sin(x)$ oscillates between the horizontal lines $y = \pm 1$, the graph of $y = \sin(x) + D$ oscillates between $y = D \pm 1$. For this reason, we sometimes refer to the constant D as the *mean* of the function $y = \sin(x) + D$. In Figure 19.3, notice that the graph of $y = \sin(x) + D$ is symmetrically split by the horizontal "mean" line $y = D$.

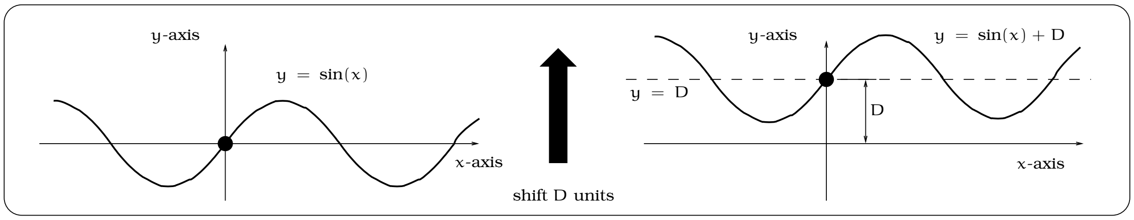

Figure 19.3: Interpreting the mean.

Next, consider the effect of horizontally shifting the graph of $y = \sin(x)$ by C units to the right. By Facts 13.3.1, the new curve is the graph of the function $y = \sin(x - C)$. Also, recall the effect of the sign of C: If C is negative, the effect of shifting C units right is the same as shifting $|C|$ units left. If the domain of $\sin(x)$ is $0 \leq x \leq 2\pi$, then the domain of $\sin(x - C)$ is $0 \leq x - C \leq 2\pi$, again by Facts 13.3.1. Rewriting this, the domain of $\sin(x - C)$ is $C \leq x \leq 2\pi + C$ and the graph will go through precisely one period on this domain. In other words, the new function $\sin(x - C)$ is still 2π-periodic. The constant C is usually called the *phase shift* of $y = \sin(x - C)$. Looking at Figure 19.4, it is possible to interpret C graphically: C will be a point where the graph crosses the horizontal axis *on its way up* from a minimum to a maximum.

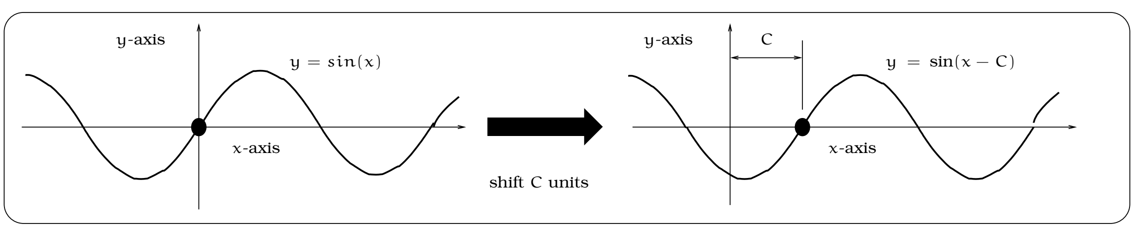

Figure 19.4: Interpreting the phase shift.

Vertically dilating the graph, either by vertical expansion or compression, leads to a new curve. The graph of this vertically dilated curve is $y = A\sin(x)$, for some positive constant A. Furthermore, if $A > 1$, the graph of $y = A\sin(x)$ is a vertically expanded version of $y = \sin(x)$, whereas, if $0 < A < 1$, then the graph of $y = A\sin(x)$ is a vertically compressed version of $y = \sin(x)$. Notice, the function $y = A\sin(x)$ is still 2π-periodic. What has changed is the band of oscillation: whereas the graph of the function $y = \sin(x)$ stays between the horizontal lines $y = \pm 1$, the graph of $y = A\sin(x)$ oscillates between the horizontal lines $y = \pm A$. We usually refer to A as the *amplitude* of the function $y = A\sin(x)$.

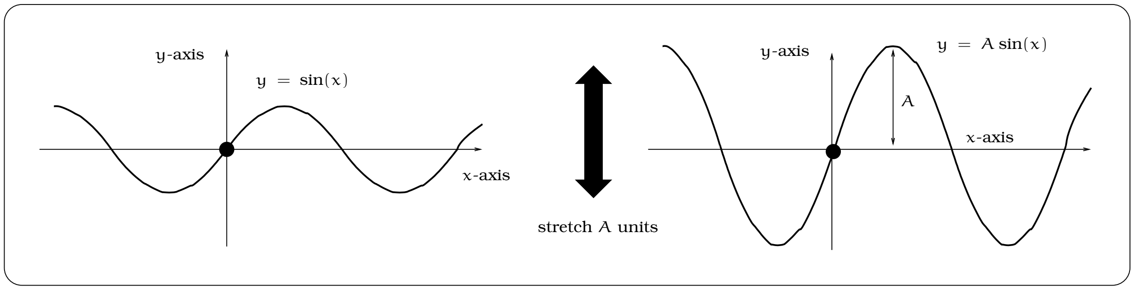

Figure 19.5: Interpreting the amplitude.

