Chapter 2

Imposing Coordinates

You find yourself visiting Spangle, WA and dinner time is approaching. A friend has recommended Tiff’s Diner, an excellent restaurant; how will you find it?

Of course, the solution to this simple problem amounts to locating a “point” on a two-dimensional map. This idea will be important in many problem solving situations, so we will quickly review the key ideas.

2.1 The Coordinate System

If we are careful, we can develop the flow of ideas underlying two-dimensional coordinate systems in such a way that it easily generalizes to three-dimensions. Suppose we start with a blank piece of paper and mark two points; let’s label these two points “P” and “Q.” This presents the basic problem of finding a foolproof method to reconstruct the picture.

The basic idea is to introduce a coordinate system for the plane (analogous to the city map grid of streets), allowing us to catalog points in the plane using pairs of real numbers (analogous to the addresses of locations in the city).

Here are the details. Start by drawing two perpendicular lines, called the horizontal axis and the vertical axis, each of which looks like a copy of the real number line. We refer to the intersection point of these two lines as the origin. Given P in the plane, the plan is to use these two axes to obtain a pair of real numbers \((x,y)\) that will give us the exact location of P. With this in mind, the horizontal axis is often called the x-axis and the vertical axis is often called the y-axis. Remember, a typical real number line (like the x-axis or the y-axis) is divided into three parts: the positive numbers, the negative numbers, and the number zero (see Figure 2.2(a)). This allows us to specify positive and negative portions of the x-axis and y-axis. Unless we say otherwise, we will always adopt the convention that the positive x-axis consists of those numbers to the right of the origin on
the $x$-axis and the positive $y$-axis consists of those numbers above the origin on the $y$-axis. We have just described the $xy$-coordinate system for the plane:

![Number line and xy-coordinate system](image)

**Figure 2.2:** Coordinates.

### 2.1.1 Going from $P$ to a Pair of Real Numbers.

Imagine a coordinate system had been drawn on our piece of paper in Figure 2.1. Let’s review the procedure of going from a point $P$ to a pair of real numbers:

1. First, draw two new lines passing through $P$, one parallel to the $x$-axis and the other parallel to the $y$-axis; call these $\ell$ and $\ell^*$, as pictured in Figure 2.3.

2. Notice that $\ell$ will cross the $y$-axis exactly once; the point on the $y$-axis where these two lines cross will be called “$y$.” Likewise, the line $\ell^*$ will cross the $x$-axis exactly once; the point on the $x$-axis where these two lines cross will be called “$x$.”

3. If you begin with two different points, like $P$ and $Q$ in Figure 2.1, you will see that the two pairs of points you obtain will be different; i.e., if $Q$ gives you the pair $(x^*,y^*)$, then either $x \neq x^*$ or $y \neq y^*$. This shows that two different points in the plane give two different pairs of real numbers and describes the process of assigning a pair of real numbers to the point $P$.

The great thing about the procedure we just described is that it is reversible! In other words, suppose you start with a pair of real numbers, say $(x,y)$. Locate the number $x$ on the $x$-axis and the number $y$ on the $y$-axis. Now draw two lines: a line $\ell$ parallel to the $x$-axis passing through the number $y$ on the $y$-axis and a line $\ell^*$ parallel to the $y$-axis passing through the number $x$ on the $x$-axis. The two lines $\ell$ and $\ell^*$ will intersect
in exactly one point in the plane, call it $P$. This procedure describes how to go from a given pair of real numbers to a point in the plane. In addition, if you start with two different pairs of real numbers, then the corresponding two points in the plane are going to be different. In the future, we will constantly be going back and forth between points in the plane and pairs of real numbers using these ideas.

**Definition 2.1.1. Coordinate System:** Every point $P$ in the $xy$-plane corresponds to a unique pair of real numbers $(x, y)$, where $x$ is a number on the horizontal $x$-axis and $y$ is a number on the vertical $y$-axis; for this reason, we commonly use the notation $P = (x,y)$.

Having specified positive and negative directions on the horizontal and vertical axes, we can now divide our two dimensional plane into four quadrants. The first quadrant corresponds to all the points where both coordinates are positive, the second quadrant consists of points with the first coordinate negative and the second coordinate positive, etc. Every point in the plane will lie in one of these four quadrants or on one of the two axes. This quadrant terminology is useful to give a rough sense of location, just as we use the terminology “Northeast, Northwest, Southwest and Southeast” when discussing locations on a map.

### 2.2 Three Features of a Coordinate System

A coordinate system involves *scaling*, *labeling* and *units* on each of the axes.

#### 2.2.1 Scaling

Sketch two $xy$ coordinate systems. In the first, make the scale on each axis the same. In the second, assume “one unit” on the $x$ axis has the same length as “two units” on the $y$ axis. Plot the points $(1,1), (-1,1), (-\frac{4}{5}, \frac{16}{25}), (-\frac{3}{5}, \frac{9}{25}), (-\frac{2}{5}, \frac{4}{25}), (-\frac{1}{5}, \frac{1}{25}), (0,0), (\frac{1}{5}, \frac{1}{25}), (\frac{2}{5}, \frac{4}{25}), (\frac{3}{5}, \frac{9}{25}), (\frac{4}{5}, \frac{16}{25}), (1,1)$.

Both pictures illustrate how the points lie on a parabola in the $xy$-coordinate system, but the aspect ratio has changed. The aspect ratio is defined by this fraction:

$$\text{aspect ratio} \overset{\text{def}}{=} \frac{\text{length of one unit on the vertical axis}}{\text{length of one unit on the horizontal axis}}.$$ 

Figure 2.5(a) has aspect ratio 1, whereas Figure 2.5(b) has aspect ratio $\frac{1}{2}$. In problem solving, you will often need to make a rough assumption about the relative axis scaling. This scaling will depend entirely on the
information given in the problem. Most graphing devices will allow you to specify the aspect ratio.

### 2.2.2 Axes Units

Sometimes we are led to coordinate systems where each of the two axes involve different types of units (labels). Here is a sample, that illustrates the power of using pictures.

**Example 2.2.1.** As the marketing director of Turboweb software, you have been asked to deliver a brief message at the annual stockholders meeting on the performance of your product. Your staff has assembled this tabular collection of data; how can you convey the content of this table most clearly?

<table>
<thead>
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<th>week</th>
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</tr>
<tr>
<td>2</td>
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<tr>
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</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

One idea is to simply flash an overhead slide of this data to the audience; this can be deadly! A better idea is to use a visual aid. Suppose we let the variable \( x \) represent the week and the variable \( y \) represent the gross sales (in thousands of dollars) in week \( x \). We can then plot the points \((x,y)\) in the xy-coordinate system; see Figure 2.6.

Notice, the units on the two axes are very different: y-axis units are “thousands of dollars” and x-axis units are “weeks.” In addition, the aspect ratio of this coordinate system is not 1. The beauty of this picture is the visual impact it gives your audience. From the coordinate plot we can get a sense of how the sales figures are dramatically increasing. In fact, this plot is good evidence you deserve a big raise!
Mathematical modeling is all about relating concrete phenomena and symbolic equations, so we want to embrace the idea of visualization. Most typically, visualization will involve plotting a collection of points in the plane. This can be achieved by providing a “list” or a “prescription” for plotting the points. The material we review in the next couple of sections makes the transition from symbolic mathematics to visual pictures go more smoothly.

2.3 **A Key Step in all Modeling Problems**

The initial problem solving or modeling step of deciding on a choice of $xy$-coordinate system is called *imposing a coordinate system*: There will often be many possible choices; it takes problem solving experience to develop intuition for a “natural” choice. This is a key step in all modeling problems.

**Example 2.3.1.** Return to the tossed ball scenario on page 1. How do we decide where to draw a coordinate system in the picture?

Figure 2.7 on page 16 shows four natural choices of $xy$-coordinate system. To choose a coordinate system we must specify the origin. The four logical choices for the origin are either the top of the cliff, the bottom of the cliff, the launch point of the ball or the landing point of the ball. So, which choice do we make? The answer is that any of these choices will work, but one choice may be more natural than another. For example, Figure 2.7(b) is probably the most natural choice: in this coordinate system, the motion of the ball takes place entirely in the first quadrant, so the $x$ and $y$ coordinates of any point on the path of the ball will be non-negative.