Finally, horizontally dilating the graph, either by horizontal expansion or compression, leads to a new curve. The equation of this horizontally dilated curve is $y = \sin(cx)$, for some constant $c > 0$. We know that $y = \sin(x)$ is a 2π-periodic function and observe that horizontally dilation still results in a periodic function, but the period will typically NOT be 2π. For future purposes, it is useful to rewrite the equation for the horizontally stretched curve in a way more directly highlighting the period. To begin with, once the horizontal stretching factor c is known, we could rewrite

$$c = \frac{2\pi}{B}, \quad \text{for some } B \neq 0.$$

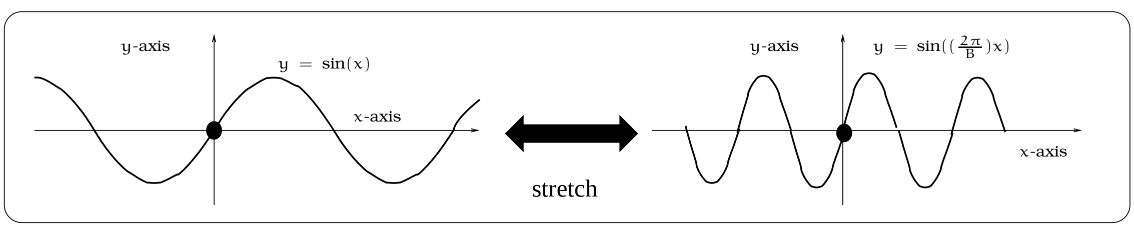

Figure 19.6: Interpreting the period.

Here is the point of this yoga with the horizontal dilating constant: If we let the values of x range over the interval $[0, B]$, then $\frac{2\pi}{B}x$ will range over the interval $[0,2\pi]$. In other words, the function $y = \sin\left(\frac{2\pi}{B}x\right)$ is B-periodic and we can read off the *period* of $y = \sin\left(\frac{2\pi}{B}x\right)$ by viewing the constant in this mysterious way. The four constructions outlined lead to a new family of functions.

Definition 19.1.1 (The Sinusoidal Function)**.** *Let* A, B, C *and* D *be fixed constants, where* A *and* B *are both positive. Then we can form the new function*

$$y = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D,$$

which is called a sinusoidal function. The four constants can be interpreted graphically as indicated:

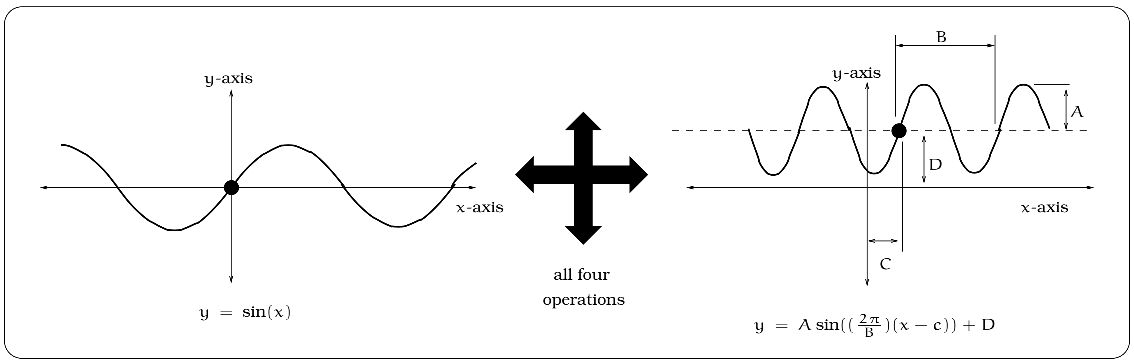

Figure 19.7: Putting it all together for the sinusoidal function.

19.1.1 How to roughly sketch a sinusoidal graph

Important Procedure 19.1.2. *Given a sinusoidal function in the standard form*

$$y = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D,$$

once the constants A, B, C, *and* D *are specified, any graphing device can produce an accurate graph. However, it is pretty straightforward to sketch a rough graph by hand and the process will help reinforce the graphical meaning of the constants* A, B, C, *and* D*. Here is a "five step procedure" one can follow, assuming we are given* A, B, C, *and* D*. It is a good idea to follow Example 19.1.3 as you read this procedure; that way it will seem a lot less abstract.*

1. *Draw the horizontal line given by the equation* $y = D$*; this line will "split" the graph of* $y = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D$ *into symmetrical upper and lower halves.*

2. *Draw the two horizontal lines given by the equations* $y = D \pm A$*. These two lines determine a horizontal strip inside which the graph of the*

sinusoidal function will oscillate. Notice, the points where the sinusoidal function has a maximum value lie on the line $y = D + A$*. Likewise, the points where the sinusoidal function has a minimum value lie on the line* $y = D - A$*. Of course, we do not yet have a prescription that tells us where these maxima (peaks) and minima (valleys) are located; that will come out of the next steps.*

3. *Since we are given the period* B*, we know these important facts: (1) The period* B *is the horizontal distance between two successive maxima (peaks) in the graph. Likewise, the period* B *is the horizontal distance between two successive minima (valleys) in the graph. (2) The horizontal distance between a maxima (peak) and the successive minima (valley) is* $\frac{1}{2}B$*.*

4. *Plot the point* (C, D)*. This will be a place where the graph of the sinusoidal function will cross the mean line* $y = D$ *on its way up from a minima to a maxima. This is not the only place where the graph crosses the mean line; it will also cross at the points obtained from* (C, D) *by horizontally shifting by any integer multiple of* $\frac{1}{2}B$*. For example, here are three places the graph crosses the mean line:* $(C, D), (C + \frac{1}{2}B, D), (C + B, D)$

5. *Finally, midway between* (C, D) *and* $(C+\frac{1}{2}B, D)$ *there will be a maxima (peak); i.e. at the point* $(C + \frac{1}{4}B, D + A)$*. Likewise, midway between* $(C + \frac{1}{2}B, D)$ *and* $(C + B, D)$ *there will be a minima (valley); i.e. at the point* $(C + \frac{3}{4}B, D - A)$*. It is now possible to roughly sketch the graph on the domain* $C \leq x \leq C + B$ *by connecting the points described. Once this portion of the graph is known, the fact that the function is periodic tells us to simply repeat the picture in the intervals* $C + B \leq x \leq C + 2B$*,* $C - B \leq x \leq C$*, etc.*

To make sense of this procedure, let's do an explicit example to see how these five steps produce a rough sketch.

Example 19.1.3. *The temperature (in °C) of Adri-N's dorm room varies during the day according to the sinusoidal function* $d(t) = 6\sin\left(\frac{\pi}{12}(t-11)\right) + 19$, *where* t *represents hours after midnight. Roughly sketch the graph of* $d(t)$ *over a 24 hour period.. What is the temperature of the room at 2:00 pm? What is the maximum and minimum temperature of the room?*

Solution. We begin with the rough sketch. Start by taking an inventory of the constants in this sinusoidal function:

$$d(t) = 6\sin\left(\frac{\pi}{12}(t-11)\right) + 19 = A\sin\left(\frac{2\pi}{B}(t-C)\right) + D.$$

Conclude that $A = 6$, $B = 24$, $C = 11$, $D = 19$. Following the first four steps of the procedure outlined, we can sketch the lines $y = D = 19$, $y = D \pm A = 19 \pm 6$ and three points where the graph crosses the mean line (see Figure19.8).

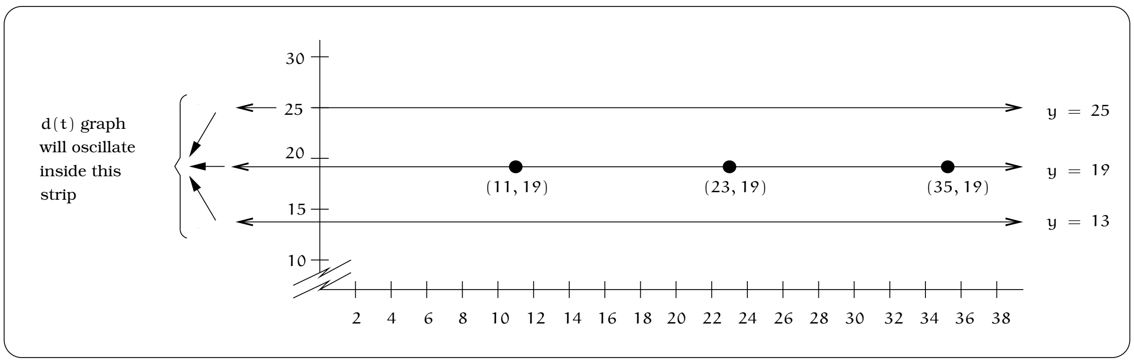

Figure 19.8: Sketching the mean D and amplitude A.

According to the fifth step in the sketching procedure, we can plot the maxima $(C + \frac{1}{4}B, D + A) = (17, 25)$ and the minima $(C + \frac{3}{4}B, D - A) = (29, 13)$. We then "connect the dots" to get a rough sketch on the domain $11 \leq t \leq 35$.