**Example 2.3.2.** Michael and Aaron are running toward each other, beginning at opposite ends of a 10,000 ft. airport runway, as pictured in Figure 2.8 on page 17. Where and when will these guys collide?

**Solution.** This problem requires that we find the “time” and “location” of the collision. Our first step is to impose a coordinate system.

We choose the coordinate system so that Michael is initially located at the point $M = \{0, 0\}$ (the origin) and Aaron is initially located at the point $A = \{10,000, 0\}$. To find the coordinates of Michael after $t$ seconds, we need to think about how distance and time are related.

Since Michael is moving at the rate of 15 ft/second, then after one second he is located 15 feet right of the origin; i.e., at the point $(15, 0)$. After 2 seconds, Michael has moved an additional 15 feet, for a total of 30 feet; so he is located at the point $(30, 0)$, etc. Conclude Michael has traveled 15t ft. **to the right** after $t$ seconds; i.e., his location is the point...
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Figure 2.7: Choices when imposing an $xy$-coordinate system.

$M(t) = (15t, 0)$. Similarly, Aaron is located 8 ft. left of his starting location after 1 second (at the point $(9, 992, 0)$), etc. Conclude Aaron has traveled $8t$ ft. to the left after $t$ seconds; i.e., his location is the point $A(t) = (10,000 - 8t, 0)$.

The key observation required to solve the problem is that the point of collision occurs when the coordinates of Michael and Aaron are equal. Because we are moving along the horizontal axis, this amounts to finding where and when the $x$-coordinates of $M(t)$ and $A(t)$ agree. This is a straightforward algebra problem:

\begin{align*}
15t &= 10,000 - 8t \\
23t &= 10,000 \\
t &= 434.78
\end{align*} \hfill (2.1)

To the nearest tenth of a second, the runners collide after 434.8 seconds. Plugging $t = 434.78$ into either expression for the position:

$M(434.8) = (15(434.8), 0) = (6,522, 0)$.
2.4 Distance

We end this Chapter with a discussion of direction and distance in the plane. To set the stage, think about the following analogy:

**Example 2.4.1.** You are in an airplane flying from Denver to New York. How far will you fly? To what extent will you travel north? To what extent will you travel east?

Consider two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ in the $xy$ coordinate system, where we assume that the units on each axis are the same; for example, both in units of “feet.” Imagine starting at the location $P$ (Denver) and flying to the location $Q$ (New York) along a straight line segment; see Figure 2.9(a). Now ask yourself this question: To what overall extent have the $x$ and $y$ coordinates changed?

To answer this, we introduce visual and notational aides into this figure. We have inserted an “arrow” pointing from the starting position $P$ to the ending position $Q$; see Figure 2.9(b). To simplify things, introduce the notation $\Delta x$ to keep track of the change in the $x$-coordinate and $\Delta y$
to keep track of the change in the $y$-coordinate, as we move from $P$ to $Q$. Each of these quantities can now be computed:

$$\Delta x = \text{change in } x\text{-coordinate going from } P \text{ to } Q$$

$$= (x\text{-coord of ending point}) - (x\text{-coord of beginning point})$$

$$= x_2 - x_1$$

$$\Delta y = \text{change in } y\text{-coordinate going from } P \text{ to } Q$$

$$= (y\text{-coord of ending point}) - (y\text{-coord of beginning point})$$

$$= y_2 - y_1.$$ 

We can interpret $\Delta x$ and $\Delta y$ using the right triangle in Figure 2.9(b). This means we can use the Pythagorean Theorem to write:

$$d^2 = (\Delta x)^2 + (\Delta y)^2,$$

that is,

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2},$$

which tells us the distance $d$ from $P$ to $Q$. In other words, $d$ is the distance we would fly if we had flown along that line segment connecting the two points. As an example, if $P = (1,1)$ and $Q = (5,4)$, then $\Delta x = 5 - 1 = 4$, $\Delta y = 4 - 1 = 3$ and $d = 5$.