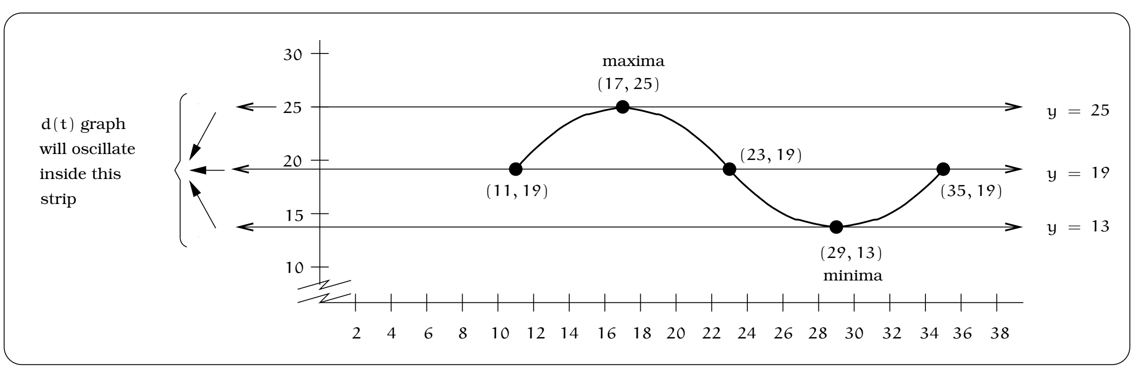

Figure 19.9: Visualizing the maximum and minimum over one period.

Finally, we can use the fact the function has period 24 to sketch the graph to the right and left by simply repeating the picture every 24 horizontal units.

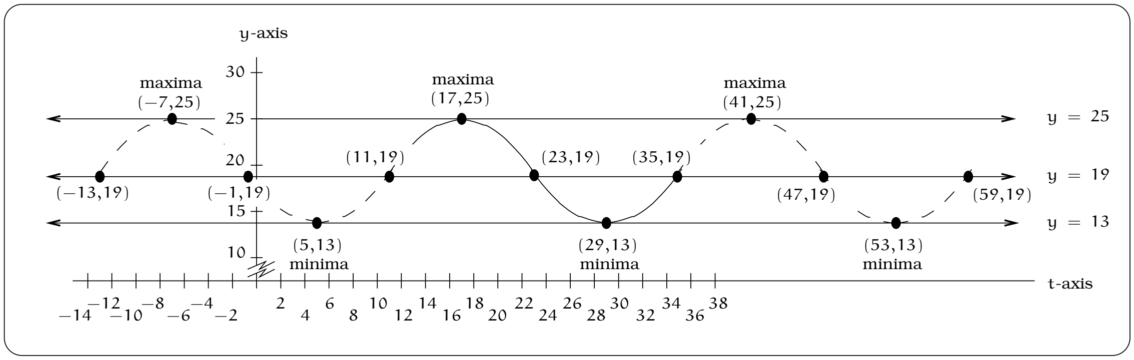

Figure 19.10: Repeat sketch for every full period.

We restrict the picture to the domain $0 \leq t \leq 24$ and obtain the computer generated graph pictured in Figure 19.11; as you can see, our rough graph is very accurate. The temperature at 2:00 p.m. is just $d(14) = 23.24^\circ$ C. From the graph, the maximum value of the function will be $D + A = 25^\circ$ C and the minimum value will be $D - A = 13^\circ$ C.

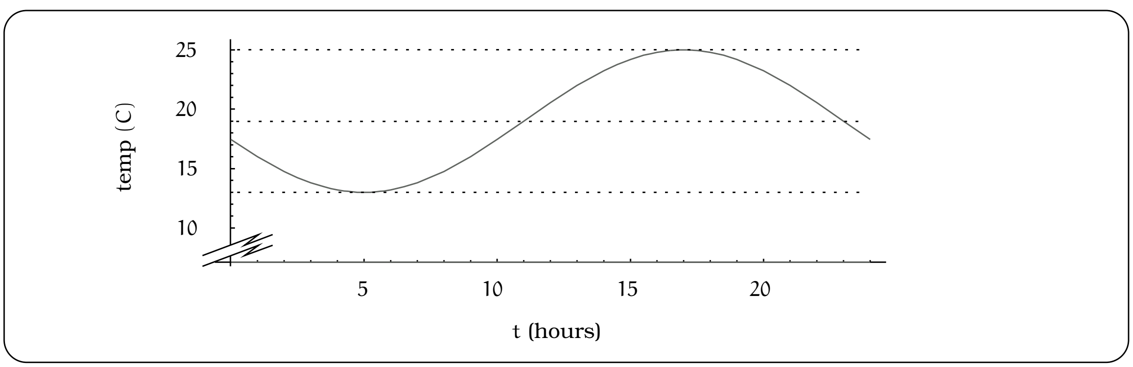

Figure 19.11: The computer generated solution.