There is a subtle idea behind the way we defined $\Delta x$ and $\Delta y$: You need to specify the “beginning” and “ending” points used to do the calculation in Equations 2.2. What happens if we had reversed the choices in Figure 2.9?

Then the quantities $\Delta x$ and $\Delta y$ will both be negative and the lengths of the sides of the right triangle are computed by taking the absolute value.
2.4. DISTANCE

of \( \Delta x \) and \( \Delta y \). As far as a distance calculation is concerned, the previous formula still works because of this algebra equality:

\[
d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{|\Delta x|^2 + |\Delta y|^2}.
\]

We will sometimes refer to \( \Delta x \) and \( \Delta y \) as directed distances in the \( x \) and \( y \) directions. The notion of directed distance becomes important in our discussion of lines in Chapter 4 and later when you learn about vectors; it is also very important in calculus.

For example, if \( P = (5,4) \) and \( Q = (1,1) \), then \( \Delta x = 1 - 5 = -4 \), \( \Delta y = 1 - 4 = -3 \) and \( d = 5 \).

**Important Fact 2.4.2** (Distance formula). If \( P = (x_1,y_1) \) and \( Q = (x_2,y_2) \) are two points in the plane, then the straight line distance between the points (in the same units as the two axes) is given by the formula

\[
d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}.
\]

(2.3)

If your algebra is a little rusty, a very common mistake may crop up when you are using the distance formula. For example,

\[
\sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} \quad \text{and} \quad \sqrt{9 + 16} = 3 + 4 \quad \text{fails}.
\]

Notice, you have an impossible situation: 5 is never equal to 7.

**Figure 2.10:** A different direction.
Example 2.4.3. Two cars depart from a four way intersection at the same time, one heading East and the other heading North. Both cars are traveling at the constant speed of 30 ft/sec. Find the distance (in miles) between the two cars after 1 hour 12 minutes. In addition, determine when the two cars would be exactly 1 mile apart.

Solution. Begin with a picture of the situation. We have indicated the locations of the two vehicles after t seconds and the distance d between them at time t. By the distance formula, the distance between them is

\[ d = \sqrt{(a - 0)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}. \]

This formula is a first step; the difficulty is that we have traded the mystery distance d for two new unknown numbers a and b. To find the coordinate a for the Eastbound car, we know the car is moving at the rate of 30 ft/sec, so it will travel 30t feet after t seconds; i.e., \( a = 30t \). Similarly, we find that \( b = 30t \). Substituting into the formula for d we arrive at

\[ d = \sqrt{(30t)^2 + (30t)^2} = \sqrt{2t^2(30)^2} = 30t\sqrt{2}. \]

First, we need to convert 1 hour and 12 minutes into seconds so that our formula can be used:

\[ 1 \text{ hr 12 min} = 1 + \frac{12}{60} \text{ hr} = 1.2 \text{ hr} = (1.2\text{hr}) \left( \frac{60 \text{ min}}{\text{hr}} \right) \left( \frac{60 \text{ sec}}{\text{min}} \right) = 4,320 \text{ sec}. \]

Substituting \( t = 4,320 \text{ sec} \) and recalling that 1 mile = 5,280 feet, we arrive at

\[ d = 129,600\sqrt{2} \text{ feet} = 183,282 \text{ feet} = 34.71 \text{ miles}. \]

For the second question, we specify the distance being 1 mile and want to find when this occurs. The idea is to set d equal to 1 mile and solve for
t. However, we need to be careful, since the units for $d$ are feet:

$$30t\sqrt{2} = d$$

$$= 5,280$$

Solving for $t$:

$$t = \frac{5,280}{30\sqrt{2}}$$

$$= 124.45 \text{ seconds}$$

$$= 2 \text{ minutes 4 seconds}.$$  

The two cars will be 1 mile apart in 2 minutes, 4 seconds. □
2.5 Exercises

**Problem 2.1.** In the following four cases, let \( P \) be the initial (starting) point and \( Q \) the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. Compute \( d = \) the distance from \( P \) to \( Q \), \( \Delta x \) and \( \Delta y \). Give your answer in exact form; e.g. \( \sqrt{2} \) is an exact answer, whereas 1.41 is an approximation of \( \sqrt{2} \).

(a) \( P = (0,0), Q = (1,1) \).
(b) \( P = (2,1), Q = (1,-1) \).
(c) \( P = (-1,2), Q = (4,-1) \).
(d) \( P = (1,2), Q = (1+3t,3+t) \), where \( t \) is a constant.

**Problem 2.2.** Start with two points \( M = (a,b) \) and \( N = (s,t) \) in the \( xy \)-coordinate system. Let \( d \) be the distance between these two points. Answer these questions and make sure you can justify your answers:

(a) TRUE or FALSE: \( d = \sqrt{(a-s)^2 + (b-t)^2} \).
(b) TRUE or FALSE: \( d = \sqrt{(a-s)^2 + (t-b)^2} \).
(c) TRUE or FALSE: \( d = \sqrt{(s-a)^2 + (t-b)^2} \).
(d) Suppose \( M \) is the beginning point and \( N \) is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is \( \Delta x \)? What is \( \Delta y \)?
(e) Suppose \( N \) is the beginning point and \( M \) is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is \( \Delta x \)? What is \( \Delta y \)?
(f) If \( \Delta x = 0 \), what can you say about the relationship between the positions of the two points \( M \) and \( N \)? If \( \Delta y = 0 \), what can you say about the relationship between the positions of the two points \( M \) and \( N \)? (Hint: Use some specific values for the coordinates and draw some pictures to see what is going on.)

**Problem 2.3.** Steve and Elsie are camping in the desert, but have decided to part ways. Steve heads North, at 6 AM, and walks steadily at 3 miles per hour. Elsie sleeps in, and starts walking West at 3.5 miles per hour starting at 8 AM.

When will the distance between them be 25 miles?

**Problem 2.4.** Erik’s disabled sailboat is floating at a stationary location 3 miles East and 2 miles North of Kingston. A ferry leaves Kingston heading due East toward Edmonds at 12 mph. At the same time, Erik leaves the sailboat in a dinghy heading due South at 10 ft/sec (hoping to intercept the ferry). Edmonds is 6 miles due East of Kingston.

(a) Compute Erik’s speed in mph and the Ferry speed in ft/sec.
(b) Impose a coordinate system and complete this table of data concerning locations (i.e., coordinates) of Erik and the ferry. Insert into the picture the locations of the ferry and Erik after 7 minutes.

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<th>Time</th>
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<tr>
<td>t hr</td>
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</tbody>
</table>

(c) Explain why Erik misses the ferry.
(d) After 10 minutes, a Coast Guard boat leaves Kingston heading due East at a speed of 25 ft/sec. Will the Coast Guard boat catch the ferry before it reaches Edmonds? Explain.

**Problem 2.5.** Suppose two cars depart from a four way intersection at the same time, one heading north and the other heading west. The car heading north travels at the steady speed of 30 ft/sec and the car heading west travels at the steady speed of 58 ft/sec.
2.5. EXERCISES

(a) Find an expression for the distance between the two cars after t seconds.
(b) Find the distance in miles between the two cars after 3 hours 47 minutes.
(c) When are the two cars 1 mile apart?

**Problem 2.6.** Allyson and Adrian have decided to connect their ankles with a bungee cord; one end is tied to each person's ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrian moves 8 ft/sec in the directions indicated.

(a) Where are the two girls located after 2 seconds?
(b) After 2 seconds, will the slack in the bungee cord be used up?
(c) Determine when the bungee cord first becomes tight; i.e., there is no slack in the line. Where are the girls located when this occurs?
(d) When will the bungee cord first touch the corner of the building? (Hint: Use a fact about "similar triangles").

**Problem 2.7.** Brooke is located 5 miles out from the nearest point A along a straight shoreline in her seakayak. Hunger strikes and she wants to make it to Kono's for lunch; see picture. Brooke can paddle 2 mph and walk 4 mph.