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19.1.2 Functions not in standard sinusoidal form

Any time we are given a trigonometric function written in the standard form

$$y = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D,$$

for constants A, B, C, and D (with A and B positive), the summary in Definition 19.1.1 tells us everything we could possibly want to know about the graph. But, there are two ways in which we might encounter a trigonometric type function that is not in this standard form:

- The constants A or B might be negative. For example, $y = -2\sin(2x-7) - 3$ and $y = 3\sin\left(-\frac{1}{2}x + 1\right) + 4$ are examples that fail to be in standard form.
- We might use the cosine function in place of the sine function. For example, something like $y = 2\cos(3x + 1) - 2$ fails to be in standard sinusoidal form.

Now what do we do? Does this mean we need to repeat the analysis that led to Definition 19.1.1? It turns out that if we use our trig identities just right, then we can move any such equation into standard form and read off the amplitude, period, phase shift and mean. In other words, equations that fail to be in standard sinusoidal form for either of these two reasons will still define sinusoidal functions. We illustrate how this is done by way of some examples:

Examples 19.1.4.

(i) *Start with* $y = -2\sin(2x-7)-3$, *then here are the steps with reference to the required identities to put the equation in standard form:*

$$\begin{aligned}
y &= -2\sin(2x-7) - 3 \\
&= 2\left(-\sin(2x-7)\right) - 3 \\
&= 2\sin(2x - 7 + \pi) + (-3) && \textit{Fact 18.2.5 on page 241} \\
&= 2\sin\left(\frac{2\pi}{\pi}\left(x - \left[\frac{7-\pi}{2}\right]\right)\right) + (-3).
\end{aligned}$$

This function is now in the standard form of Definition 19.1.1, so it is a sinusoidal function with phase shift $C = \frac{7-\pi}{2} = 1.93$, *mean* $D = -3$, *amplitude* $A = 2$ *and period* $B = \pi$.

(ii) *Start with* $y = 3\sin(-\frac{1}{2}x+1)+4$, *then here are the steps with reference to the required identities to put the equation in standard form:*

$$\begin{aligned}
y &= 3\sin\left(-\frac{1}{2}x + 1\right) + 4 \\
&= 3\sin\left(-\left(\frac{1}{2}x - 1\right)\right) + 4 \\
&= 3\left(-\sin\left(\frac{1}{2}x - 1\right)\right) + 4 && \textit{Fact 18.2.4 on page 241} \\
&= 3\sin\left(\frac{1}{2}x - 1 + \pi\right) + 4 && \textit{Fact 18.2.5 on page 241} \\
&= 3\sin\left(\frac{2\pi}{4\pi}(x - [2 - 2\pi])\right) + 4
\end{aligned}$$

This function is now in the standard form of Definition 19.1.1, so it is a sinusoidal function with phase shift $C = 2 - 2\pi$*, mean* $D = 4$*, amplitude* $A = 3$ *and period* $B = 4\pi$*.*

(iii) Start with $y = 2\cos(3x+1)-2$*, then here are the steps to put the equation in standard form. A key simplifying step is to use the identity:* $\cos(t) = \sin(\frac{\pi}{2} + t)$*.*

$$\begin{aligned} y &= 2\cos(3x+1) - 2 \\ &= 2\sin\left(\frac{\pi}{2} + 3x + 1\right) - 2 \\ &= 2\sin\left(3x - \left[-1 - \frac{\pi}{2}\right]\right) + (-2) \\ &= 2\sin\left(\frac{2\pi}{\left(\frac{2\pi}{3}\right)}\left(x - \frac{1}{3}\left[-1 - \frac{\pi}{2}\right]\right)\right) + (-2) \end{aligned}$$

This function is now in the standard form of Definition 19.1.1, so it is a sinusoidal function with phase shift $C = \frac{1}{3}[-1 - \frac{\pi}{2}]$*, mean* $D = -2$*, amplitude* $A = 2$ *and period* $B = \frac{2\pi}{3}$*.*

19.2 Examples of sinusoidal behavior

Problems involving sinusoidal behavior come in two basic flavors. On the one hand, we could be handed an explicit sinusoidal function

$$y = A\sin\left(\frac{2\pi}{B}(x - C)\right) + D$$

and asked various questions. The answers typically require either direct calculation or interpretation of the constants. Example 19.1.3 is typical of this kind of problem. On the other hand, we might be told a particular situation is described by a sinusoidal function and provided some data or a graph. In order to further analyze the problem, we need a “formula”, which means finding the constants A, B, C, and D. This is a typical scenario in a “mathematical modeling problem”: the process of observing data, THEN obtaining a mathematical formula. To find A, take half the difference between the largest and smallest values of $f(x)$. The period B is most easily found by measuring the distance between two successive maxima (peaks) or minima (valleys) in the graph. The mean D is the average of the largest and smallest values of $f(x)$. The phase shift C (which is usually the most tricky quantity to get your hands on) is found by locating a “reference point”. This “reference point” is a location where the graph crosses the mean line $y = D$ *on its way up* from a minimum to a maximum. The funny thing is that the phase shift C is NOT unique; there are an infinite number of correct choices. One choice that will work is $C = (x\text{-coordinate of a maximum}) - \frac{B}{4}$. Any other choice of C will differ from this one by a multiple of the period B.