(a) If she paddles along a straight line course to the shore, find an expression that computes the total time to reach lunch in terms of the location where Brooke beaches the boat.
(b) Determine the total time to reach Kono's if she paddles directly to the point "A".
(c) Determine the total time to reach Kono's if she paddles directly to Kono's.
(d) Do you think your answer to (b) or (c) is the minimum time required for Brooke to reach lunch?
(e) Determine the total time to reach Kono's if she paddles directly to a point on the shore half way between point "A" and Kono's. How does this time compare to the times in parts (b) and (c)? Do you need to modify your answer to part (d)?

**Problem 2.8.** A spider is located at the position (1,2) in a coordinate system, where the units on each axis are feet. An ant is located at the position (15,0) in the same coordinate system. Assume the location of the spider after t minutes is \( s(t) = (1 + 2t, 2 + t) \) and the location of the ant after t minutes is \( a(t) = (15 - 2t, 2t) \).

(a) Sketch a picture of the situation, indicating the locations of the spider and ant at times \( t = 0,1,2,3,4,5 \) minutes. Label the locations of the bugs in your picture, using the notation \( s(0), s(1), ..., s(5), a(0), a(1), ..., a(5) \).
(b) When will the x-coordinate of the spider equal 5? When will the y-coordinate of the ant equal 5?
(c) Where is the spider located when its y-coordinate is 3?
(d) Where is each bug located when the y-coordinate of the spider is twice as large as the y-coordinate of the ant?
(e) How far apart are the bugs when their x-coordinates coincide? Draw a picture, indicating the locations of each bug when their x-coordinates coincide.

(f) A sugar cube is located at the position (9,6). Explain why each bug will pass through the position of the sugar cube. Which bug reaches the sugar cube first?

(g) Find the speed of each bug along its line of motion; which bug is moving faster?

Problem 2.9. A Ferrari is heading south at a constant speed on Broadway (a north/south street) at the same time a Mercedes is heading west on Aloha Avenue (an east/west street). The Ferrari is 624 feet north of the intersection of Broadway and Aloha, at the same time that the Mercedes is 400 feet east of the intersection. Assume the Mercedes is traveling at the constant speed of 32 miles/hour. Find the speed of the Ferrari so that a collision occurs in the intersection of Broadway and Aloha.

Problem 2.10. Two planes flying opposite directions (North and South) pass each other 80 miles apart at the same altitude. The Northbound plane is flying 200 mph (miles per hour) and the Southbound plane is flying 150 mph. How far apart are the planes in 20 minutes? When are the planes 300 miles apart?

Problem 2.11. Here is a list of some algebra problems with "solutions." Some of the solutions are correct and some are wrong. For each problem, determine: (i) if the answer is correct, (ii) if the steps are correct, (iii) identify any incorrect steps in the solution (noting that the answer may be correct but some steps may not be correct).

(a) If $x \neq 1$,
\[
\frac{x^2 - 1}{x + 1} = \frac{x^2 + (-1)1}{x + 1} = \frac{x^2 - 1}{x} = x - 1
\]

(b) 
\[
(x + y)^2 - (x - y)^2 = (x^2 + y^2) - x^2 - y^2 = 0
\]

(c) If $x \neq 4$,
\[
\frac{9(x - 4)^2}{3x - 12} = \frac{3^2(x - 4)^2}{3x - 12} = \frac{(3x - 12)^2}{3x - 12} = 3x - 12.
\]

Problem 2.12. Assume $\alpha, \beta$ are nonzero constants. Solve for $x$.

(a) $\alpha x + \beta = \frac{1}{\alpha x - \beta}$

(b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{x}$

(c) $\alpha + \frac{1}{\beta} = \frac{1}{x}$

Problem 2.13. Simplify as far as possible.

(a) $(1 - t)^2 + (2 + 2t)^2$

(b) $(t + 1)^2 + (-t - 1)^2 - 2$

(c) $\frac{1}{t+1} - \frac{1}{t+1}$ (write as a single fraction)

(d) $\sqrt{(2 + t)^2 + 4t^2}$