$$\begin{aligned}
A &= \frac{\text{max value } - \text{min value}}{2} \\
B &= \text{distance between two successive peaks (or valleys)} \\
C &= x\text{-coordinate of a maximum} - \frac{B}{4} \\
D &= \frac{\text{max value } + \text{min value}}{2}.
\end{aligned}$$

Example 19.2.1. *Assume that the number of hours of daylight in Seattle is given by a sinusoidal function* $d(t)$ *of time. During 1994, assume the longest day of the year is June 21 with 15.7 hours of daylight and the shortest day is December 21 with 8.3 hours of daylight. Find a formula* $d(t)$ *for the number of hours of daylight on the* t^{th} *day of the year.*

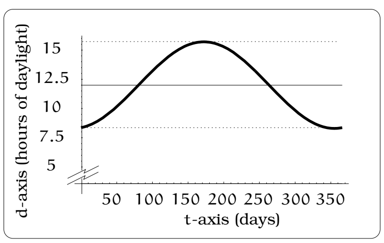


Figure 19.12: Hours of daylight in Seattle in 1994.

Solution. Because the function $d(t)$ is assumed to be sinusoidal, it has the form $y = A\sin\left(\frac{2\pi}{B}(t - C)\right) + D$, for constants A, B, C, and D. We simply need to use the given information to find these constants. The largest value of the function is 15.7 and the smallest value is 8.3. Knowing this, from the above discussion we can read off :

$$D = \frac{15.7 + 8.3}{2} = 12 \qquad A = \frac{15.7 - 8.3}{2} = 3.7.$$

To find the period, we need to compute the time between two successive maximum values of $d(t)$. To find this, we can simply double the time length of one-half period, which would be the length of time between successive maximum and minimum values of $d(t)$. This gives us the equation

$$B = 2(\text{days between June 21 and December 21}) = 2(183) = 366.$$

Locating the final constant C requires the most thought. Recall, the longest day of the year is June 21, which is day 172 of the year, so

$$C = (\text{day with max daylight}) - \frac{B}{4} = 172 - \frac{366}{4} = 80.5.$$

In summary, this shows that

$$d(t) = 3.7\sin\left(\frac{2\pi}{366}(t - 80.5)\right) + 12.$$

A rough sketch, following the procedure outlined above, gives this graph on the domain $0 \leq t \leq 366$; we have included the mean line $y = 12$ for reference. $\square$

We close with the example that started this section.

Example 19.2.2. *The depth of a migrating salmon below the water surface changes according to a sinusoidal function of time. The fish varies between 1 and 5 feet below the surface of the water. It takes the fish 1.571 minutes to move from its minimum depth to its successive maximum depth. It is located at a maximum depth when* $t = 4.285$ *minutes. What is the formula for the function* $d(t)$ *that predicts the depth of the fish after* t *minutes? What was the depth of the salmon when it was first spotted? During the first 10 minutes, how many times will the salmon be exactly 4 feet below the surface of the water?*

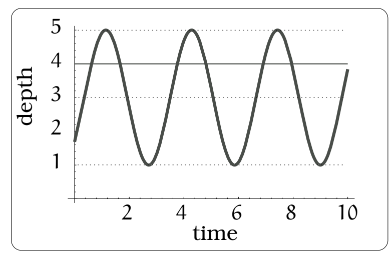


Figure 19.13: Depth of a migrating salmon.

Solution. We know that $d(t) = A\sin(\frac{2\pi}{B}(x - C)) + D$, for appropriate constants A, B, C, and D. We need to use the given information to extract these four constants. The amplitude and mean are easily found using the above formulas:

$$A = \frac{\text{max depth } - \text{min depth}}{2} = \frac{5-1}{2} = 2$$
$$D = \frac{\text{max depth } + \text{min depth}}{2} = \frac{5+1}{2} = 3.$$

The period can be found by noting that the information about the time between a successive minimum and maximum depth will be half of a period (look at the picture in Figure 19.13):

$$B = 2(1.571) = 3.142$$

Finally, to find C we

$$C = \text{(time of maximum depth)} - \frac{B}{4} = 4.285 - \frac{3.142}{4} = 3.50.$$

The formula is now

$$d(t) = 2\sin\left(\frac{2\pi}{3.142}(t - 3.5)\right) + 3 = 2\sin(2t - 7) + 3$$

The depth of the salmon when it was first spotted is just

$$d(0) = 2\sin(-7) + 3 = 1.686 \text{ feet}.$$

Finally, graphically, the last question amounts to determining how many times the graph of $d(t)$ crosses the line $y = 4$ on the domain [0,10]. This can be done using Figure 19.13. A simultaneous picture of the two graphs is given, from which we can see the salmon is exactly 4 feet below the surface of the water six times during the first 10 minutes.

□

19.3 Summary

- A **sinusoidal** function is one of the form

  $$f(t) = A \sin\left(\frac{2\pi}{B}(t - C)\right) + D$$

  where A, B, C, and D are constants.

  - A is the *amplitude* of the function; this is half the vertical distance between a high point and a low point on its graph.
  - B is the *period* of the function; this is the horizontal distance between two consecutive high points (or low points) on its graph.
  - C is the *phase shift* of the function; it is multi-valued, but one choice for C is a value of t at which the function is increasing and equal to D.
  - D is the *mean value* of the function; it is the y-value of the horizontal line about which the graph of the function is balanced.

- The graph of a sinusoidal function is a shifted, scaled version of the graph of $y = \sin t$.

19.4 Exercises

Problem 19.1. Find the amplitude, period, a phase shift and the mean of the following sinusoidal functions.

(a) $y = \sin(2x - \pi) + 1$

(b) $y = 6\sin(\pi x) - 1$

(c) $y = 3\sin(x + 2.7) + 5.2$

(d) $y = 5.6\left(\sin\left(\frac{2}{3}x - 7\right) - 12.1\right)$

(e) $y = 2.1\sin\left(\frac{x}{\pi} + 44.3\right) - 9.8$

(f) $y = 3.9\left(\sin(22.34(x + 18)) - 11\right)$

(g) $y = 11.2\sin\left(\frac{5}{\pi}(x - 9.2)\right) + 8.3$

Problem 19.2. A weight is attached to a spring suspended from a beam. At time $t = 0$, it is pulled down to a point 10 cm above the ground and released. After that, it bounces up and down between its minimum height of 10 cm and a maximum height of 26 cm, and its height $h(t)$ is a sinusoidal function of time t. It first reaches a maximum height 0.6 seconds after starting.

(a) Follow the procedure outlined in this section to sketch a rough graph of $h(t)$. Draw at least two complete cycles of the oscillation, indicating where the maxima and minima occur.

(b) What are the mean, amplitude, phase shift and period for this function?

(c) Give four different possible values for the phase shift.

(d) Write down a formula for the function $h(t)$ in standard sinusoidal form; i.e. as in 19.1.1 on Page 254.

(e) What is the height of the weight after 0.18 seconds?

(f) During the first 10 seconds, how many times will the weight be exactly 22 cm above the floor? (Note: This problem does not require inverse trigonometry.)

Problem 19.3. A respiratory ailment called "Cheyne-Stokes Respiration" causes the volume per breath to increase and decrease in a sinusoidal manner, as a function of time. For one particular patient with this condition, a machine begins recording a plot of volume per breath versus time (in seconds). Let $b(t)$ be a function of time t that tells us the volume (in liters) of a breath that starts at time t. During the test, the smallest volume per breath is 0.6 liters and this first occurs for a breath that starts 5 seconds into the test. The largest volume per breath is 1.8 liters and this first occurs for a breath beginning 55 seconds into the test.

(a) Find a formula for the function $b(t)$ whose graph will model the test data for this patient.

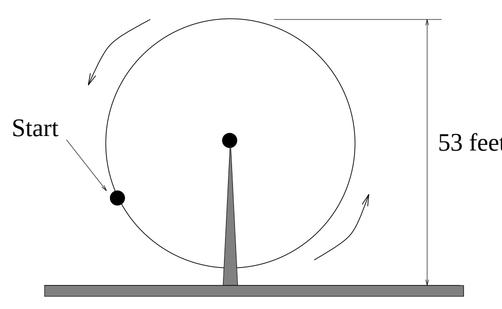
(b) If the patient begins a breath every 5 seconds, what are the breath volumes during the first minute of the test?

Problem 19.4. Suppose the high tide in Seattle occurs at 1:00 a.m. and 1:00 p.m. at which time the water is 10 feet above the height of low tide. Low tides occur 6 hours after high tides. Suppose there are two high tides and two low tides every day and the height of the tide varies sinusoidally.

(a) Find a formula for the function $y = h(t)$ that computes the height of the tide above low tide at time t. (In other words, $y = 0$ corresponds to low tide.)

(b) What is the tide height at 11:00 a.m.?

Problem 19.5. Your seat on a Ferris Wheel is at the indicated position at time $t = 0$.



Let t be the number of seconds elapsed after the wheel begins rotating counterclockwise. You find it takes 3 seconds to reach the top, which is 53 feet above the ground. The wheel is rotating 12 RPM and the diameter of the wheel is 50 feet. Let $d(t)$ be your height above the ground at time t.

(a) Argue that $d(t)$ is a sinusoidal function, describing the amplitude, phase shift, period and mean.

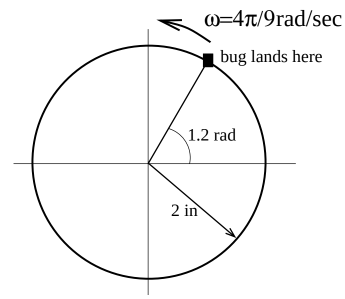
(b) When are the first and second times you are exactly 28 feet above the ground?

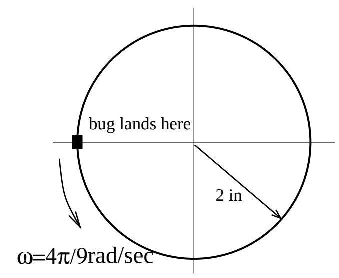
(c) After 29 seconds, how many times will you have been exactly 28 feet above the ground?

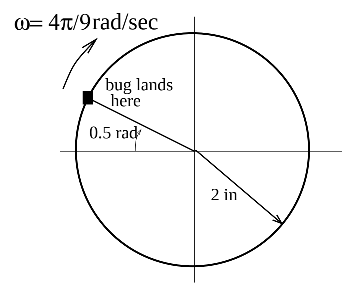
Problem 19.6. In Exercise 17.12, we studied the situation below: A bug has landed on the rim of a jelly jar and is moving around the rim. The location where the bug initially lands is described and its angular speed is given. Impose a coordinate system with the origin at the center of the circle of motion. In each of the cases, the earlier exercise found the coordinates $P(t)$ of the bug at time t. For each of the scenarios below, answer these two questions:

(a) Both coordinates of $P(t) = (x(t), y(t))$ are sinusoidal functions in the variable t. Sketch a rough graph of the functions $x(t)$ and $y(t)$ on the domain $0 \leq t \leq 9$.

(b) Use the graph sketches to help you find the the amplitude, mean, period and phase shift for each function. Write $x(t)$ and $y(t)$ in standard sinusoidal form.







Problem 19.7. The voltage output(in volts) of an electrical circuit at time t seconds is given by the function

$$V(t) = 2^{3\sin(5\pi t - 3\pi)+1}.$$

(a) What is the initial voltage output of the circuit?

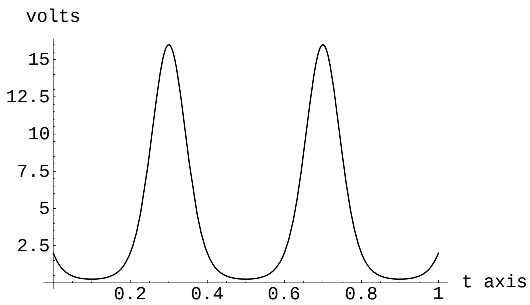
(b) Is the voltage output of the circuit ever equal to zero? Explain.

(c) The function $V(t) = 2^{p(t)}$, where $p(t) = 3\sin(5\pi t - 3\pi) + 1$. Put the sinusoidal function $p(t)$ in standard form and sketch the graph for $0 \leq t \leq 1$. Label the coordinates of the extrema on the graph.

(d) Calculate the maximum and minimum voltage output of the circuit.

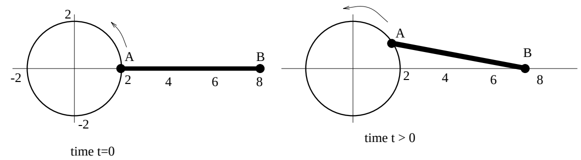
(e) During the first second, determine when the voltage output of the circuit is 10 volts.

(f) A picture of the graph of $y = V(t)$ on the domain $0 \le t \le 1$ is given; label the coordinates of the extrema on the graph.



(g) Restrict the function $V(t)$ to the domain $0.1 \le t \le 0.3$; explain why this function has an inverse and find the formula for the inverse rule. Restrict the function $V(t)$ to the domain $0.3 \le t \le 0.5$; explain why this function has an inverse and find the formula for the inverse rule.

Problem 19.8. A six foot long rod is attached at one end A to a point on a wheel of radius 2 feet, centered at the origin. The other end B is free to move back and forth along the x-axis. The point A is at $(2,0)$ at time $t = 0$, and the wheel rotates counterclockwise at 3 rev/sec.



(a) As the point A makes one complete revolution, indicate in the picture the direction and range of motion of the point B.

(b) Find the coordinates of the point A as a function of time t.

(c) Find the coordinates of the point B as a function of time t.

(d) What is the x-coordinate of the point B when $t = 1$? You should be able to find this two ways: with your function from part (c), and using some common sense (where is point A after one second?).

(e) Is the function you found in (c) a sinusoidal function? Explain.